SOLVED PAPER-2

MATHS- IB

Time: 3 Hours | Max. Marks : 75

SECTION -A

I. Answer ALL the following Very Short Answer Questions: $10 \times 2 = 20$

1. Transform the equation x+y+1=0 into normal form.
2. Evaluate $\underset{x\to 2}{Lt}([x]+x)$
3. Show that the points A(3, –2, 4), B(1,1,1), C(–1,4,–2) are collinear.
4. Reduce the equation 4x – 4y + 2z + 5 = 0 of the plane to the intercept form.
5. Compute $\underset{x\to 0}{Lt}\dfrac{3^x-1}{\sqrt{1+x}-1}$
6. Find $\underset{x\to\infty}{Lt}\dfrac{8|x|+3x}{3|x|-2x}$
7. If $y = x^2e^x\sin x$, then find $\dfrac{dy}{dx}$
8. Find the derivative of $Sec^{-1}\left(\dfrac{1}{2x^2-1}\right)$
9. The side of a square is increased from 3 cm to 3.01 cm. Find the approximate increase in its area
10. Verify Rolle's theorem for the function x^2-1 on [–1, 1]

SECTION-B

II. Answer any FIVE of the following Short Answer Questions: $5 \times 4 = 20$

11. A(5,3) and B(3,–2) are two fixed points. Find the equation of locus of P, so that the area of ΔPAB is 9 sq.units.
12. Show that the axes are to be rotated through an angle of $\dfrac{1}{2}Tan^{-1}\left(\dfrac{2h}{a-b}\right)$ so as to remove the xy term from the equation $ax^2+2hxy+by^2=0$
13. Find the image of (1,2) in the straight line 3x+4y–1=0
14. Show that $f(x)=\begin{cases}\dfrac{\cos ax-\cos bx}{x^2} & \text{if } x\neq 0\\ \dfrac{1}{2}\left(b^2-a^2\right) & \text{if } x=0\end{cases}$, is continuous at 0.
15. Find the derivative of cotx from the first Principle.
16. A particle moving along a straight line has the relation $s = t^3 + 2t + 3$, connecting the distance s describe by the particle in time t. Find the velocity and acceleration of the particle at t=4 sec.
17. Find the equations of the tangent and the normal to the curve $y=x^3+4x^2$ at (–1,3)

SECTION-C

III. Answer any FIVE of the following Long Answer Questions: $5 \times 7 = 35$

18. Find the circumcenter of the triangle whose vertices are (1,3), (0,–2) and (–3, 1)
19. Find the centroid and area of the triangle formed by $2y^2-xy-6x^2=0$ and x+y+4=0
20. Write down the equation of the pair of lines joining the origin to the point of intersection of the line 6x–y+8=0 with the pair of lines $3x^2+4xy-4y^2-11x+2y+6=0$. Show that the lines so obtained make equal angles with the coordinate axes.
21. Find the angle between the lines whose direction cosines satisfy the equation $l+m+n=0$, $l^2+m^2-n^2=0$
22. Find the derivative of $(\sin x)^{\log x}+x^{\sin x}$.
23. If the tangent at any point P on the curve $x^my^n=a^{m+n}$, mn≠0 meets the coordinate axes in A,B then show that AP:BP is a constant.
24. Show that when the curved surface of a is right circular cylinder inscribed in a sphere of radius R is maximum, then the height of the cylinder is $\sqrt{2}$**R**

SOLUTIONS

SECTION-A

1. Transform the equation x + y + 1 = 0 into normal form.

Sol: The normal form of a line is $x\cos\alpha + y\sin\alpha = p$

The given equation is $x+y+1=0 \Rightarrow x+y=-1 \Rightarrow -x-y=1$

Dividing by $\sqrt{(-1)^2+(-1)^2}=\sqrt{2}$, we have

$$-\frac{1}{\sqrt{2}}x-\frac{1}{\sqrt{2}}y=\frac{1}{\sqrt{2}} \Rightarrow x\cos 225^\circ + y\sin 225^\circ = \frac{1}{\sqrt{2}}$$

---

2. Evaluate $\underset{x\to 2}{\text{Lt}}([x]+x)$

Sol: If $x \to 2+$ then $[x]=2$ and if $x\to 2-$ then $[x]=1$

R.H.L $=\underset{x\to 2+}{\text{Lt}}[x]+x=2+2=4$ L.H.L $=\underset{x\to 2-}{\text{Lt}}[x]+x=1+2=3$

R.H.L$\neq$L.H.L. Hence given limit does not exist.

---

3. Show that the points A(3,–2,4), B(1,1,1), C(–1,4,–2) are collinear.

Sol: The given points are A(3,–2,4), B(1,1,1), C(–1,4,–2)

Now, $AB=\sqrt{(3-1)^2+(-2-1)^2+(4-1)^2}=\sqrt{4+9+9}=\sqrt{22}$

$BC=\sqrt{(1-1)^2+(1-4)^2+(1+2)^2}=\sqrt{4+9+9}=\sqrt{22}$

$AC=\sqrt{(3+1)^2+(-2-4)^2+(4+2)^2}=\sqrt{16+36+36}=\sqrt{88}=\sqrt{4\times 22}=2\sqrt{22}$

Here, $AB+BC=\sqrt{22}+\sqrt{22}=2\sqrt{22}=AC$

$\therefore$ A,B,C are collinear

---

4. Reduce the equation 4x–4y+2z+5=0 of the plane to the intercept form.

Sol: The equation of a plane in the intercept form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

The given equation is $4x-4y+2z+5=0 \Rightarrow 4x-4y+2z=-5$

$$\Rightarrow \frac{4x}{-5}-\frac{4y}{-5}+\frac{2z}{-5}=1 \Rightarrow \frac{x}{-5/4}+\frac{y}{5/4}+\frac{z}{-5/2}=1$$

---

5. Evaluate $\underset{x\to 0}{\text{Lt}}\frac{3^x-1}{\sqrt{1+x}-1}$

Sol: $\underset{x\to 0}{\text{Lt}}\frac{3^x-1}{\sqrt{1+x}-1}=\underset{x\to 0}{\text{Lt}}\left(\frac{3^x-1}{x}\right)\left(\frac{x}{\sqrt{1+x}-1}\right)$

$$= \underset{x\to 0}{Lt}\frac{3^x-1}{x}.\underset{x\to 0}{Lt}\frac{x}{\sqrt{1+x}-1} = \log 3.\underset{x\to 0}{Lt}\frac{x}{\sqrt{1+x}-1}$$

$$= \log 3.\underset{x\to 0}{Lt}\frac{x(\sqrt{1+x}+1)}{(\sqrt{1+x}-1)(\sqrt{1+x}+1)} = \log 3.\underset{x\to 0}{Lt}\frac{x(\sqrt{1+x}+1)}{1+x-1} = \log 3.\underset{x\to 0}{Lt}\frac{x(\sqrt{1+x}+1)}{x}$$

$$= \log 3.\underset{x\to 0}{Lt}(\sqrt{1+x}+1) = \log 3.(\sqrt{1+0}+1) = (\log 3)(1+1) = 2\log 3$$

---

6. Find $\underset{x\to\infty}{Lt}\frac{8|x|+3x}{3|x|-2x}$

Sol: If $x\to\infty$ then $x>0$. Hence $|x|=x$

$$\therefore \underset{x\to\infty}{Lt}\frac{8|x|+3x}{3|x|-2x} = \underset{x\to\infty}{Lt}\frac{8x+3x}{3x-2x} = \underset{x\to\infty}{Lt}\frac{11x}{x} = \underset{x\to\infty}{Lt}\ 11 = 11$$

---

7. If $y=x^2e^x\sin x$, then find $\frac{dy}{dx}$

Sol: $$\frac{dy}{dx} = x^2e^x\frac{d}{dx}\sin x + x^2\sin x\frac{d}{dx}e^x + e^x\sin x\frac{d}{dx}x^2$$

$$= x^2e^x(\cos x) + x^2\sin x(e^x) + e^x\sin x(2x)$$

$$= x^2e^x\sin x\left(\frac{\cos x}{\sin x}+1+\frac{2}{x}\right) = x^2e^x\sin x\left(\cot x+1+\frac{2}{x}\right)$$

---

8. Find the derivative of $Sec^{-1}\left(\frac{1}{2x^2-1}\right)$

Sol: Put $x=\cos\theta$ then $2x^2-1=2\cos^2\theta-1=\cos 2\theta$

$$\therefore y = Sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = Sec^{-1}(\sec 2\theta) = 2\theta = 2Cos^{-1}x$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(2Cos^{-1}x) = 2\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{-2}{\sqrt{1-x^2}}$$

---

9. The side of a square is increased from 3 cm to 3.01 cm. Find the approximate increase in the area of the square.

Sol: Let x denotes the side of the square.

Also $x = 3$ and $\Delta x = 3.01-3=0.01$

Area of the square $A = x^2$.

$$\Rightarrow \Delta A = \frac{dA}{dx}\Delta x = (2x)\Delta x = 2(3)(0.01) = 0.06 \text{ sq.cm}$$

10. Verify Rolle's theorem for the function x^2-1 on $[-1,1]$ IPE'14

Sol : Given $f(x) = x^2-1 \Rightarrow f'(x) = 2x$

f(x) is (i) continuous on $[-1,1]$ and (ii) differentiable in $(-1,1)$

(iii) $f(-1) = (-1)^2-1 = 1-1 = 0$; $f(1) = 1^2-1 = 1-1 = 0$

$\Rightarrow f(-1) = f(1)$

So, from Rolle's theorem, $f'(c)=0 \Rightarrow 2c=0 \Rightarrow c=0$

$\therefore\ c=0 \in (-1,1)$

Hence, Rolle's theorem is verified.

SECTION – B

11. A(5,3) and B(3,–2) are two fixed points. Find the equation of locus of P, so that the area of triangle PAB is 9 sq.units.

Sol : Given that A=(5,3), B=(3,–2) and P(x, y) be a point on the locus.

From the given condition, area of ΔPAB=9

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x-5 & x-3 \\ y-3 & y+2 \end{vmatrix} = 9 \Rightarrow \left|(x-5)(y+2)-(y-3)(x-3)\right| = 2(9)$$

$$\Rightarrow | xy+2x-5y-10-(xy-3y-3x+9)| = 18 \Rightarrow |5x-2y-19| = 18$$

$$\Rightarrow 5x-2y-19 = \pm 18 \Rightarrow 5x-2y-19 = 18 \text{ (or) } 5x-2y-19 = -18$$

$$\Rightarrow 5x-2y-37 = 0 \text{ (or) } 5x-2y-1 = 0 \Rightarrow (5x-2y-37)(5x-2y-1) = 0$$

$\therefore$ The equation of locus of P(x,y) is $(5x-2y-37)(5x-2y-1)=0$

---

12. Show that the axes are to be rotated through an angle of $\frac{1}{2}\text{Tan}^{-1}\left(\frac{2h}{a-b}\right)$ so as to remove the xy term from the equation $ax^2+2hxy+by^2=0$

Sol: Let the axes be rotated through an angle θ then $x=X\cos\theta-Y\sin\theta$ and $y=Y\cos\theta+X\sin\theta$

$\therefore$ the transformed equation of the given equation is

$$a(X\cos\theta-Y\sin\theta)^2+2h(X\cos\theta-Y\sin\theta)(Y\cos\theta+X\sin\theta)+b(Y\cos\theta+X\sin\theta)^2=0$$

$$\Rightarrow a(X^2\cos^2\theta-2XY\cos\theta\sin\theta+Y^2\sin^2\theta)+2h(X^2\cos\theta\sin\theta+XY\cos^2\theta-XY\sin^2\theta-Y^2\sin\theta\cos\theta)$$
$$+b(Y^2\cos^2\theta+2XY\sin\theta\cos\theta+X^2\sin^2\theta)=0 \ \ldots\ldots (1)$$

Now, to remove the XY term , we have to equate the coefficient of XY term to zero

$\Rightarrow -2a\cos\theta\sin\theta+2h\cos^2\theta-2h\sin^2\theta+2b\sin\theta\cos\theta=0$ (the coefficients of XY from (1) are collected)

$$\Rightarrow 2\sin\theta\cos\theta(b-a)+2h(\cos^2\theta-\sin^2\theta)=0 \Rightarrow \sin2\theta(b-a)+2h\cos2\theta=0 \Rightarrow \sin2\theta(a-b)=2h\cos2\theta$$

$$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = \frac{2h}{a-b}, a \neq b \Rightarrow \tan 2\theta = \frac{2h}{a-b} \Rightarrow 2\theta = \text{Tan}^{-1}\frac{2h}{a-b} \Rightarrow \theta = \frac{1}{2}\text{Tan}^{-1}\left(\frac{2h}{a-b}\right)$$

13. Find the image of (1,2) in the straight line 3x+4y–1=0

Sol: Let (h,k) be the image of (1,2) w.r.to 3x+4y–1=0

Here $(x_1,y_1)=(1,2)$, a=3, b=4, c = –1.

$$\therefore \frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2} \Rightarrow \frac{h-1}{3}=\frac{k-2}{4}=\frac{-2[3(1)+4(2)-1]}{3^2+4^2}$$

$$\Rightarrow \frac{h-1}{3}=\frac{k-2}{4}=\frac{-2(10)}{25}=-2\left(\frac{2}{5}\right)=-\frac{4}{5}$$

Now, $\frac{h-1}{3}=-\frac{4}{5} \Rightarrow h-1=-\frac{12}{5} \Rightarrow h=1-\frac{12}{5}=\frac{5-12}{5}=-\frac{7}{5}$

Also $\frac{k-2}{4}=-\frac{4}{5} \Rightarrow k-2=-\frac{16}{5} \Rightarrow k=2-\frac{16}{5}=\frac{10-16}{5}=\frac{-6}{5}$

$\therefore$ the image is $(h,k)=\left(-\frac{7}{5},-\frac{6}{5}\right)$

---

14. Show that $f(x)=\begin{cases}\frac{\cos ax-\cos bx}{x^2} & \text{if } x\neq 0\\ \frac{1}{2}(b^2-a^2) & \text{if } x=0\end{cases}$, is continuous at 0.

Sol: Given that $f(0)=\frac{1}{2}(b^2-a^2)$

$$\underset{x\to 0}{Lt}\frac{\cos ax-\cos bx}{x^2}=\underset{x\to 0}{Lt}\frac{2\sin\left(\frac{ax+bx}{2}\right)\sin\left(\frac{bx-ax}{2}\right)}{x^2} \quad \left(\because \cos C-\cos D=2\sin\frac{C+D}{2}\sin\frac{D-C}{2}\right)$$

$$=2\underset{x\to 0}{Lt}\left(\frac{\sin\left(\frac{a+b}{2}\right)x}{x}\right)\left(\frac{\sin\left(\frac{b-a}{2}\right)x}{x}\right)=2\left(\underset{x\to 0}{Lt}\frac{\sin\left(\frac{a+b}{2}\right)x}{x}\right)\left(\underset{x\to 0}{Lt}\frac{\sin\left(\frac{b-a}{2}\right)x}{x}\right)$$

$$=2\left(\frac{a+b}{2}\right)\left(\frac{b-a}{2}\right)=\frac{b^2-a^2}{2} \quad \left(\because \underset{x\to 0}{Lt}\frac{\sin kx}{x}=k\right)$$

$\therefore \underset{x\to 0}{Lt} f(x)=f(0)$. Hence f(x) is continuous at x=0

---

15. Find the derivative of cotx from first principle.

Sol: f(x+h)–f(x)=cot(x+h)–cotx

$$=\frac{\cos(x+h)}{\sin(x+h)}-\frac{\cos x}{\sin x}=\frac{\cos(x+h).\sin x-\sin(x+h).\cos x}{\sin(x+h).\sin x}=\frac{-\sin(x+h-x)}{\sin(x+h)\sin x}=\frac{-\sinh}{\sin(x+h)\sin x}$$

$$\therefore f'(x)=\underset{h\to 0}{Lt}\frac{f(x+h)-f(x)}{h}=\underset{h\to 0}{Lt}\left(\frac{-\sinh}{h}\right).\underset{h\to 0}{Lt}\frac{1}{\sin(x+h).\sin x}=-1\left(\frac{1}{\sin x.\sin x}\right)=-\operatorname{cosec}^2 x$$

---

16. A particle moving along a straight line has the relation $s=t^3+2t+3$, connecting the distance s describe by the particle in time t. Find the velocity and acceleration of the particle at t=4 sec.

Sol : The distance-time relation is given by $s=t^3+2t+3$

$\Rightarrow$ Velocity $v=\frac{ds}{dt}=3t^2+2$, Acceleration $a=\frac{dV}{dt}=6t$

(i) Velocity at t=4 is $3(4)^2+2=3(16)+2=48+2=50$ units/sec

(ii) Acceleration at t=4 is 6(4) = 24 units /sec^2

17. Find the equations of the tangent and the normal to the curve $y=x^3+4x^2$ at (–1,3)

Sol: Equation of the given curve is $y=x^3+4x^2 \Rightarrow \frac{dy}{dx}=3x^2+8x$

At P(–1,3), slope of the tangent $m=3(-1)^3+8(-1)=3-8=-5$

Equation of the tangent at P(–1,3) is $y-y_1=m(x-x_1)$

$\Rightarrow y-3=-5(x+1)=-5x-5 \Rightarrow 5x+y+2=0$

Equation of the normal at P is $y-y_1=-\frac{1}{m}(x-x_1) \Rightarrow y-3=\frac{1}{5}(x+1)$

$\Rightarrow 5y-15=x+1 \Rightarrow x-5y+16=0$

SECTION-C

18. Find the circumcenter of the triangle whose vertices are (1,3), (0,–2) and (–3, 1)

Sol: Let S(x,y) be the circumcentre of the triangle ABC where A=(1,3), B=(0, –2), C=(–3,1)

We know that SA=SB=SC i.e., SA=SB and SB=SC $\Rightarrow SA^2=SB^2$ and $SB^2=SC^2$

Now, $SA^2=SB^2 \Rightarrow (x-1)^2+(y-3)^2=(x-0)^2+(y+2)^2$

$\Rightarrow (x^2-2x+1)+(y^2-6y+9)=x^2+(y^2+4y+4) \Rightarrow -2x-10y+6=0 \Rightarrow -2(x+5y-3)=0 \Rightarrow x+5y-3=0$ ....(1)

Also $SB^2=SC^2 \Rightarrow (x-0)^2+(y+2)^2=(x+3)^2+(y-1)^2$

$\Rightarrow x^2+(y^2+4y+4)=(x^2+6x+9)+(y^2-2y+1) \Rightarrow 6x-6y+6=0 \Rightarrow 6(x-y+1)=0 \Rightarrow x-y+1=0$ ....(2)

Solving (1) and (2), we get S; $x+5y-3=0$

$x-y+1=0$

$6y-4=0 \Rightarrow 6y=4 \Rightarrow y=2/3$

$(2) \Rightarrow x-\frac{2}{3}+1=0 \Rightarrow x=\frac{2}{3}-1=\frac{2-3}{3}=-\frac{1}{3}$

$\therefore$ the circumcentre of the given triangle is S (x,y) = (1/3, 2/3)

---

19. Find the centroid and area of the triangle formed by $2y^2-xy-6x^2=0$ and $x+y+4=0$

Sol: First we find the points of intersection of $2y^2-xy-6x^2=0$ .....(1), $x+y+4=0$......(2)

$(2) \Rightarrow x+y+4=0 \Rightarrow y=-(x+4)$ ....(3)

Substituting in (1), we have $2(x+4)^2+x(x+4)-6x^2=0$

$\Rightarrow 2(x^2+8x+16)+x^2+4x-6x^2=0 \Rightarrow 2x^2+16x+32+x^2+4x-6x^2=0$

$\Rightarrow -3x^2+20x+32=0 \Rightarrow 3x^2-20x-32=0$

$\Rightarrow 3x^2-24x+4x-32=0 \Rightarrow 3x(x-8)+4(x-8)=0$

$\Rightarrow (3x+4)(x-8)=0 \Rightarrow x=-4/3$ or $x=8$

If x= –4/3 then $(3) \Rightarrow y = -\left(\frac{-4}{3}+4\right) = \frac{4}{3} - 4 = \frac{4-12}{3} = -\frac{8}{3}$

$\Rightarrow$ a point of intersection $A = \left(-\frac{4}{3}, -\frac{8}{3}\right)$

If x=8 then (3) $\Rightarrow$ y = –(8+4) = –12

$\therefore$ the other point of intersection is B(8,–12)

Also, the pair of lines $2y^2-xy-6x^2=0$ intersect at O(0,0)

$\therefore$ centroid of the triangle formed by O(0,0), A(–4/3, –8/3), B(8,–12) is

$$\left(\frac{0-\frac{4}{3}+8}{3}, \frac{0-\frac{8}{3}-12}{3}\right) = \left(\frac{-4+24}{3(3)}, \frac{-8-36}{3(3)}\right) = \left(\frac{20}{9}, -\frac{44}{9}\right)$$

$$\text{Area of } \Delta OAB = \frac{1}{2}|x_1y_2 - x_2y_1| = \frac{1}{2}\left|\left(-\frac{4}{3}\right)(-12) - \left(-\frac{8}{3}\right)(8)\right| = \frac{1}{2}\left|\frac{48}{3}+\frac{64}{3}\right|$$

$$= \frac{1}{2}\left|\frac{112}{3}\right| = \frac{56}{3} \text{ sq.units}$$

---

20. Write down the equation of the pair of lines joining the origin to the point of intersection of the line 6x–y+8=0 with the pair of lines $3x^2+4xy-4y^2-11x+2y+6=0$. Show that the lines so obtained make equal angles with the coordinate axes.

AP 15

Sol: The given line is 6x–y+8=0 $\Rightarrow$ 6x–y = –8 $\Rightarrow \frac{6x-y}{-8}=1$ ......(1)

Now, we homogenise the equation $3x^2+4xy-4y^2-11x+2y+6=0$ using (1)

$\Rightarrow 3x^2+4xy-4y^2-11x(1)+24(1)+6(1^2)=0$

$$\Rightarrow 3x^2+4xy-4y^2-11x\left(\frac{6x-y}{-8}\right)+2y\left(\frac{6x-y}{-8}\right)+6\left(\frac{6x-y}{-8}\right)^2=0$$

$\Rightarrow 64(3x^2+4xy-4y^2)+8.11x(6x-y)-8.2y(6x-y)+6(6x-y)^2=0$

$\Rightarrow 64(3x^2+4xy-4y^2)+88(6x^2-xy)-16(6xy-y^2)+6(36x^2-12xy+y^2)=0$

$\Rightarrow (192x^2+256xy-256y^2)+(528x^2-88xy)-96xy+16y^2+(216x^2-72xy+6y^2)=0$

$\Rightarrow 936x^2-234y^2=0 \Rightarrow 234(4x^2-y^2)=0 \Rightarrow 4x^2-y^2=0$

Now, the equation of angular bisectors of $4x^2-y^2=0$ is given by

$h(x^2-y^2)-(a-b)xy=0 \Rightarrow 0(x^2-y^2)-(4-1)xy=0 \Rightarrow -3xy=0 \Rightarrow xy=0$

$\Rightarrow$ x=0 or y=0

$\therefore$ the pair of lines $4x^2-y^2=0$ are equally incilined to the coordinate axes.

---

21. Find the angle between the lines whose direction cosines satisfy the equation l+m+n=0, $l^2+m^2-n^2=0$

Sol: Given that l+m+n=0 .....(1), $l^2+m^2-n^2=0$ ......(2)

(1) $\Rightarrow l=-(m+n)$

$\therefore$ (2) $\Rightarrow (-(m+n))^2+m^2-n^2=0 \Rightarrow m^2+n^2+2mn+m^2-n^2=0$

$\Rightarrow 2m^2+2nm=0 \Rightarrow m^2+nm=0 \Rightarrow m(m+n)=0 \Rightarrow$ m=0 or m+n=0 $\Rightarrow$ m=0 or m=–n

If m=0 then (1) $\Rightarrow l+0+n=0 \Rightarrow l=-n$

$\therefore l$: m : n = –n : 0 : n $\Rightarrow$ –1 : 0 : 1

$\therefore$ The d.r's of a line are (–1,0,1) ......(3)

If m=–n then (1) $\Rightarrow l-n+n=0 \Rightarrow l=0$

$\therefore l : m : n = 0 : -n : n \Rightarrow 0 : -1 : 1$

$\therefore$ The d.r's of the other line are (0,–1,1) ......(4)

If θ is the acute angle between the 2 lines then from (3) & (4)

$$\cos\theta = \frac{|(-1)(0)+(0)(-1)+1(1)|}{\sqrt{((-1)^2+0^2+1^2)(0^2+(-1)^2+1^2)}} = \frac{1}{2} = \cos\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$$

---

22. Find the derivative of $(\sin x)^{\log x} + x^{\sin x}$.

Sol: Let $y=(\sin x)^{\log x}+x^{\sin x}$.

and $u=(\sin x)^{\log x}$, $v=x^{\sin x} \Rightarrow y=u+v$

$u=(\sin x)^{\log x}$

$\Rightarrow \log u=\log(\sin x)^{\log x}$

$\Rightarrow \log u=\log x(\log\sin x)$

differentiating w.r.t x

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \log x\left(\frac{1}{\sin x}(\cos x)\right)+\log\sin x\left(\tfrac{1}{x}\right)$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = u\left(\log x.\cot x+\frac{\log\sin x}{x}\right)$$

$$\Rightarrow \frac{du}{dx} = \sin x^{\log x}\left(\log x\cot x+\frac{\log\sin x}{x}\right)$$

$v=x^{\sin x}$

$\Rightarrow \log v=\log x^{\sin x}$

$\Rightarrow \log v=\sin x\log x$

differentiating w.r.t x

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \sin x\left(\frac{1}{x}\right)+\log x\cos x$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left(\frac{\sin x}{x}+\log x\cos x\right)$$

$$\Rightarrow \frac{dv}{dx} = v\left(\frac{\sin x}{x}+\log x\cos x\right)$$

$$\Rightarrow \frac{dv}{dx} = x^{\sin x}\left(\frac{\sin x}{x}+\log x\cos x\right)$$

$$\therefore (1) \Rightarrow \frac{dy}{dx} = \sin x^{\log x}\left(\log x\cot x+\frac{\log\sin x}{x}\right)+x^{\sin x}\left(\frac{\sin x}{x}+\log x\cos x\right)$$

---

23. If the tangent at any point P on the curve $x^m y^n = a^{m+n}$, mn≠0 meets the coordinate axes in A,B then show that AP:BP is a constant.

Sol: Let $P(x_1,y_1)$ be the point on the curve $x^m y^n = a^{m+n}$

Now differentiating the above equation w.r.to x

$$x^m\left(ny^{n-1}\frac{dy}{dx}\right)+y^n(mx^{m-1})=0 \Rightarrow \frac{dy}{dx} = \frac{-my^n x^{m-1}}{nx^m y^{n-1}} = -\frac{m}{n}\left(\frac{y}{x}\right)$$

$$\Rightarrow \text{slope } m = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{m}{n}\left(\frac{y_1}{x_1}\right)$$

$\therefore$ the equation of the tangent at $P(x_1,y_1)$ with slope $-\frac{m}{n}\left(\frac{y_1}{x_1}\right)$ is

$$y-y_1 = -\frac{m}{n}\left(\frac{y_1}{x_1}\right)(x-x_1) \Rightarrow nx_1(y-y_1) = -my_1(x-x_1)$$

$$\Rightarrow nx_1y - nx_1y_1 = -my_1x+mx_1y_1 \Rightarrow (my_1)x+(nx_1)y-x_1y_1(n+m)=0$$

The x – intercept of the above line is $-\frac{\text{const. term}}{\text{coeff. of x}} = \frac{x_1y_1(n+m)}{my_1} \Rightarrow A = \left(\frac{(m+n)x_1}{m}, 0\right)$

and the y – intercept is $-\frac{\text{constant term}}{\text{coefficient of y}} = \frac{x_1y_1(m+n)}{nx_1} \Rightarrow B = \left(0, \frac{(m+n)y_1}{n}\right)$

Now, AP:PB=$(x_1-x) : (x-x_2)$

$= \frac{(m+n)x_1}{m} - x_1 : (x_1 - 0) = \frac{mx_1 + nx_1 - mx_1}{m} : x_1 = \frac{nx_1}{n} : x_1 = \frac{n}{m} : 1 = n : m$

$\therefore$ AP : PB = n : m, which is a constant

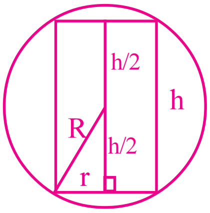
---

24. Show that when the curved surface of a is right circular cylinder inscribed in a sphere of radius R is maximum, then the height of the cylinder is $\sqrt{2}R$

Sol: Given that radius of the sphere is R.

Let h,r denote the height and base radius of the cylinder

From the diagram, $\left(\frac{h}{2}\right)^2 + r^2 = R^2 \Rightarrow r^2 = R^2 - \frac{h^2}{4}$



$\Rightarrow r = \sqrt{R^2 - \frac{h^2}{4}}$ ....(1)

The curved surface area of the cylinder A=2πrh

$= 2\pi h\sqrt{R^2 - \frac{h^2}{4}}$ [From (1)]

$= 2\pi\sqrt{R^2h^2 - \frac{h^2}{4}}$

Let $f(h) = R^2h^2 - \frac{h^4}{4} \Rightarrow f'(h) = R^2(2h) - \frac{4h^3}{4} = 2R^2h - h^3$

$\therefore f'(h)=2R^2h-h^3$ ....(2)

The extreme values of f(h) are attained when $f'(h)=0$

$\Rightarrow 2R^2h-h^3=0 \Rightarrow h(2R^2-h^2)=0$

Since, $h\neq0$, $2R^2-h^2=0 \Rightarrow h^2 = 2R^2 \Rightarrow h = \sqrt{2}R$

From (2), $f''(h)=2R^2-3h^2$

$\Rightarrow f''(\sqrt{2}R) = 2R^2 - 3(2R^2) = 2R^2 - 6R^2 = -4R^2 < 0$

$\therefore f''(\sqrt{2}R) < 0$

Hence, f(h) is maximum at $h = \sqrt{2}R$

$\therefore$ the curved surface area of the cylinder is maximum, when its height $h = \sqrt{2}\,R$