SOLVED PAPER-1

MATHS-1A

Time: 3 Hours Max. Marks : 75

(Board of Intermediate Education -Model Paper)

SECTION -A

I. Answer ALL the following Very Short Answer Questions: $10 \times 2 = 20$

1. If $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ and f:A→B is a surjection defined by f(x) = cosx then find B.
2. Find the domain of the real -valued function $f(x) = \frac{1}{\log(2-x)}$
3. A certain bookshop has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80, Rs.60 and Rs.40 each respectively. Find the total amount the bookshop will receive by selling all the books, using matrix algebra.
4. If $A = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$, then find A+A' and AA'.
5. Show that the points whose position vectors are $-2\bar{a}+3\bar{b}+5\bar{c}, \bar{a}+2\bar{b}+3\bar{c}, 7\bar{a}-\bar{c}$ are collinear when $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors.
6. Let $\bar{a} = 2\bar{i}+4\bar{j}-5\bar{k}$, $\bar{b} = \bar{i}+\bar{j}+\bar{k}$, $\bar{c} = \bar{j}+2\bar{k}$. Find unit vector in the opposite direction of $\bar{a}+\bar{b}+\bar{c}$
7. If $\bar{a} = \bar{i}+2\bar{j}-3\bar{k}$ and $\bar{b} = 3\bar{i}-2\bar{j}+2\bar{k}$, then show that $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$ are perpendicular to each other.
8. Prove that $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \cot 36^\circ$
9. Find the period of the function defined by f(x) = tan(x + 4x + 9x+.....+ n^2x).
10. If sinh x = 3 then show that $x = \log_e\left(3+\sqrt{10}\right)$.

SECTION -B

II. Answer any FIVE of the following Short Answer Questions: $5 \times 4 = 20$

11. Show that $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = (a-b)(b-c)(c-a)$.
12. Let A B C D E F be regular hexagon with centre 'O' . Show that $\overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF} = 3\overline{AD} = 6\overline{AO}$.
13. If $\bar{a} = \bar{i}-2\bar{j}-3\bar{k}$, $\bar{b} = 2\bar{i}+\bar{j}-\bar{k}$ and $\bar{c} = \bar{i}+3\bar{j}-2\bar{k}$, find $\bar{a}\times(\bar{b}\times\bar{c})$.
14. If A is not an integral multiple of $\frac{\pi}{2}$,Provethat (i) tan A+cotA=2cosec2A (ii) cotA–tanA= 2cot 2A
15. Solve: $2\cos^2\theta - \sqrt{3}\sin\theta + 1 = 0$.
16. Prove that $\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(4\tan^{-1}\frac{1}{3}\right)$
17. In ΔABC, Prove that $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2}$.

SECTION -C

III. Answer any FIVE of the following Long Answer Questions: $5 \times 7 = 35$

18. Let f:A→B, g:B→C be bijections. Then prove that $(gof)^{-1} = f^{-1}og^{-1}$.
19. By using mathematical induction show that $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+.....(\text{upto 'n' terms}) = \frac{n}{3n+1}, \forall n \in N$,
20. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$ than find $(A')^{-1}$.
21. Solve the following equations by Gauss-Jordan method 3x+4y+5z=18, 2x–y+8z=13 and 5x–2y+7z=20.
22. If A=(1,–2,–1), B= (4,0,–3), C=(1,2,–1) and D= (2,–4,–5), find the distance between $\overline{AB}$ and $\overline{CD}$.
23. If A, B, C are angles of a triangle, then porve that $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2} = 1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$.
24. In a ΔABC, If a = 13, b = 14, c = 15, find R, r, r_1, r_2 and r_3.

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SOLUTIONS

SECTION -A

1. If $A=\left\{0,\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3},\frac{\pi}{2}\right\}$ and f: A → B is a surjection defined by f(x) = cosx then find B.

Sol: Given $A=\left\{0,\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3},\frac{\pi}{2}\right\}$ and f(x) =cosx

$f(0)=\cos(0)=1;\ f\left(\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2};\ f\left(\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}};f\left(\frac{\pi}{3}\right)=\cos\frac{\pi}{3}=\frac{1}{2};\ f\left(\frac{\pi}{2}\right)=\cos\frac{\pi}{2}=0$

$\therefore B=f(A)=\left\{1,\frac{\sqrt{3}}{2},\frac{1}{\sqrt{2}},\frac{1}{2},0\right\}$ [∵ For a surjection, Range f(A)= Codomain B]

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2. Find the domain of the real function $f(x)=\frac{1}{\log(2-x)}$

Sol: Given f(x) is defined when $2-x>0$ and $(2-x)\neq 1$

$\Rightarrow x-2<0$ and $2-x\neq 1$ $\Rightarrow x<2$ and $x\neq 1$ $\Rightarrow x\in(-\infty,2)-\{1\}$

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3. A certain book shop has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs.80, Rs.60, Rs.40 each respectively. Using matrix algebra, find the total value of the books in the shop.

Sol: Number of 3 types of books is expressed by the row matrix A

Chem Phy Eco

$$A=\begin{bmatrix}10\times12 & 8\times12 & 10\times12\end{bmatrix}=\begin{bmatrix}120 & 96 & 120\end{bmatrix}$$

Selling price of 3 types of books is expressed by the column matrix B.

$$B=\begin{bmatrix}80\\60\\40\end{bmatrix}\begin{matrix}\text{Chemistry}\\\text{Physics}\\\text{Economics}\end{matrix}$$

Total value of the books in the shop is given by AB

$$AB=[120\quad 96\quad 120]\begin{bmatrix}80\\60\\40\end{bmatrix}=[120\times80+96\times60+120\times40]=[9600+5760+4800]=[20160]$$

∴ Total value of the books = Rs. 20160

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4. If $A=\begin{bmatrix}2 & -4\\-5 & 3\end{bmatrix}$ then find A+A' and AA'.

Sol: $A=\begin{bmatrix}2 & -4\\-5 & 3\end{bmatrix}\Rightarrow A'=\begin{bmatrix}2 & -5\\-4 & 3\end{bmatrix}$ $\therefore A+A'=\begin{bmatrix}2 & -4\\-5 & 3\end{bmatrix}+\begin{bmatrix}2 & -5\\-4 & 3\end{bmatrix}=\begin{bmatrix}2+2 & -4-5\\-5-4 & 3+3\end{bmatrix}=\begin{bmatrix}4 & -9\\-9 & 6\end{bmatrix}$

$$AA'=\begin{bmatrix}2 & -4\\-5 & 3\end{bmatrix}\begin{bmatrix}2 & -5\\-4 & 3\end{bmatrix}=\begin{bmatrix}4+16 & -10-12\\-10-12 & 25+9\end{bmatrix}=\begin{bmatrix}20 & -22\\-22 & 34\end{bmatrix}$$

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5. Show that the points whose P.V are $-2\bar{a}+3\bar{b}+5\bar{c}, \bar{a}+2\bar{b}+3\bar{c}, 7\bar{a}-\bar{c}$ are collinear, where $\bar{a},\bar{b},\bar{c}$ are non-coplanar vectors.

Sol: Given points $\overline{OP}=-2\bar{a}+3\bar{b}+5\bar{c}, \overline{OQ}=\bar{a}+2\bar{b}+3\bar{c}, \quad \overline{OR}=7\bar{a}-\bar{c}$, where O is origin

Now, $\overline{PQ}=\overline{OQ}-\overline{OP}=(\bar{a}+2\bar{b}+3\bar{c})-(-2\bar{a}+3\bar{b}+5\bar{c})=3\bar{a}-\bar{b}-2\bar{c}$

$\overline{QR}=\overline{OR}-\overline{OQ}=(7\bar{a}-\bar{c})-(\bar{a}+2\bar{b}+3\bar{c})=6\bar{a}-2\bar{b}-4\bar{c}=2(3\bar{a}-\bar{b}-2\bar{c})=2\overline{PQ}$

$\therefore \overline{QR}=2\overline{PQ}$, which is in the form $\overline{QR}=t\overline{PQ}, t\in R$

Hence the vectors $\overline{QR}, \overline{PQ}$ are collinear $\Rightarrow$ the points P, Q, R are collinear.

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6. Let $\bar{a}=2\bar{i}+4\bar{j}-5\bar{k}, \bar{b}=\bar{i}+\bar{j}+\bar{k}, \bar{c}=\bar{j}+2\bar{k}$. Find the unit vector in the opposite direction of $\bar{a}+\bar{b}+\bar{c}$

Sol: Given $\bar{a}=2\bar{i}+4\bar{j}-5\bar{k}, \bar{b}=\bar{i}+\bar{j}+\bar{k}, \bar{c}=0\bar{i}+\bar{j}+2\bar{k}$, then

$\bar{a}+\bar{b}+\bar{c}=(2\bar{i}+4\bar{j}-5\bar{k})+(\bar{i}+\bar{j}+\bar{k})+(0\bar{i}+\bar{j}+2\bar{k})=3\bar{i}+6\bar{j}-2\bar{k}$

$\Rightarrow |\bar{a}+\bar{b}+\bar{c}|=\sqrt{3^2+6^2+(-2)^2}=\sqrt{9+36+4}=\sqrt{49}=7$

$$\therefore \text{ Required unit vector }=\frac{-(\bar{a}+\bar{b}+\bar{c})}{|\bar{a}+\bar{b}+\bar{c}|}=\frac{-(3\bar{i}+6\bar{j}-2\bar{k})}{7}$$

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7. If $\bar{a}=\bar{i}+2\bar{j}-3\bar{k}, \bar{b}=3\bar{i}-\bar{j}+2\bar{k}$ then show that $\bar{a}+\bar{b}, \bar{a}-\bar{b}$ are perpendicular.

Sol: $\bar{a}+\bar{b}=(\bar{i}+2\bar{j}-3\bar{k})+(3\bar{i}-\bar{j}+2\bar{k})=4\bar{i}+\bar{j}-\bar{k}$

$\bar{a}-\bar{b}=(\bar{i}+2\bar{j}-3\bar{k})-(3\bar{i}-\bar{j}+2\bar{k})=-2\bar{i}+3\bar{j}-5\bar{k}$

Now, $(\bar{a}+\bar{b}).(\bar{a}-\bar{b})=(4\bar{i}+\bar{j}-\bar{k}).(-2\bar{i}+3\bar{j}-5\bar{k})=4(-2)+1(3)+(-1)(-5)=-8+3+5=-8+8=0$

$\therefore (\bar{a}+\bar{b})$ and $(\bar{a}-\bar{b})$ are perpendicular

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8. Prove that $\dfrac{\cos 9^\circ+\sin 9^\circ}{\cos 9^\circ-\sin 9^\circ}=\cot 36^\circ$

Sol: $$\text{L.H.S}=\frac{\cos 9^\circ+\sin 9^\circ}{\cos 9^\circ-\sin 9^\circ}=\frac{\frac{\cos 9^\circ}{\cos 9^\circ}+\frac{\sin 9^\circ}{\cos 9^\circ}}{\frac{\cos 9^\circ}{\cos 9^\circ}-\frac{\sin 9^\circ}{\cos 9^\circ}}=\frac{1+\tan 9^\circ}{1-\tan 9^\circ}=\frac{\tan 45^\circ+\tan 9^\circ}{1-\tan 45^\circ\tan 9^\circ}=\tan(45^\circ+9^\circ)=\tan 54^\circ$$

$=\tan(90^\circ-36^\circ)=\cot 36^\circ=$ R.H.S

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9. Find the period of $\tan(x+4x+9x+....+n^2x)$ (n any positive integer)

Sol: Given $\tan(x+4x+9x+....+n^2x)$

$$=\tan(1+4+9+....+n^2)x=\tan(1^2+2^2+3^2+....+n^2)x=\tan\left[\frac{n(n+1)(2n+1)}{6}\right]x$$

Period of $\tan(kx)=\dfrac{\pi}{|k|}$ $\qquad \therefore$ Period $=\dfrac{\pi}{\frac{n(n+1)(2n+1)}{6}}=\dfrac{6\pi}{n(n+1)(2n+1)}$

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10. If sinhx=3 then show that $x=\log(3+\sqrt{10})$

Sol: Given $\sinh x=3$, then $x=\text{Sinh}^{-1}(3)=\log_e(3+\sqrt{3^2+1})$ $\qquad \left[\because \text{Sinh}^{-1}x=\log_e\left(x+\sqrt{x^2+1}\right)\right]$

$=\log_e(3+\sqrt{9+1})=\log_e(3+\sqrt{10})$

$\therefore x=\log_e(3+\sqrt{10})$. Hence proved

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SECTION -B

11. Show that $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = (a-b)(b-c)(c-a)$

Sol: L.H.S = $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = \begin{vmatrix} bc & b+c & 1 \\ c(a-b) & a-b & 0 \\ b(a-c) & a-c & 0 \end{vmatrix} (\because R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1)$

$$= (a-b)(a-c)\begin{vmatrix} bc & b+c & 1 \\ c & 1 & 0 \\ b & 1 & 0 \end{vmatrix} = (a-b)(a-c)(c-b) = (a-b)(b-c)(c-a) = \text{R.H.S}$$

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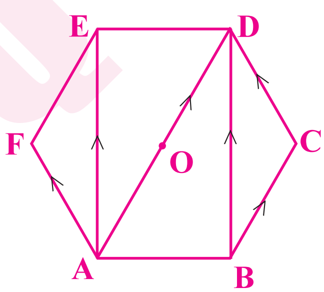
12. If ABCDEF is a regular hexagon with centre O, then prove that
$\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\overline{AD} = 6\overline{AO}$

Sol: Given ABCDEF is a regular hexagon with centre 'O'.



$\therefore\ (\overline{AB}+\overline{AC})+(\overline{AD})+(\overline{AE}+\overline{AF})$

$= (\overline{AB}+\overline{AC})+(\overline{AD})+(\overline{BD}+\overline{CD})\quad [\because \overline{AE}=\overline{BD}, \overline{AF}=\overline{CD}]$

$= (\overline{AB}+\overline{BD})+\overline{AD}+(\overline{AC}+\overline{CD}) = (\overline{AD})+(\overline{AD})+\overline{AD} = 3\overline{AD}$

$= 3(2\overline{AO})\quad [\because \overline{AD} = 2\overline{AO}]$

$= 6\overline{AO}$

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13. If $\bar{a} = \bar{i} - 2\bar{j} - 3\bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} - \bar{k}$ and $\bar{c} = \bar{i} + 3\bar{j} - 2\bar{k}$, find $\bar{a}\times(\bar{b}\times\bar{c})$.

Sol: Given $\bar{a} = \bar{i} - 2\bar{j} - 3\bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} - \bar{k}$ **and** $\bar{c} = \bar{i} + 3\bar{j} - 2\bar{k}$

Now, $\bar{b}\times\bar{c} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 1 & -1 \\ 1 & 3 & -2 \end{vmatrix} = \bar{i}(-2+3) - \bar{j}(-4+1) + \bar{k}(6-1) = \bar{i} + 3\bar{j} + 5\bar{k}$

$\therefore \bar{a}\times(\bar{b}\times\bar{c}) = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -2 & -3 \\ 1 & 3 & 5 \end{vmatrix} = \bar{i}(-10+9) - \bar{j}(5+3) + \bar{k}(3+2) = -\bar{i} - 8\bar{j} + 5\bar{k}$

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14. If A is not an integral multiple of $\frac{\pi}{2}$, prove that

(i) tan A + cot A = 2cosec2A **(ii) cot A – tan A = 2cot 2A**

Sol: (i) $\tan A + \cot A = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} = \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} = \frac{1}{\sin A.\cos A} = \frac{2}{2\sin A\cos A} = \frac{2}{\sin 2A} = 2\operatorname{cosec} 2A$

(ii) $\cot A - \tan A = \frac{1}{\tan A} - \tan A = \frac{1-\tan^2 A}{\tan A} = 2\left(\frac{1-\tan^2 A}{2\tan A}\right) = \frac{2}{\tan 2A} = 2\cot 2A$

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15. Solve $2\cos^2\theta-\sqrt{3}\sin\theta+1=0$

Sol: Given equation is $2\cos^2\theta-\sqrt{3}\sin\theta+1=0$

$\Rightarrow 2(1-\sin^2\theta)-\sqrt{3}\sin\theta+1=0 \Rightarrow 2-2\sin^2\theta-\sqrt{3}\sin\theta+1=0$

$\Rightarrow 2\sin^2\theta+\sqrt{3}\sin\theta-3=0 \;\Rightarrow\; 2\sin^2\theta+2\sqrt{3}\sin\theta-\sqrt{3}\sin\theta-(\sqrt{3})^2=0$

$\Rightarrow 2\sin\theta(\sin\theta+\sqrt{3})-\sqrt{3}(\sin\theta+\sqrt{3})=0$

$\Rightarrow (\sin\theta+\sqrt{3})(2\sin\theta-\sqrt{3})=0 \Rightarrow (\sin\theta+\sqrt{3})=0 \Rightarrow \sin\theta=-\sqrt{3}$ which has no solution

(or) $(2\sin\theta-\sqrt{3})=0 \Rightarrow 2\sin\theta=\sqrt{3} \Rightarrow \sin\theta=\dfrac{\sqrt{3}}{2}$

So $\sin\theta=\dfrac{\sqrt{3}}{2}=\sin\dfrac{\pi}{3}$, here P.V is $\alpha=\dfrac{\pi}{3}$

$\therefore$ General solution is given by $\theta=n\pi+(-1)^n\alpha,\ n\in Z \Rightarrow \theta=n\pi+(-1)^n\dfrac{\pi}{3}, n\in Z$

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16. Prove that $\cos\left(2\mathrm{Tan}^{-1}\dfrac{1}{7}\right)=\sin\left(4\mathrm{Tan}^{-1}\dfrac{1}{3}\right)$

Sol: We take $\mathrm{Tan}^{-1}\dfrac{1}{7}=\alpha \Rightarrow \tan\alpha=\dfrac{1}{7}$ and $\mathrm{Tan}^{-1}\dfrac{1}{3}=\beta \Rightarrow \tan\beta=\dfrac{1}{3}$, then

$$\cos\left(2\mathrm{Tan}^{-1}\frac{1}{7}\right)=\cos 2\alpha=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}=\frac{1-\frac{1}{49}}{1+\frac{1}{49}}=\frac{48}{50}=\frac{24}{25} \quad ......(1)$$

Also, $\tan\beta=\dfrac{1}{3} \Rightarrow \tan 2\beta=\dfrac{2\tan\beta}{1-\tan^2\beta}=\dfrac{2.\frac{1}{3}}{1-\frac{1}{9}}=\dfrac{2}{3}\times\dfrac{9}{8}=\dfrac{3}{4}$

Hence, $\sin\left(4\mathrm{Tan}^{-1}\dfrac{1}{3}\right)=\sin(4\beta)=\sin(2(2\beta)) = \dfrac{2\tan 2\beta}{1+\tan^2 2\beta}=\dfrac{2.\frac{3}{4}}{1+\frac{9}{16}}=\dfrac{3}{2}\times\dfrac{16}{25}=\dfrac{24}{25}$ ....(2)

$\therefore$ from (1) and (2), $\cos\left(2\mathrm{Tan}^{-1}\dfrac{1}{7}\right)=\sin\left(4\mathrm{Tan}^{-1}\dfrac{1}{3}\right)$

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17. In ΔABC, prove that $\tan\left(\dfrac{B-C}{2}\right)=\dfrac{b-c}{b+c}\cot\dfrac{A}{2}$

Sol: $\dfrac{b-c}{b+c}=\dfrac{2R\sin B-2R\sin C}{2R\sin B+2R\sin C}=\dfrac{2R(\sin B-\sin C)}{2R(\sin B+\sin C)}=\dfrac{\sin B-\sin C}{\sin B+\sin C}$

$$=\frac{2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}=\cot\left(\frac{B+C}{2}\right)\tan\left(\frac{B-C}{2}\right)$$

$\therefore \dfrac{b-c}{b+c}=\tan\dfrac{A}{2}\tan\left(\dfrac{B-C}{2}\right)$ $\left[\because \text{In } \Delta ABC, \cot\left(\dfrac{B+C}{2}\right)=\tan\dfrac{A}{2}\right]$

$\Rightarrow \tan\left(\dfrac{B-C}{2}\right)=\dfrac{b-c}{b+c}\cot\dfrac{A}{2}$

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SECTION -C

18. If $f:A\to B$, $g:B\to C$ are two bijective functions then prove that $(gof)^{-1}=f^{-1}og^{-1}$

Sol: Part -1: Given that $f: A\to B$, $g: B\to C$ are two bijective functions, then

(i) $gof: A\to C$ is bijection $\Rightarrow (gof)^{-1}: C\to A$ is also a bijection

(ii) $f^{-1}: B\to A$, $g^{-1}: C\to B$ are both bijections $\Rightarrow (f^{-1}og^{-1}): C\to A$ is also a bijection.

So, $(gof)^{-1}$ and $f^{-1}og^{-1}$, both have same domain 'C'

Part -2: Given $f: A\to B$ is bijection, then $f(a)=b \Rightarrow a=f^{-1}(b)$ ....(1) [Here $a\in A$, $b\in B$]

$g: B\to C$ is bijection, then $g(b)=c \Rightarrow b=g^{-1}(c)$ ....(2) [Here $b\in B$, $c\in C$]

$gof: A\to C$ is bijection, then $gof(a)=c \Rightarrow a=(gof)^{-1}(c)$ ....(3)

Now, $(f^{-1}og^{-1})(c)=f^{-1}[g^{-1}(c)]=f^{-1}(b)=a$ ....(4) [From (1) & (2)]

$\therefore (gof)^{-1}(c)=(f^{-1}og^{-1})(c), \forall c\in C$ [from (3) & (4)]

Hence, we proved that $(gof)^{-1}=f^{-1}og^{-1}$

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19. Prove that $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+.....+n$ terms $=\frac{n}{3n+1}$

Sol: To find n^{th} term:

1,4,7... are in A.P with $a=1$, $d=3$ $\therefore T_n=a+(n-1)d \Rightarrow T_n=1+(n-1)3=1+3n-3=3n-2$

4,7,10... are in A.P with $a=4$, $d=3$ $\therefore T_n=4+(n-1)3=3n+1$

$\therefore n^{th}$ term is $T_n=\frac{1}{(3n-2)(3n+1)}$

Let, $S(n):\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+.....+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$

Step 1: L.H.S of $S(1)=\frac{1}{1.4}=\frac{1}{4}$; R.H.S of $S(1)=\frac{1}{3.1+1}=\frac{1}{4}$

$\therefore$ L.H.S = R.H.S.

So, S(1) is true

Step 2: Assume that S(k) is true ,for $k\in N$

$S(k):\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+.....+\frac{1}{(3k-2)(3k+1)}=\frac{k}{3k+1}$ .....(1)

Step 3: We show that S(k+1) is true

$(k+1)^{th}$ term $=\frac{1}{(3(k+1)-2)(3(k+1)+1)}=\frac{1}{(3k+1)(3k+4)}$

On adding $(k+1)^{th}$ term to both sides of (1), we get

$$\text{L.H.S}=\left[\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+.....+\frac{1}{(3k-2)(3k+1)}\right]+\frac{1}{(3k+1)(3k+4)}$$

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$$=\frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)}=\frac{k(3k+4)+1}{(3k+1)(3k+4)}=\frac{3k^2+4k+1}{(3k+1)(3k+4)}$$

$$=\frac{(k+1)\,\cancel{(3k+1)}}{\cancel{(3k+1)}\,(3k+4)}=\frac{k+1}{3k+4}=\frac{k+1}{3k+3+1}=\frac{k+1}{3(k+1)+1}=\text{R.H.S}$$

$\therefore$ L.H.S =R.H.S.

So, S(k + 1) is true whenever S(k) is true

Hence, by P.M.I the given statement is true, for all $n\in N$

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20. If $A=\begin{bmatrix}1 & -2 & 3\\ 0 & -1 & 4\\ -2 & 2 & 1\end{bmatrix}$ then find $(A')^{-1}$.

Sol: Given that $A=\begin{bmatrix}1 & -2 & 3\\ 0 & -1 & 4\\ -2 & 2 & 1\end{bmatrix}\Rightarrow A'=\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}\Rightarrow \det A'=1\begin{vmatrix}-1 & 4\\ 2 & 1\end{vmatrix}+0-2\begin{vmatrix}-2 & -1\\ 3 & 4\end{vmatrix}$

$$=1(-1-8)+0-2(-8+3)=-9+10=1$$

The cofactor matrix of A'= $\begin{bmatrix}\begin{vmatrix}-1 & 2\\ 4 & 1\end{vmatrix} & -\begin{vmatrix}-2 & 2\\ 3 & 1\end{vmatrix} & \begin{vmatrix}-2 & -1\\ 3 & 4\end{vmatrix}\\ -\begin{vmatrix}0 & -2\\ 4 & 1\end{vmatrix} & \begin{vmatrix}1 & -2\\ 3 & 1\end{vmatrix} & -\begin{vmatrix}1 & 0\\ 3 & 4\end{vmatrix}\\ \begin{vmatrix}0 & -2\\ -1 & 2\end{vmatrix} & -\begin{vmatrix}1 & -2\\ -2 & 2\end{vmatrix} & \begin{vmatrix}1 & 0\\ -2 & -1\end{vmatrix}\end{bmatrix}=\begin{bmatrix}-9 & 8 & -5\\ -8 & 7 & -4\\ 2 & 2 & -1\end{bmatrix}\Rightarrow \text{Adj}A'=\begin{bmatrix}-9 & -8 & 2\\ 8 & 7 & 2\\ -5 & -4 & -1\end{bmatrix}$ -----(1)

Now, $(A')^{-1}=\frac{1}{\det A}(\text{Adj}A)=\frac{1}{1}\begin{bmatrix}-9 & -8 & 2\\ 8 & 7 & 2\\ -5 & -4 & -1\end{bmatrix}=\begin{bmatrix}-9 & -8 & 2\\ 8 & 7 & 2\\ -5 & -4 & -1\end{bmatrix}$

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21. Solve the equations 3x + 4y + 5z = 18; 2x − y + 8z = 13; 5x − 2y + 7z = 20 by Gauss-Jordan method.

Sol: Given equations in the matrix equation form: AX = D, where

$$A=\begin{bmatrix}3 & 4 & 5\\ 2 & -1 & 8\\ 5 & -2 & 7\end{bmatrix},\quad D=\begin{bmatrix}18\\ 13\\ 20\end{bmatrix},\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},$$

Augmented matrix $[AD]=\begin{bmatrix}3 & 4 & 5 & 18\\ 2 & -1 & 8 & 13\\ 5 & -2 & 7 & 20\end{bmatrix}$

$$\sim\begin{bmatrix}1 & 5 & -3 & 5\\ 2 & -1 & 8 & 13\\ 5 & -2 & 7 & 20\end{bmatrix}(R_1\to R_1-R_2)$$

$$\sim\begin{bmatrix}1 & 5 & -3 & 5\\ 0 & -11 & 14 & 3\\ 0 & -27 & 22 & -5\end{bmatrix}\begin{matrix}\\ (R_2\to R_2-2R_1)\\ (R_3\to R_3-5R_1)\end{matrix}$$

$$\sim\begin{bmatrix}1 & 5 & -3 & 5\\ 0 & 1 & -26 & -25\\ 0 & -27 & 22 & -5\end{bmatrix}(R_2\to -5R_2+2R_3)$$

$$\sim\begin{bmatrix}1 & 0 & 127 & 130\\ 0 & 1 & -26 & -25\\ 0 & 0 & -680 & -680\end{bmatrix}\begin{matrix}(R_1\to R_1-5R_2)\\ \\ (R_3\to R_3+27R_2)\end{matrix}$$

$$\sim\begin{bmatrix}1 & 0 & 127 & 130\\ 0 & 1 & -26 & -25\\ 0 & 0 & 1 & 1\end{bmatrix}(R_3\to R_3\div(-680)) \quad \sim\begin{bmatrix}1 & 0 & 0 & 3\\ 0 & 1 & 0 & 1\\ 0 & 0 & 1 & 1\end{bmatrix}\begin{matrix}(R_1\to R_1-127R_3)\\ (R_2\to R_2+26R_3)\\ \\ \end{matrix}$$

$\therefore\ x=3, y=1, z=1$

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22. If A = (1, –2, –1), B = (4, 0, –3), C = (1, 2, –1) and D = (2, –4, –5) then find the distance between $\overline{AB}$ and $\overline{CD}$.

Sol: Given points A = (1,–2,–1), B = (4,0,–3), C = (1,2,–1), D = (2,–4,–5)

$\overline{OA}=\bar{i}-2\bar{j}-\bar{k}$, $\overline{OB}=4\bar{i}-3\bar{k}$, $\overline{OC}=\bar{i}+2\bar{j}-\bar{k}$, $\overline{OD}=2\bar{i}-4\bar{j}-5\bar{k}$

(i) Vector equation of the line $\overline{AB}$ is $\bar{r}=\bar{a}+t\bar{b}, t\in R$, where

$\bar{a}=\overline{OA}=\bar{i}-2\bar{j}-\bar{k}$ & $\bar{b}=\overrightarrow{AB}=\overline{OB}-\overline{OA}=(4\bar{i}-3\bar{k})-(\bar{i}-2\bar{j}-\bar{k})=3\bar{i}+2\bar{j}-2\bar{k}$

(ii) Vector equation of the line $\overline{CD}$ is $\bar{r}=\bar{c}+s\bar{d}, s\in R$, where

$\bar{c}=\overline{OC}=\bar{i}+2\bar{j}-\bar{k}$ and $\bar{d}=\overline{CD}=\overline{OD}-\overline{OC}=(2\bar{i}-4\bar{j}-5\bar{k})-(\bar{i}+2\bar{j}-\bar{k})=\bar{i}-6\bar{j}-4\bar{k}$

So, $\bar{a}-\bar{c}=(\bar{i}-2\bar{j}-\bar{k})-(\bar{i}+2\bar{j}-\bar{k})=-4\bar{j}$

$$\bar{b}\times\bar{d}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 3 & 2 & -2\\ 1 & -6 & -4\end{vmatrix}=\bar{i}(-8-12)-\bar{j}(-12+2)+\bar{k}(-18-2)=-20\bar{i}+10\bar{j}-20\bar{k}$$

Now, $(\bar{a}-\bar{c}).(\bar{b}\times\bar{d})=(-4\bar{j}).(-20\bar{i}+10\bar{j}-20\bar{k})=-4(10)=-40$

Also, $|\bar{b}\times\bar{d}|=\sqrt{(-20)^2+10^2+(-20)^2}=\sqrt{400+100+400}=\sqrt{900}=30$

$\therefore$ Shortest distance(SD) $=\dfrac{|(\bar{a}-\bar{c}).(\bar{b}\times\bar{d})|}{|\bar{b}\times\bar{d}|}=\dfrac{|-40|}{30}=\dfrac{40}{30}=\dfrac{4}{3}$ units

23. If A,B,C are angles of a triangle, then P.T $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

Sol: L.H.S= $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}$

$$=\left[\frac{1-\cos A}{2}+\frac{1-\cos B}{2}-\frac{1-\cos C}{2}\right] \qquad \left[\because \sin^2\frac{\theta}{2}=\frac{1-\cos\theta}{2}\right]$$

$$=\frac{1}{2}[1-(\cos A+\cos B)+\cos C]$$

$$=\frac{1}{2}\left[1-2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)+\left(1-2\sin^2\frac{C}{2}\right)\right]\left[\because \cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right);\cos\theta=1-2\sin^2\frac{\theta}{2}\right]$$

$$=\frac{1}{2}\left[2-2\cos\left(90^\circ-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right)-2\sin^2\frac{C}{2}\right] \qquad \left(\because \frac{A+B}{2}+\frac{C}{2}=90^\circ\right)$$

$$=\frac{1}{2}\left[2-2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)-2\sin^2\frac{C}{2}\right] \qquad (\because \cos(90^\circ-\theta)=\sin\theta)$$

$$=\frac{\not 2}{\not 2}\left[1-\sin\frac{C}{2}\cos\frac{A-B}{2}-\sin^2\frac{C}{2}\right] \qquad \text{(taking out '2' common)}$$

$$=1-\sin\frac{C}{2}\left(\cos\left(\frac{A-B}{2}\right)+\sin\frac{C}{2}\right)=1-\sin\frac{C}{2}\left(\cos\left(\frac{A-B}{2}\right)+\sin\left(90^\circ-\frac{A+B}{2}\right)\right) \qquad \left(\because \frac{A+B}{2}+\frac{C}{2}=90^\circ\right)$$

$$=1-\sin\frac{C}{2}\left(\cos\left(\frac{A-B}{2}\right)+\cos\frac{A+B}{2}\right) \qquad (\because \sin(90^\circ-\theta)=\cos\theta)$$

$$=1-\sin\frac{C}{2}\left(2\cos\frac{A}{2}\cos\frac{B}{2}\right) \qquad (\because \cos(A+B)+\cos(A-B)=2\cos A\cos B)$$

$$=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}=\text{R.H.S}$$

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24. In a ΔABC if a=13, b=14, c=15 then show that $R=\frac{65}{8}, r=4, r_1=\frac{21}{2}, r_2=12, r_3=14$

Sol: Given $a=13, b=14, c=15$, then $2s=a+b+c=13+14+15=42 \Rightarrow \not 2s=\not 42 \Rightarrow s=21$

Now $\Delta=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{21(21-13)(21-14)(21-15)}$

$$=\sqrt{21\times(8)(7)(6)}=\sqrt{(3\times7)(4\times2)(7)(3\times2)}=\sqrt{3^2\times4^2\times7^2}=3\times4\times7=84$$

(i) $R=\frac{abc}{4\Delta}=\frac{13\times14\times15}{4\times84}=\frac{65}{8}$ (ii) $r=\frac{\Delta}{s}=\frac{84}{21}=4$;

(iii) $r_1=\frac{\Delta}{s-a}=\frac{84}{21-13}=\frac{84}{8}=\frac{21}{2}$ (iv) $r_2=\frac{\Delta}{s-b}=\frac{84}{21-14}=\frac{84}{7}=12$

(v) $r_3=\frac{\Delta}{s-c}=\frac{84}{21-15}=\frac{84}{6}=14$

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