PRACTICE MAKES A MAN PERFECT

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PRACTICE KEEPS A STUDENT TOP

IPE SCANNER

Total Textual Bits

- IPE SCANNER includes All Textual Solved Examples & Exercises Q's.
- All VSAQ individual topics are arranged in the order of Simple to Complex.
- Practice without peeping for solutions enhances Grip & Confidence levels.
- All together this IPE Scanner is a confidence booster.
- "Spot- Answers" are provided to avoid 'waste of time' for checking answers.
- This IPE Scanner is very useful to EAPCET, IIT-JEE Aspirants, because practising this IPE Scanner strengthens the basic 'funda' of the textual content, which is a strong pre-requisite for entrance exams.

IPE SCANNER

EXCLUSIVE FOR REAL TOPPERS

TOTAL TEXTUAL 2 MARK QUESTIONS [VSAQ]

I. STRAIGHT LINES

1) SLOPES OF LINES, PARALLEL & PERPENDICULAR LINES

1) Find the slope of the straight line passing through the points (8, 1), (–1, 7)

2) Find the slope of the straight line passing through the points (–p, q), (q, –p)

3) Find the value of x, if the slope of the line passing through (2, 5), (x, 3) is 2. **[AP,TS M 18]**

4) Find the value of y, if the line joining (3, y) and (2, 7) is parallel to the line joining the points (–1, 4) and (0, 6) **[TS M 17]**

5) Find the slopes of the lines (i) parallel to and (ii) perpendicular to the line passing through (6,3) and (–4,5)

6) Find the slope of the lines x+y=0, x–y=0

7) Find the value of k, if the straight lines 6x–10y+3=0, kx –5y+8 =0 are parallel. **[AP J 17]**

8) Find the value of p, if the straight lines 3x + 7y –1 =0 and 7x –py + 3= 0 are mutually perpendicular. **[AP J 19][TS M 16,19]**

9) Find the value of k, if the straight lines y–3kx+4=0 and (2k–1)x–(8k–1)y–6=0 are perpendicular.

2) POINT - SLOPE FORM & TWO POINT FORM

10) Find the equation of the straight line,which makes 135° with the X-axis in the positive direction and pass through the point (3,–2).

11) Find the equation of the straight line, whose inclination is 135° and pass through the point(–2,3).

12) Find the equation of the straight line, which makes 150° with the X-axis in the positive direction and pass through the point (–2,–1).

13) Find the equation of the straight line, which makes $\pi/3$ with the X-axis in the positive direction and pass through the point (1,2).

14) Find the equation of the straight line,which makes $\pi/4$ with the X-axis in the positive direction and pass through the point (0,0).

15) Find the equation of the line passing through the points (1,–2) and (–2,3)

ANSWERS

1) –2/3 **2)** –1 **3)** 1 **4)** 9 **5)** (i) –1/5 (ii) 5 **6)** –1,1 **7)** 3 **8)** 3

9) 1/6 or –1 **10)** x+y–1=0 **11)** x+y–1=0 **12)** $x+\sqrt{3}y+(2+\sqrt{3})=0$

13) $\sqrt{3}x-y+(2-\sqrt{3})=0$ **14)** y=x **15)** 5x+3y+1=0

16) Find the equation of the line containing the points (1,2), (1,–2).

17) Find the equation of the line containing the points (2,–3), (0,–3). **[TS M 22]**

18) Find the equation of the straight line passing through the points $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ **[AP J 15]**

19) Prove that the points (1, 11), (2, 15), (–3, –5) are collinear and find the equation of the straight line containing them.

20) Prove that the points (a, b+c), (b, c+a) and (c, a+b) are collinear and find the equation of the straight line containing them.

21) Find the equation of the straight line parallel to the line 2x+3y+7 =0 and passing through the point (5, 4).

22) Find the equation of the straight line perpendicular to the line 5x–3y+1=0 and passing through the point (4, –3). **[TS J 16] [TS M 15]**

23) Find the equation of the straight line passing through the point (4,–3) and perpendicular to the line passing through the points (1, 1) and (2, 3).

24) Find the equation of the straight line passing through A (–1,3) and (i) parallel (ii) perpendicular to the straight line passing through B(2,–5),C(4, 6).

25) Find the equations of the straight lines passing through (1,3) and (i) parallel to (ii) perpendicular to the line passing through the points (3,–5) and (–6,1).

26) Find the equation of the line perpendicular line 3x+4y+6=0 and making intercept –4 on X-axis.

27) The angle made by a straight line with the positive X-axis in the positive direction is 60º and Y-intercept cut off by it is 3. Find the equation of the line.

28) The angle made by a straight line with the positive X-axis in the positive direction is 150º and Y-intercept cut off by it is 2. Find the equation of the line.

29) Find the equation of the straight line with inclination $\theta = \text{Tan}^{-1}\left(\frac{2}{3}\right)$ and y-intercept 3

30) Find the angle made by the straight line $y = -\sqrt{3}x + 3$ with the positive direction of the X- axis.

31) Find the angle which the straight line $y = \sqrt{3}x - 4$ makes with the Y- axis. **[AP,TS M 19]**

ANSWERS

16) x=1 **17)** y+3=0 **18)** $2x - y(t_1 + t_2) + 2at_1t_2 = 0$ **19)** 4x–y+7=0

20) x+y=a+b+c **21)** 2x+3y–22=0 **22)** 3x+5y+3=0 **23)** x+2y+2=0

24) i) 11x–2y+17=0 ii)2x+11y–31=0 **25)** (i) 2x+3y–11=0 (ii) 3x–2y+3=0 **26)** 4x–3y+16=0

27) $\sqrt{3}x - y + 3 = 0$ **28)** $x+\sqrt{3}y-2\sqrt{3}=0$ **29)** 2x–3y+9=0 **30)** θ=120º **31)** 30º

3) SLOPE - INTERCEPT FORM & INTERCEPT FORM

32) Reduce the equation 3x+4y=5 into (i) Slope intercept form (ii) Intercept form **[TS M 22]**

33) Transform the equation x+y+1=0 into (i) slope intercept form (ii) intercept form **[AP M 17]**

34) Transform the equation $\sqrt{3}x + y = 4$ into (i) slope intercept form (ii) intercept form

35) Transform the equation 4x–3y+12=0 into (i) slope intercept form (ii) intercept form **[IPE' 14][AP M 16]**

36) Find the equation of the straight line passing through (–4,5) and cutting off equal intercepts on the coordinate axes. **[AP M 20] [TS J 16,18] [TS M 15] [AP J 18]**

37) Find the equation of the straight line passing through the point (–2, 4)and making intercepts, whose sum is zero **[AP J 17] [TS J 15] [AP M 15]**

38) Find the equation of the straight line passing through the point (2,3) and making intercepts, whose sum is zero.

39) Find the equation of the straight line passing through the origin and making equal angles with the co-ordinate axes.

40) Find the sum of the squares of the intercepts of the line 4x–3y = 12 on the axes of co-ordinates. **[AP M 18]**

41) Find the area of the triangle formed by the line 3x–4y+12=0 with the coordinate axes.**[AP M 15]**

42) Find the area of the triangle formed by x–4y+2=0 with the coordinate axes. **[AP M 20]**

43) Find the equation of the straight line passing through the points (–1,2), (5,–1) and also find the area of the triangle formed by it with the axes of coordinates.

44) Find the value of a if the area of the triangle formed by the lines x=0,y=0, 3x+4y=a is 6 sq.units. **[AP J 18]**

45) Find the condition for the points (a,0),(h,k) and (0,b) when $ab \neq 0$ to be collinear.

46) Find the equation of the straight line passing through the point (3,–4) and making X andY-intercepts which are in the ratio 2:3.

47) If the product of the intercepts made by the straight line x tanα + ysecα = 1, $\left(0 \le \alpha < \frac{\pi}{2}\right)$, on the co-ordinates axes is equal to sinα, find α.

ANSWERS

32) (i) $y = \frac{-3}{4}x + \frac{5}{4}$ (ii) $\frac{x}{(5/3)} + \frac{y}{(5/4)} = 1$ **33)** (i) y=(–1)x+(–1) (ii) $\frac{x}{(-1)} + \frac{y}{(-1)} = 1$

34) (i) $y = -\sqrt{3}x + 4$ (ii) $\frac{x}{4/\sqrt{3}} + \frac{y}{4} = 1$ **35)** (i) $y = \frac{4}{3}x + 4$ (ii) $\frac{x}{-3} + \frac{y}{4} = 1$

36) x+y –1= 0 **37)** x–y+6=0 **38)**x–y+1=0 **39)**y=±x **40)**25

41) 6 sq.units **42)**1/2 sq.units **43)** i) x+2y–3=0 ii) 9/4 sq.units **44)** ±12

45) hb+ka=ab **46)** 3x+2y=1 **47)** 45°

4) NORMAL FORM, SYMMETRIC FORM & PARAMETRIC EQUATIONS

48) Transform the equation x+y+1=0 into Normal form **[TS M 18]**

49) Reduce the equation x+y–2=0 into (i) Intercept form (ii) Normal form

50) Reduce the equation $\sqrt{3}x + y + 10 = 0$ into (i) Slope intercept form (ii) Normal form **[AP 23]**

51) Transform the equation 3x+4y+12=0 into (i) Intercept form (ii) Normal form **[TS 17,22]**

52) Write the equation of the line whose length of the normal (Perpendicular) from the origin is p=5 and inclination is $\alpha=60^\circ$

53) Find the equation of the straight line whose distance from the origin is 4, if the normal ray from the origin to the straight line makes an angle of 135° with the positive direction of the X-axis.

54) A straight line whose inclination with the positive direction of the X–axis measured in the anti-clockwise sense is $\pi/3$ makes positive intercept on the Y–axis. If the straight line is at a distance of 4 from the origin, find its equation.

55) Write the equation of the line in the symmetric form whose slope is $\sqrt{3}$ and passing through the point (2,3).

56) Write the parametric equations of the straight line passing through the point (3,2) and making an angle of 135° with the positive direction of the X-axis in the positive direction.

57) A straight line passing through A(–2,1) makes an angle of 30° with the positive direction of the X-axis. Find the points on the straight line whose distance from A is 4 units.

58) A straight line passing through A(1,–2) makes an angle $\text{Tan}^{-1}\frac{4}{3}$ with the positive direction of the X-axis. Find the points on the straight line whose distance from A is 5 units.

ANSWERS

48) $x\cos 225^\circ + y\sin 225^\circ = \frac{1}{\sqrt{2}}$ **49)** (i) $\frac{x}{2}+\frac{y}{2}=1$ (ii) $x\cos\frac{\pi}{4}+y\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

50) (i) $y=-\sqrt{3}x-10$ (ii) $x\cos 210^\circ + y\sin 210^\circ = 5$

51) (i) $\frac{x}{(-4)}+\frac{y}{(-3)}=1$ (ii) $x\cos\alpha+y\sin\alpha=12/5$, where $\alpha=\text{Tan}^{-1}(4/3)$

52) $x+\sqrt{3}y=10$ **53)** $x-y+4\sqrt{2}=0$ **54)** $\sqrt{3}x-y+8=0$ **55)** $\frac{x-2}{\cos\frac{\pi}{3}}=\frac{y-3}{\sin\frac{\pi}{3}}$

56) $x=3-\frac{r}{\sqrt{2}}, y=2+\frac{r}{\sqrt{2}}$ **57)** $(-2+2\sqrt{3}, 3),(-2-2\sqrt{3}, -1)$ **58)** $(4,2), (-2,-6)$

5) INTERSECTION OF LINES, LINE RATIOS

59) Find the point of intersection of the lines $4x+8y-1=0$, $2x-y+1=0$

60) Find the value of p, if the straight lines $x+p=0, y+2=0, 3x+2y+5=0$ are concurrent. **[AP M 22][TS J 15,17] [TS M 17,20]**

61) Find the point of concurrence of the set of lines $(k+1)x+(k+2)y+5=0$

62) Find the point of concurrence of the set of lines $(2+5k)x-3(1+2k)y+(2-k)=0$

63) Find the ratios in which (i) the X-axis and (ii) the Y-axis divide the line segment AB joining the points A(2,–3) and B(3, –6). **[TS M 22]**

64) Find the ratio in which the line $2x+3y-20=0$ divides the line joining the points (2,3) and (2,10).

65) Find the ratio in which the straight line $2x+3y-5=0$ divides the line joining the points (0,0) and (–2,1).

66) Find the ratio in which the straight line $3x-4y=7$ divides the line joining the points (2,–7) and (–1,3).

67) State whether the points A(3,2), B(–4,–3) lie on the same side or opposite sides of the line $2x-3y+4=0$.

68) State whether the points A(2,–1), B(1,1) lie on the same side or opposite sides of the line $3x+4y=6$.

69) Find the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$

6) DISTANCE BETWEEN PARALLELS, ANGLE BETWEEN LINES

70) Find the distance between the parallel lines $5x-3y-4=0$, $10x-6y-9=0$.

71) Find the distance between the parallel lines $3x+4y-3=0$, $6x+8y-1=0$. **[AP M 19]**

72) Find the distance between the parallel lines $5x-3y-4=0$, $10x-6y-9=0$ **[TS J 18]**

73) Find the perpendicular distance from the point (3,4) to the straight line $3x-4y+10=0$ **[AP M 16]**

74) Find the perpendicular distance from the point (–3,4) to the straight line $5x-12y=2$

ANSWERS

59) $\left(\frac{-7}{20},\frac{3}{10}\right)$ **60)** 1/3 **61)** (5,–5) **62)** (5, 4) **63)** –1 : 2, –2 : 3

64) 1:2 **65)** –5 : 6 **66)** 27:22 **67)** Same Side **68)** Opposite sides

69) $\left(\frac{ab}{a+b},\frac{ab}{a+b}\right)$ **70)** 1 **71)** 1/2 **72)** $\frac{1}{\sqrt{136}}$ **73)** 3/5 **74)** 5

75) Find the length of the perpendicular drawn from the point (0,0) to the straight line x–3y–4=0

76) Find the length of the perpendicular from the point (–2,–3) to the straight line 5x–2y+4=0 **[TS 16,20]**

77) Find the angle between the straight lines 3x+5y=7, 2x–y+4=0

78) Find the angle between the lines $\sqrt{3}x + y + 1 = 0$ and x + 1 =0.

79) Find the angle between the lines 2x+y+4 = 0 and y–3x = 7.

80) Find the angle between the straight lines y=4–2x, y=3x+7

81) Find the angle between the lines $y = -\sqrt{3}x + 5,\ y = \frac{1}{\sqrt{3}}x - \frac{2}{\sqrt{3}}$

82) Find the angle between the lines ax+by = a+b, a(x–y)+b(x+y) = 2b.

7) ALL MIXTURE QUESTIONS

83) Write the equation of the reflection of the line x=1 in the Y-axis.

84) Write the equations of the straight lines parallel to X-axis and (i) at a distance of 3 units above the X- axis and (ii) at a distance of 4 units below the X-axis.

85) Write the equations of the straight lines parallel to Y-axis and (i) at a distance of 2 units from the Y-axis to the right of it, (ii) at a distance of 5 units from the Y-axis to the left of it.

86) (–4,5) is a vertex of a square and one of its diagonals is 7x –y+8 = 0. Find the equation of the other diagonal.

87) A(10,4), B(–4,9) and C(–2,-1) are the vertices of a triangle. Find the equation of the median through A

88) A(10,4), B(–4,9) and C(–2,–1) are the vertices of a triangle. Find the equation of the altitude through B

89) A(10,4), B(–4,9) and C(–2,-1) are the vertices of a triangle. Find the equation of the perpendicular bisector of the side $\overleftrightarrow{AB}$.

90) If the portion of a straight line intercepted between the axes of co-ordinates is bisected at (2p, 2q), write the equation of the straight line.

ANSWERS

75) $\frac{4}{\sqrt{10}}$ **76)** 0 **77)** $\text{Cos}^{-1}\frac{1}{\sqrt{130}}$ **78)** 30° **79)** π/4 **80)** π/4

81) π/2 **82)** π/4 **83)** x+1=0 **84)** y=3, y= –4 **85)** x=2, x= –5

86) x+7y–31=0 **87)** y=4. **88)** 12x+5y+3=0 **89)** 28x–10y–19=0 **90)** $\frac{x}{p} + \frac{y}{q} = 4$

91) The intercepts of a straight line on the axes of co-ordinates are a and b.If p is the length of the perpendicular drawn from the origin to this line. Write the value of p in terms of a and b.

92) Find the equation of the straight line passing through the point of intersection of the lines x+y+1 = 0 and 2x–y+5 = 0 and containing the point (5,–2).

93) Find the value of 'p' if the lines 4x–3y–7=0, 2x+py+2=0 and 6x+5y–1=0 are concurrent.

94) If a,b,c are arithmetic progression, then show that the equation ax+by+c=0 represents a family of concurrent lines and find the point of concurrency.

95) If 3a+2b+4c=0, then show that the equation ax+by+c=0 represents a family of concurrent straight lines and find the point of concurrency.

96) Find the set of values of 'a' if the points (1,2) and (3,4) lie to the same side of the straight line 3x–5y+a = 0.

97) Find the values of k, if the angle between the straight lines kx+y+9=0 and 3x–y+4=0 is $\pi/4$.

98) Find the incentre of the triangle whose vertices are $(1,\sqrt{3}), (2,0)$ and (0,0)

99) Find the foot of the perpendicular from (–1,3) on the straight line 5x–y–18=0

100) Find the image of (1,–2) w.r.t. the straight line 2x–3y+5=0.

II) 3D-COORDINATES

1) CENTROID OF TRIANGLE, TETRAHEDRON, DISTANCE BETWEEN POINTS, COLLINEAR POINTS

101) Find the centroid of triangle whose vertices are (5, 4, 6), (1,–1,3) and (4,3,2).

102) Find the coordinates of the vertex 'C' of ΔABC if its centroid is the origin and the vertices A,B are (1,1,1) and (–2,4,1) respectively. **[TS M 15][TS J 18][AP M 20]**

103) If (x_1,y_1,z_1) and (x_2,y_2,z_2) are two vertices and (α,β,γ) is the centroid of a triangle, find the third vertex of the triangle.

104) The centroid of the tetrahedron whose vertices are(2, 3, –4) , (–3, 3, –2), (–1, 4, 2), (3, 5, 1). **[TS J 16]**

105) If (3,2,–1), (4,1,1), (6,2,5) are 3 vertices and (4,2,2) is the centroid of a tetrahedron, find the 4^{th} vertex of that tetrahedron. **[AP 17,22][AP M, J 15]**

ANSWERS

91) $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$ **92)** 3x+7y–1=0 **93)**4 **94)** (1,–2) **95)** (3/4, 1/2) **96)**$(-\infty,7)\cup(11,\infty)$

97) 2,–1/2. **98)** $\left(1,\frac{1}{\sqrt{3}}\right)$ **99)** (4,2) **100)** (–3,4) **101)** $\left(\frac{10}{3},2,\frac{11}{3}\right)$

102)(1, –5,–2) **103)** C=$(3\alpha-x_1-x_2,\ 3\beta-y_1-y_2,\ 3\gamma-z_1-z_2)$ **104)** $\left(\frac{1}{4},\frac{15}{4},\frac{-3}{4}\right)$ **105)** (3,3,3)

106) Find the fourth vertex of the parallelogram whose consecutive vertices are (2,4,–1),(3,6,–1) and (4, 5, 1) **[TS M 17]**

107) Show that the points (1,2,3), (2,3,1), (3,1,2) form an equilateral triangle. **[AP M 16,18]**

108) Show that the points A(–4,9,6), B(–1,6,6), C(0,7,10) form a right angled isosceles triangle.

109) Show that ABCD is a square where A,B,C,D are the points (0,4,1), (2,3,–1), (4,5,0) and (2,6,2) respectively.

110) Show that the points(4,7,8),(2,3,4),(–1,–2,1),(1,2,5) are vertices of a parallelogram.

111) Find the distance between the mid point of the line segment $\overline{AB}$ and the point (3,–1,2)where A = (6,3,–4), B = (–2,–1,2) **[TS M 16]**

112) If M(α,β,γ) is the mid point of the line segment joining the points A(x_1,y_1,z_1) and B then find B.

113) If the distance between the points (5,–1,7) and (x, 5, 1) is 9 units, find the values of x.**[AP M 19]**

114) Show that the points (1, 2, 3), (7, 0, 1), (–2, 3, 4) are collinear

115) Show that the points (5,4,2), (6,2,–1) and (8,–2,–7) are collinear. **[AP J 17]**

116) Show that the points A(3, –2, 4), B(1, 1, 1), C(–1, 4, 2) are collinear **[TS M 22]**

117) Show that the points A(3,2,–4), B(5,4,–6) and C(9, 8, –10) are collinear and find the ratio in which B divides $\overline{AC}$

118) Find the ratio in which the XZ-plane divides the line joining A(–2,3,4) and B(1,2,3)

[TS M 18,19,22] **[IPE'14][TS J 15, 17]**

119) Find the ratio in which YZ-plane divides the line joining A(2,4,5), B(3,5,–4). Also find the point of intersection.

2) D.C'S & DR'S

120) Find the dr's and dc'r of the line joining the points (4,–7,3), (6,–5,2).

121) Find the direction cosines of the line joining the points (–4,1,7),(2,–3,2)

122) A line makes angles 90º, 60º, 30º with the positive direction of X, Y, Z axes respectively. Find its direction cosines.

123) If the d.c's of a line are (1/c, 1/c, 1/c), then find c.

124) Find the angle between the lines whose direction ratios are (1,1,2) $(\sqrt{3}, -\sqrt{3}, 0)$.

125) O is the origin, P(2,3,4), Q(1,k,1) are points such that $\overline{OP} \perp \overline{OQ}$ then find k.

ANSWERS

106) (3,3,1) **111)** $\sqrt{14}$ units **112)**$(2\alpha-x_1, 2\beta-y_1, 2\gamma-z_1)$ **113)**8 or 2 **117)** 1:2

118) –3:2 **119)** –2:3, (0,2,23) **120)** (–2, –2, 1) (or) (2, 2,–1); $\pm\left(\frac{2}{3},\frac{2}{3},\frac{-1}{3}\right)$

121) $\pm\left(\frac{-6}{\sqrt{77}},\frac{4}{\sqrt{77}},\frac{5}{\sqrt{77}}\right)$ **122)** $(0,\frac{1}{2},\frac{\sqrt{3}}{2})$ **123)** $\pm\sqrt{3}$ **124)** 90º **125)**–2

126) Show that the line joining the points A(2,3,–1) and B(3,5,–3) is perpendicular to the line joining C(1,2,3) and D(3,5,7).

127) For what value of x the line joining A(4,1,2), B(5,x,0) is perpendicular to the line joining C(1,2,3), D(3,5,7).

128) Find the d.c's of a line that makes equal angles with the axes.

129) A ray makes angles $\pi/3$, $\pi/3$ with $\overline{OX}$ and $\overline{OY}$ respectively. Find the angle made by it with $\overrightarrow{OZ}$

130) If a line makes angles α,β,γ with the positive directions of X,Y,Z axes then find the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

3) MISCELLANEOUS

131) If H,G,S and I respectively denote orthocentre, centroid, circumcentre and in-centre of a triangle formed by the points (1,2,3), (2,3,1) and (3,1,2, then find H,G,S,I.

132) Find the incentre of the triangle formed by the points (0,0,0), (3,0,0), (0,4,0).

133) A,B,C are three points OX, OY,OZ respectively, at distances a, b, c from the origin 'O'.Find the coordinates of the point which is equidistant from A,B,C and 'O'.

134) 'P' is a variable point which moves such that 3PA= 2PB. If A (–2, 2,3) and B(13, –3, 13) prove that P satisfies the equation $x^2+y^2+z^2+28x-12y+10z-247=0$

135) Show that the locus of the point whose distance from Y-axis is thrice distance from (1,2,–1)is $8x^2+9y^2+8z^2-18x-36y+18z+54=0$

136) A(5,4,6), B(1,–1,3), C(4,3, 2) are three points. Find the coordinates of the point in which the bisector of $\angle BAC$ meets the side BC.

137) If the point (1, 2, 3) is changed to the point (2,3, 1) through translation of axes, find the new origin.

138) Find the ratio in which the point P(5,4,–6) divides the line segment joining the points A (3, 2 –4) and B(9, 8 –10). Also find the harmonic conjugate of P.

ANSWERS

127) 3 **128)** $\pm\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ **129)** $\gamma=\frac{\pi}{4},\frac{3\pi}{4}$ **130)** 2 **131)** (2,2,2), (2,2,2) (2,2,2), (2,2,2)

132) (1,1,0) **133)** $\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$ **136)** $\left(\frac{23}{8},\frac{3}{2},\frac{19}{8}\right)$ **137)** (–1, –1, 2) **138)** 1:2, (–3,–4,2)

III)THE PLANE

139) Find the equation of the plane which makes intercepts 1,2,4 on the x,y,z-axes respectively. **[AP M 22]**

140) Write the equation of the plane 4x–4y+2z+5=0 in the intercept form. **[AP M 19]**

141) Reduce the equation x+2y–3z–6=0 of plane to the normal form. **[TS M 19]**

142) Find a traid of d.c's of the normal to plane x+2y+2z–4=0 **[TS 15,16,22]**

143) Find the angle between the planes 2x–y+z=6 and x+y+2z=7 **[AP M 15,17]**

144) Find the angle between the planes x+2y+2z–5=0, 3x+3y+2z–8=0 **[TS M 15,17]**

145) Find the constant k so that the planes x–2y+kz=0, 2x+5y–z=0 are at right angles.

146) Find the intercepts of 4x+3y–2z+2=0 on the coordinate axes **[AP,TS M 18]**

147) Find the equation of the plane through the point (α,β,γ) and parallel to the plane ax+by+cz=0

148) Find the equation of the plane passing through the point (1,1,1) and parallel to the plane x+2y+3z–7=0 **[AP J 17, 18]**

149) Find the equation of the plane passing through the point (1,2,–3) and parallel to the plane 2x–3y+6z=0

150) Find the equation of the plane passing through the point (–2,1,3) and having (3,–5,4) as d.r.'s of its normal. **[AP M 20]**

151) Find the equation of the plane through (–1,6,2) and perpendicular to the join of (1,2,3) and (–2,3,4).

152) Find the equation of the plane if the foot of the perpendicular from origin to the plane is A(1,3,–5)

153) Find the equation of the plane if the foot of the perpendicular from origin to the plane is A(2,3,–5) **[TS J 17]**

154) Find the equation of the plane passing through the point (2,3,4) and perpendicular to the x-axis.

155) Find the equation to the plane parallel to the ZX-plane and passing through (0,4,4).

156) Show that 2x+3y+7=0 represents a plane perpendicular to XY-plane.

157) Find the equation of the plane bisecting the line segment joining (2,0,6), (–6,2,4)and perpendicular to it.

158) Show that the plane through (1,1,1), (1,–1,1) and (–7,–3,–5) is parallel to the Y-axis.

159) Find the equation to the plane through the points (0,–1,–1), (4,5,1) and (3,9,4).

ANSWERS

139) 4x+2y+z=4 **140)** $\frac{x}{(-5/4)}+\frac{y}{(5/4)}+\frac{z}{(-5/2)}=1$ **141)** $\left(\frac{1}{\sqrt{14}}\right)x+\left(\frac{2}{\sqrt{14}}\right)y+\left(\frac{-3}{\sqrt{14}}\right)z=\frac{6}{\sqrt{14}}$

142) $\pm\left(\frac{1}{3},\frac{2}{3},\frac{2}{3}\right)$ **143)** $\pi/3$ **144)** $\text{Cos}^{-1}\frac{13}{3\sqrt{22}}$ **145)** –8

146) –1/2, –2/3, 1 **147)** a(x–α)+b(y–β)+c(z–γ)=0 **148)** x+2y+3z–6=0

149) 2x–3y+6z+22=0 **150)** 3x–5y+4z–1=0 **151)** 3x–y–z+11=0

152) x+3y–5z–35=0 **153)** 2x+3y–5z–38=0 **154)** x=2

155) y=4 **157)** 8x–2y+2z+8=0 **159)** 5x–7y+11z+4=0

IV) LIMITS & CONTINUITY

1) DIRECT SUBSTITUTION

160) Compute $\lim_{x\to 1}(x^2+2x+3)$

161) Evaluate $\lim_{x\to 1}(x+2)(2x+1)$

162) Compute $\lim_{x\to 0}\frac{1}{x^2-3x+2}$

163) Compute $\lim_{x\to 3}\frac{1}{x+1}$

164) Compute $\lim_{x\to 1}\frac{2x+1}{3x^2-4x+5}$

165) Compute $\lim_{x\to 1}\frac{x^2+2}{x^2-2}$

166) Compute $\lim_{x\to 0}\left(\frac{x-1}{x^2+4}\right)$

167) Evaluate $\lim_{x\to -3}\frac{1}{x+2}$

168) Evaluate $\lim_{x\to 2}\frac{x^2-5}{4x+10}$

169) Compute $\lim_{x\to 0} x^{\frac{3}{2}},\ (x>0)$

170) Compute $\lim_{x\to 0}\left(\sqrt{x}+x^{\frac{5}{2}}\right),\ (x>0)$

171) Compute $\lim_{x\to 3}\frac{x^2-9}{x^3-6x^2+9x+1}$

172) Compute $\lim_{x\to 2}\left(\frac{2}{x+1}-\frac{3}{x}\right)$

2) FACTORISATION & RATIONALISATION

173) Compute $\lim_{x\to a}\frac{x^2-a^2}{x-a}$

174) Evaluate $\lim_{x\to 2}\frac{x-2}{x^3-8}$

175) Evaluate $\lim_{x\to 3}\frac{x^2-8x+15}{x^2-9}$ **[AP, TS M 16][TS J 18]**

176) Evaluate $\lim_{x\to 3}\frac{x^3-3x^2}{x^2-5x+6}$

177) Evaluate $\lim_{x\to 3}\frac{x^3-6x^2+9x}{x^2-9}$

178) Evaluate $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$

179) Evaluate $\lim_{x\to 1}\frac{(2x-1)(\sqrt{x}-1)}{(2x^2+x-3)}$

180) Evaluate $\lim_{x\to 2}\frac{2x^2-7x-4}{(2x-1)(\sqrt{x}-2)}$

181) Evaluate $\lim_{x\to 2}\left\{\frac{1}{x-2}-\frac{4}{x^2-4}\right\}$ **[AP J 16]**

182) Evaluate $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$ **[TS 22]**

ANSWERS

160)6 **161)**9 **162)**1/2 **163)**1/4 **164)**3/4 **165)**–3
166)–1/4 **167)**–1 **168)** –1/18 **169)**0 **170)**0 **171)**0
172)–5/6 **173)**2a **174)**1/12 **175)**–1/3 **176)**9 **177)**0
178)108/7 **179)**1/10 **180)** $\frac{5(2+\sqrt{2})}{3}$ **181)**1/4 **182)**1/2

3)STANDARD LIMITS (i) $\lim_{x\to0}\frac{e^x-1}{x}=1$ (ii) $\lim_{x\to0}\frac{a^x-1}{x}=\log_e a$ (iii) $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$

183) Show that $\lim_{x\to0}\frac{e^{3x}-1}{x}=3$ **[TS J 15][AP M 18]** **184)**Evaluate $\lim_{x\to0}\frac{e^{7x}-1}{x}$ **[AP M 17]**

185) Evaluate $\lim_{x\to0}\frac{e^{\sin x}-1}{x}$ **186)**Evaluate $\lim_{x\to0}\frac{a^x-1}{b^x-1}$ **[TS M 19][AP M 15]**

187) Evaluate $\lim_{x\to0}\frac{e^x-1}{\sqrt{1+x}-1}$ **[TS J 18] [AP 16,20] [TS M 15]**

188) Evaluate $\lim_{x\to0}\frac{3^x-1}{\sqrt{1+x}-1}$ **[AP 23]** **189)**Evaluate $\lim_{x\to0}\frac{e^{3+x}-e^3}{x}$ **[AP M 19][MP]**

190) Evaluate $\lim_{x\to3}\frac{e^x-e^3}{x-3}$ **191)**Evaluate $\lim_{x\to1}\frac{\log_e x}{x-1}$ **[TS J 17]**

192) Compute $\lim_{x\to0}\frac{\log_e(1+5x)}{x}$ **[TS M 19]** **193)** Evaluate $\lim_{x\to0}\frac{(1+x)^{3/2}-1}{x}$

194) Evaluate $\lim_{x\to0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$ **195)** Evaluate $\lim_{x\to0}\frac{(1+x)^{1/8}-(1-x)^{1/8}}{x}$

4) LIMITS AT INFINITY & INFINITE LIMITS

196) Compute $\lim_{x\to\infty}\frac{x^2+5x+2}{2x^2-5x+1}$ **[AP M 17]** **197)** Evaluate $\lim_{x\to\infty}\frac{11x^3-3x+4}{13x^3-5x^2-7}$ **[AP 18,20]**

198) Evaluate $\lim_{x\to\infty}\frac{3x^2+4x+5}{2x^3+3x-7}$ **[AP J 15]** **199)** Compute $\lim_{x\to\infty}\frac{6x^2-x+7}{x+3}$

200) Compute $\lim_{x\to\infty}\frac{3x^5-1}{4x^2+1}$ **201)** Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$

202) Compute $\lim_{x\to\infty}\left(\sqrt{x+1}-\sqrt{x}\right)$ **203)** Compute $\lim_{x\to\infty}\frac{\sqrt{x^2+6}}{2x^2-1}$

ANSWERS

184)7 **185)**1 **186)** $\log_b a$ **187)**2 **188)**2log3 **189)** e^3
190) e^3 **191)**1 **192)**5 **193)**3/2 **194)**2/3 **195)**1/4
196)1/2 **197)**11/13 **198)**0 **199)** ∞ **200)**∞ **201)**1/2
202)0 **203)**0

204) Compute $\underset{x\to\infty}{Lt}\ e^{-x^2}$

205) Compute $\underset{x\to-\infty}{Lt}\ \frac{2x^2-x+3}{x^2-2x+5}$

206) Compute $\underset{x\to-\infty}{Lt}\left(\frac{2x+3}{\sqrt{x^2-1}}\right)$

207) Find $\underset{x\to-\infty}{Lt}\ \frac{5x^3+4}{\sqrt{2x^4+1}}$

208) Compute $\underset{x\to2}{Lt}\ \frac{x^2+2x-1}{x^2-4x+4}$

209) Evaluate $\underset{x\to3}{Lt}\ \frac{x^2+3x+2}{x^2-6x+9}$ **[AP M 19]**

210) Compute $\underset{x\to1-}{Lt}\ \frac{1+5x^3}{1-x^2}$

5) LIMITS OF TRIGONOMETRIC FUNCTIONS

211) Show that $\underset{x\to0}{Lt}\ \frac{\tan x}{x}=1$

212) Evaluate $\underset{x\to1}{Lt}\ \frac{\sin(x-1)}{x^2-1}$

213) Evaluate $\underset{x\to a}{Lt}\ \frac{\tan(x-a)}{x^2-a^2}$ **[TS M 15]**

214) Compute $\underset{x\to0}{Lt}\ \frac{\sin ax}{\sin bx}$ **[TS M 18]**

215) Compute $\underset{x\to0}{Lt}\ \frac{\sin ax}{x\cos x}$ **[AP M 20]**

216) Evaluate $\underset{x\to0}{Lt}\ \frac{1-\cos 2mx}{\sin^2 nx}$ **[TS J 15]**

217) Evaluate $\underset{x\to0}{Lt}\ \frac{1-\cos mx}{1-\cos nx}$

218) Evaluate $\underset{x\to0}{Lt}\ \frac{\cos ax-\cos bx}{x^2}$

219) Find $\underset{x\to0}{Lt}\ \frac{\sin(a+bx)-\sin(a-bx)}{x}$

220) Evaluate $\underset{x\to0}{Lt}\ \frac{1-\cos x}{x}$

221) Evaluate $\underset{x\to0}{Lt}\ \frac{\sec x-1}{x^2}$

222) Evaluate $\underset{x\to\pi/2}{Lt}\ \frac{\cos x}{\left(x-\frac{\pi}{2}\right)}$

223) Evaluate $\underset{x\to0}{Lt}\ \frac{x(e^x-1)}{1-\cos x}$

224) Evaluate $\underset{x\to0}{Lt}\ \frac{e^x-\sin x-1}{x}$ **[TS M J 16]**

ANSWERS

204) 0 **205)** 2 **206)** –2 **207)** –∞ **208)** ∞ **209)** ∞
210) ∞ **211)** 1/2 **212)** 1/2 **213)** 1/2a **214)** a/b **215)** a
216) $\frac{2m^2}{n^2}$ **217)** $\frac{m^2}{n^2}$ **218)** $\frac{b^2-a^2}{2}$ **219)** 2b.cosa **220)** 0 **221)** 1/2
222) –1 **223)** 2 **224)** 0

225) Find $\lim_{x\to a}\left(\frac{x\sin a - a\sin x}{x-a}\right)$

226) Find $\lim_{x\to a}\left[\frac{\sin(x-a)\tan^2(x-a)}{(x^2-a^2)^2}\right]$

227) Evaluate $\lim_{x\to 0}\frac{\log(1+x^3)}{\sin^3 x}$

228) Compute $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$

6) MISCELLANEOUS MODELS

229) Show that $\lim_{x\to 0+}\left(\frac{2|x|}{x}+x+1\right)=3$. **[MP][AP M 15]** **230)** Show that $\lim_{x\to 2-}\frac{|x-2|}{x-2}=-1$ **[TS 22]**

231) Find $\lim_{x\to\infty}\frac{8|x|+3x}{3|x|-2x}$ **[TS 17,20][AP 22]**

232) If $f(x)=\frac{|x|}{x}$ then show that $\lim_{x\to 0} f(x)$ does not exist.

233) Compute $\lim_{x\to 2+}([x]+x)$ and $\lim_{x\to 2-}([x]+x)$ **[AP 23] [TS J 17]**

234) Let f:R→R be defined by $f(x)=\begin{cases}2x-1 & \text{if } x\le 3\\ 5 & \text{if } x\ge 3\end{cases}$ Show that $\lim_{x\to 3} f(x)=5$

235) Let f:R→R be defined by $f(x)=\begin{cases}x^2, & x\le 1\\ 2x-1 & x>1\end{cases}$, find $\lim_{x\to 1} f(x)$

236) Let f:R→R be defined by $f(x)=\begin{cases}1-x & \text{if } x\le 1\\ 1+x & \text{if } x>1\end{cases}$; find $\lim_{x\to 1} f(x)$

237) Find whether the limit of f(x) exists or not at x =2, where $f(x)=\begin{cases}\frac{x}{2}, & \text{if } x<2\\ \frac{x^2}{3} & \text{if } x\ge 2\end{cases}$

238) Find whether the limit of f(x) exists or not at x =3, where $f(x)=\begin{cases}x+2, \text{if } -1<x\le 3\\ x^2 \quad \text{if } 3<x<5\end{cases}$

239) Find $\lim_{x\to 2} f(x)$ where $f(x)=\begin{cases}x^2 & \text{if } x\le 1\\ x & \text{if } 1<x\le 2\\ x-3 & \text{if } x>2\end{cases}$ **240)** Find $\lim_{x\to 0} f(x)$ where $f(x)=\begin{cases}x-1 & \text{if } x<0\\ 0 & \text{if } x=0\\ x+1 & \text{if } x>0\end{cases}$

ANSWERS

225) sina–acosa **226)** 0 **227)**1 **228)**π **231)**11

233)4,3 **235)** 1 **236)**does not exist. **237)**does not exist

238)does not exist **239)**does not exist. **240)**does not exist.

241) Find the right and left hand limit of $f(x)=\begin{cases}1 & \text{if } x<0\\ 2x+1 & \text{if } 0\le x<1;\\ 3x & \text{if } x>1\end{cases}$ for $x=1$

242) If $f(x)=-\sqrt{25-x^2}$ then find $\underset{x\to 1}{Lt}\dfrac{f(x)-f(1)}{x-1}$

243) If $f(x)=\sqrt{1+2x}$, $x\in\left[-\dfrac{1}{2},\infty\right)$ then compute $\underset{x\to\left(\frac{-1}{2}\right)+}{Lt} f(x)$. Hence find $\underset{x\to\frac{-1}{2}}{Lt} f(x)$

244) Show that $\underset{x\to(-2)-}{Lt}\sqrt{x^2-4}=0=\underset{x\to 2+}{Lt}\sqrt{x^2-4}$

245) Evaluate $\underset{\mathbf{x\to 0}}{\mathbf{Lt}}\ \mathbf{x^2 sin}\dfrac{1}{x}$

246) Compute $\underset{x\to 0}{Lt}\ x^2\cos\dfrac{2}{x}$

247) Show that $\underset{x\to 0-}{Lt}\ x^3\cos\dfrac{3}{x}=0$

248) Show that $\underset{x\to 3}{Lt}\dfrac{x-3}{\sqrt{|x^2-9|}}=0$

249) Evaluate $\underset{x\to\infty}{Lt}\dfrac{x^2-\sin x}{x^2-2}$

250) Compute $\underset{x\to\infty}{Lt}\dfrac{2+\sin x}{x^2+3}$

251) Compute $\underset{x\to\infty}{Lt}\dfrac{2+\cos^2 x}{x+2007}$

252) Compute $\underset{x\to-\infty}{Lt}\dfrac{6x^2-\cos 3x}{x^2+5}$

253) Compute $\underset{x\to\infty}{Lt}\dfrac{\cos x+\sin^2 x}{x+1}$

254) Evaluate $\underset{x\to 1}{Lt}\left[\dfrac{x-1}{x^2-x}-\dfrac{1}{x^3-3x^2+x}\right]$

255) Find $\underset{x\to a}{Lt}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

256) Evaluate $\underset{x\to 0}{Lt}\dfrac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$

ANSWERS

241) L.H.L=3=R.H.L. **242)** $\dfrac{1}{\sqrt{24}}$ **243)** 0 **245)** 0 **246)** 0 **249)** 1

250) 0 **251)** 0 **252)** 6 **253)** 0 **254)** 2 **255)** $\dfrac{2}{3\sqrt{3}}$ **256)** 1/2

7) CONTINUITY

257) Is f defined by $f(x)=\begin{cases}\frac{\sin 2x}{x} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$ continuous at 0? **[TS J 16][AP J 17]**

258) Is the function f, defined by $f(x)=\begin{cases}x^2 & \text{if } x\le 1\\ x & \text{if } x>1\end{cases}$ continuous on R? **[AP J 15]**

259) Is f given by $f(x)=\begin{cases}\frac{x^2-9}{x^2-2x-3} & \text{if } 0<x<5 \text{ and } x\neq 3\\ 1.5 & \text{if } x=3\end{cases}$, continuous at the point 3?

260) If f is given by $f(x)=\begin{cases}k^2x-k & \text{if } x\ge 1\\ 2 & \text{if } x<1\end{cases}$ is a continuous function on R, then find k.

261) Check the continuity of $f(x)=\begin{cases}\frac{1}{2}(x^2-4) & \text{if } 0<x<2\\ 0 & \text{if } x=2\\ 2-8x^{-3} & \text{if } x>2\end{cases}$ at 2.

262) Discuss the continuity of $f(x)=\begin{cases}\frac{x-1}{\sqrt{x}-1} & \text{if } x>1\\ 5-3x & \text{if } -2\le x\le 1\\ \frac{6}{x-10} & \text{if } x<-2\end{cases}$

263) Show that f(x)=sinx is continuous on R.

264) Show that $f(x)=\frac{x-|x|}{x}$, $(x\neq 0)$, is continuous on R\{0}

265) Show that the function f(x)=$[\cos(x^{10}+1)]^{1/3}$,x∈R is a continuous function.

266) Define continuous function at a point.

ANSWERS

254)0 **255)**6 **256)** 0 **257)** not continous at 0 **258)** continuous on R

259) continuous at 3 **260)**k=2 or –1 **261)** not continuous at 2 **262)** not continuous at –2

266) A function f(x) is said to be continuous at x =a if(i) f(a) exists (ii) $f(a)=\lim_{x\to a+} f(x)=\lim_{x\to a-} f(x)$

V) DIFFERENTIATION

1) DERIVATIVE OF COMPOSITE FUNCTIONS

267) If $f(x)=(ax+b)^n$, then find $f'(x)$ **268)** Find the derivative of $\sqrt{x}+2x^{\frac{3}{4}}+3x^{\frac{5}{6}}$

269) Find the derivative of $(x^3+6x^2+12x-13)^{100}$. **270)** If $f(x)=1+x+x^2+....+x^{100}$ find $f'(1)$. **[TS M 19]**

271) Find the derivative of $\sqrt{2x-3}+\sqrt{7-3x}$ **[TS M 15]** **272)** Find the derivative of $\cot^n x$

273) Find the derivative of $\text{cosec}^4 x$ **274)** Find the derivative of $\frac{1-\cos 2x}{1+\cos 2x}$ **[AP 20]**

275) If $f(x) = \sin(\log x)$ then find $f'(x)$ **[AP 18,22]** **276)** Find the derivative of $y = \cos(\log x+e^x)$ **[AP 22]**

277) Find the derivative of $\tan(e^x)$ **278)** If $y = \log(\tan 5x)$, then find $\frac{dy}{dx}$

279) Find the derivative of $\log(\sec x+\tan x)$ **[IPE'14] [AP J 17]**

280) Find the derivative of $\log\left(\frac{x^2+x+2}{x^2-x+2}\right)$ **281)** If $y=\log(\cosh 2x)$, then find $\frac{dy}{dx}$

282) Find the derivative of $\log(\sin(\log x))$ **283)** Find the derivative of $y = \sin[\cos(x^2)]$

284) Find the derivative of $y=\cos[\log(\cot x)]$. **285)** Find $\frac{d}{dx}(\sec\sqrt{\tan x})$]

286) Find the derivative of $\log(\cot(1-x^2))$ **287)** Find the derivative of $y=\log_7(\log x)$

288) Find the derivative of 7^{x^3+3x}. **[AP J 16]** **289)** Find the derivative of $20^{\log(\tan x)}$.

ANSWERS

267) $an(ax+b)^{n-1}$ **268)** $\frac{1}{2}\left[x^{-1/2}+3x^{-1/4}+5x^{-1/6}\right]$

269) $100(x^3+6x^2+12x-13)^{99}(3x^2+12x+12)$ **270)** 5050 **271)** $\frac{1}{\sqrt{2x-3}}-\frac{3}{2\sqrt{7-3x}}$

272) $-n\cot^{n-1}x\,\text{cosec}^2 x$ **273)** $-4\,\text{cosec}^4 x\cot x$ **274)** $2\tan x\sec^2 x$ **275)** $\frac{\cos(\log x)}{x}$

276) $-\sin(\log x+e^x)\left(\frac{1}{x}+e^x\right)$ **277)** $e^x\sec^2(e^x)$ **278)** $10\,\text{cosec}\,10x$ **279)** $\sec x$

280) $\frac{2x+1}{x^2+x+2}-\frac{2x-1}{x^2-x+2}$ **281)** $2\tanh 2x$ **282)** $\frac{\cot(\log x)}{x}$

283) $-\cos[\cos(x^2)].\sin(x^2)2x$ **284)** $\sin[\log(\cot x)](\text{csc}\,x)(\sec x)$.

285) $\frac{(\sec\sqrt{\tan x})(\tan\sqrt{\tan x})\sec^2 x}{2\sqrt{\tan x}}$ **286)** $4x\,\text{cosec}(2(1-x^2))$ **287)** $\frac{\log_7 e}{x\log_e x}$

288) $7^{x^3+3x}(\log 7)(3x^2+3)$ **289)** $2\log(20).20^{\log(\tan x)}.\text{cosec}(2x)$

2) DERIVATIVES ON UV, U/V MODEL

290) Find the derivative of $f(x)=(x^2-3)(4x^3+1)$. **[APJ 15]**

291) Find the derivative of $\left(\sqrt{x}+1\right)(x^2-4x+2)$

292) Find the derivative of $(\sqrt{x}-3x)\left(x+\frac{1}{x}\right)$

293) Find the derivative of $f(x)=e^x(x^2+1)$.

294) Find the derivative of $(4+x^2)e^{2x}$

295) Find the derivative of $5\sin x+e^x\log x$.

296) Find the derivative of $5^x+\log x+x^3e^x$.

297) If $f(x)=e^{2x}.\log x$, $(x>0)$ then find $f'(x)$

298) Find the derivative of $y=e^{2x}\log(3x+4)$

299) Find the derivative of $y=(ax+b)^n.(cx+d)^m$.

300) Find the derivative of $\sin mx.\cos nx$.

301) Find the derivative of $y=\sin^m x.\cos^n x$.

302) Find the derivative of $e^x+\sin x\cos x$.

303) Find the derivative of $\left(\frac{1}{x}-x\right)^3 e^x$

304) If $f(x)=a^x.e^{x^2}$ then find $f'(x)$ **[MP]**

305) Find the derivative of $ax^{2n}\log x+bx^n e^{-x}$

306) If $f(x)=xe^x\sin x$ then find $f'(x)$. **[TS J 18]**

ANSWERS

290) $20x^4-36x^2+2x$

291) $\left(\sqrt{x}+1\right)(2x-4)+\frac{1}{2\sqrt{x}}(x^2-4x+2)$

292) $\frac{3}{2}\sqrt{x}+\frac{1}{x\sqrt{x}}-6x$

293) $(x+1)^2 e^x$

294) $2e^{2x}(x^2+x+4)$

295) $5\cos x+e^x\left(\frac{1}{x}+\log x\right)$

296) $5^x.\log 5+\frac{1}{x}+x^3.e^x+3x^2e^x$

297) $e^{2x}.\frac{1}{x}+\log x(e^{2x})(2)$

298) $e^{2x}\left(\frac{3}{3x+4}+2\log(3x+4)\right)$

299) $(ax+b)^{n-1}(cx+d)^{m-1}[cm(ax+b)+an(cx+d)]$

300) $m.\cos mx.\cos nx-n.\sin mx.\sin nx$

301) $m.\cos^{n+1}x.\sin^{m-1}x-n.\sin^{m+1}x.\cos^n x$

302) $e^x+\cos 2x$

303) $e^x\left(\frac{1}{x}-x\right)^3-3e^x\left(\frac{1}{x}-x\right)^2\left(1+\frac{1}{x^2}\right)$

304) $a^x.e^{x^2}(2x+\log a)$

305) $ax^{2n-1}+2anx^{2n-1}\log x-bx^n e^{-x}+bnx^{n-1}e^{-x}$

306) $e^x(x\cos x+x\sin x+\sin x)$

307) If $y=x^2e^x\sin x$, then find $\frac{dy}{dx}$

308) If $f(x)=x^2 2^x \log x$, find $f'(x)$

309) Find the derivative of $x^n n^x \log(nx)$

310) If $y=\frac{2x+3}{4x+5}$ then find $\frac{dy}{dx}$ **[AP J 15]**

311) Find the derivative of $y=\frac{ax+b}{cx+d}, [|c|+|d|\neq 0]$

312) If $y=\frac{a-x}{a+x}, (x\neq -a)$ then find $\frac{dy}{dx}$ **[TS M 18]**

313) Find the derivative of $\frac{1-x\sqrt{x}}{1+x\sqrt{x}}$

314) Find the derivative of $y=\frac{1}{ax^2+bx+c}$

315) Find the derivative of $y=\frac{px^2+qx+r}{ax+b}$

316) Find the derivative of $y=\frac{\cos x}{\sin x+\cos x}$

317) Find the derivative of $y=\frac{\sin(x+a)}{\cos x}$

318) Find the derivative of $y=\frac{\sin(ax+b)}{\cos(cx+d)}$

319) If $f(x)=\sqrt{\frac{1+x^2}{1-x^2}}$, then find $f'(x)$

320) Find the derivative of $f(x)=\frac{x\cos x}{\sqrt{1+x^2}}$

321) If $y=\frac{x\,\mathrm{Sin}^{-1}x}{\sqrt{1-x^2}}$ find $\frac{dy}{dx}$

322) Find the derivative of $\frac{x(1+x^2)}{\sqrt{1-x^2}}$

ANSWERS

307) $x^2e^x\sin x\left(\cot x+1+\frac{2}{x}\right)$

308) $x\,2^x[\log x^2+x\log x\log 2+1]]$

309) $x^{n-1}n^x.(1+n\log(nx)+x\log n.\log(nx))$

310) $2(4x+5)^{-2}$

311) $\frac{ad-bc}{(cx+d)^2}$

312) $\frac{-2a}{(a+x)^2}$

313) $\frac{-3\sqrt{x}}{(1+x\sqrt{x})^2}$

314) $\frac{-(2ax+b)}{(ax^2+bx+c)^2}$

315) $\frac{apx^2+2bpx+(bq-ar)}{(ax+b)^2}$

316) $\frac{-1}{1+\sin 2x}$

317) $\frac{\cos a}{\cos^2 x}$

318) $\frac{a.\cos(ax+b).\cos(cx+d)+c.\sin(ax+b)\sin(cx+d)}{\cos^2(cx+d)}$

319) $\frac{2x}{(\sqrt{1-x^4})(1-x^2)}$

320) $(1+x^2)^{-3/2}[\cos x-x(1+x^2)\sin x]$

321) $\frac{1}{(1-x^2)^{3/2}}\left[x\sqrt{1-x^2}+\mathrm{Sin}^{-1}x\right]$

322) $\frac{1+3x^2-2x^4}{(1-x^2)^{3/2}}$

3) TRIGONOMETRIC & INVERSE TRIGONOMETRIC FUNCTIONS

323) If $y=\text{Sin}^{-1}(\cos x)$ then find $\frac{dy}{dx}$ **[TS J 16]**

324) If $y=\text{Sin}^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$

325) Find derivative of $\text{Sin}^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.to x **[TS M,J 15]**

326) Find the derivative of $\text{Sin}^{-1}(3x-4x^3)$ w.r.to x. **[TS M 16]**

327) Find the derivative of $y=e^{\text{Sin}^{-1}x}$ **[AP J 18]**

328) If $y = e^{a\sin^{-1}x}$ then prove that $\frac{dy}{dx}=\frac{ay}{\sqrt{1-x^2}}$ **[TS J 18]**

329) Find the derivative of $\log(\text{Sin}^{-1}e^x)$

330) Find the derivative of $y = (\sin x)^2(\text{Sin}^{-1}x)^2$.

331) Find the derivative of $\text{Cos}^{-1}(4x^3-3x)$ w.r.to x. **[IPE'14]**

332) Find the derivative of $\text{Sec}^{-1}\left(\frac{1}{2x^2-1}\right)$ **[AP M 17]**

333) **If** $y=\text{Cosec}^{-1}\left(e^{2x+1}\right)$, find $\frac{dy}{dx}$

334) If $y=\text{Tan}^{-1}(\log x)$ then find $\frac{dy}{dx}$ **[AP M 19] [TS J 15]**

335) Find the derivative of $\text{Tan}^{-1}(\sec x+\tan x)$.

336) Find the derivative of $\text{Tan}^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.to x **[AP M 15]**

ANSWERS

323) -1 **324)** $\frac{1}{2\sqrt{x-x^2}}$ **325)** $\frac{2}{1+x^2}$ **326)** $\frac{3}{\sqrt{1-x^2}}$ **327)** $\frac{e^{\text{Sin}^{-1}x}}{\sqrt{1-x^2}}$

329) $\frac{e^x}{\text{Sin}^{-1}e^x\sqrt{1-e^{2x}}}$ **330)** $(\text{Sin}^{-1}x)^2(\sin 2x)+2\sin^2 x.\frac{\text{Sin}^{-1}x}{\sqrt{1-x^2}}$ **331)** $\frac{-3}{\sqrt{1-x^2}}$

332) $\frac{-2}{\sqrt{1-x^2}}$ **333)** $\frac{-2}{\sqrt{\left(e^{2x+1}\right)^2-1}}$ **334)** $\left(\frac{1}{1+\log^2 x}\right)\frac{1}{x}$ **335)** $1/2$ **336)** $\frac{2}{1+x^2}$

337) If $y = \text{Tan}^{-1}(\cos\sqrt{x})$ then find $\frac{dy}{dx}$

338) Find the derivative of $\text{Tan}^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$.**[AP J 18]**

339) Find the derivative of $\text{Tan}^{-1}\left(\frac{a-x}{1+ax}\right)$

340) Find the derivative of $y=\sin[\text{Tan}^{-1}(e^x)]$

341) Find the derivative of $y = \text{Tan}^{-1}\left(\tanh\frac{x}{2}\right)$

342) Find the derivative of $\text{Tan}^{-1}\left(\frac{3a^2x-x^3}{a(a^2-3x)^2}\right)$

343) Find the derivative of $y = x\,\text{Tan}^{-1}x$.

344) Find the derivative of $y=\sin x.(\text{Tan}^{-1}x)^2$

345) Find the derivative of $y=\text{Cot}^{-1}(\text{cosec}3x)$

346) If $y = (\text{Cot}^{-1}x^3)^2$ then find $\frac{dy}{dx}$ **[TS M 18][IPE 13]**

347) Find the derivative of $\text{Sinh}^{-1}\left(\frac{3x}{4}\right)$

348) Find the derivative of $y = \text{Sinh}^{-1}\left(\frac{1-x}{1+x}\right)$.

ANSWERS

337) $-\frac{\sin\sqrt{x}}{2\sqrt{x}(1+\cos^2\sqrt{x})}$ **338)** 1/2 **339)** $-\frac{1}{1+x^2}$ **340)** $\cos[\tan^{-1}(e^x)]\frac{e^x}{1+e^{2x}}$

341) $\frac{\text{sech}^2\frac{x}{2}}{2\left(1+\tanh^2\frac{x}{2}\right)}$ **342)** $\frac{3a}{x^2+a^2}$ **343)** $\frac{x}{1+x^2}+\text{Tan}^{-1}x$

344) $\frac{2\sin x\,\text{Tan}^{-1}x}{1+x^2}+\cos x.(\text{Tan}^{-1}x)^2$ **345)** $\frac{3\,\text{cosec}3x.\cot 3x}{1+\text{cosec}^2 3x}$ **346)** $\frac{-6x^2\cot^{-1}x^3}{1+x^6}$

347) $\frac{3}{\sqrt{9x^2+16}}$ **348)** $\frac{-\sqrt{2}}{(1+x)\sqrt{1+x^2}}$

4) LOGARITHMIC DIFFERENTIATION

349) Find the derivative of x^x.

350) Find the derivative of $y=x^y$

351) If $y=(\tan x)^{\sin x}$ then find $\frac{dy}{dx}$

352) Find the derivative of $(\log x)^{\tan x}$

353) If $y = (x^x)^x$ = then find dy/dx

5) DIFFERENTIATION OF IMPLICIT & PARAMETRIC FUNCTIONS

354) Find $\frac{dy}{dx}$ if $x^3+y^3-3axy=0$

355) Find $\frac{dy}{dx}$ if $2x^2-3xy+y^2+x+2y-8=0$ **[TS M 16]**

356) Find the derivative of $x^4+y^4-a^2xy=0$ w.r.to x

357) Find $\frac{dy}{dx}$ if $x=a\cos^3 t$, $y=a\sin^3 t$. **[AP M 16] [IPE 12]**

358) If $y=e^t+\cos t$, $x=\log t+\sin t$ then find $\frac{dy}{dx}$ **[AP J 17]**

359) If $x=3\cos t-2\cos^3 t$, $y=3\sin t-2\sin^3 t$ then find $\frac{dy}{dx}$

360) If $x = a\left[\cos t + \log\tan\left(\frac{t}{2}\right)\right]$, $y = a\sin t$ then find $\frac{\mathbf{dy}}{\mathbf{dx}}$

361) Differentiate $f(x)=e^x$ w.r.to $g(x)=\sqrt{x}$.

362) Differentiate $f(x)=e^{\sin x}$, w.r.to $g(x)=\sin x$.

363) Differentiate $f(x) =\log_a x$ w.r.to $g(x) = a^x$.

ANSWERS

349) $x^x(1+\log x)$ **350)** $\frac{y^2}{x(1-y\log x)}$ **351)** $\tan x^{\sin x}(\sec x+\cos x(\log\tan x))$

352) $(\log x)^{\tan x}\left(\frac{\tan x}{x\log x}+\log(\log x).\sec^2 x\right)$ **353)** $(x^x)^x[x+2x\log x]$ **354)** $\frac{ay-x^2}{y^2-ax}$

355) $\frac{3y-4x-1}{2y-3x+2}$ **356)** $\frac{a^2y-4x^3}{4y^3-a^2}$ **357)** $-\tan t$ **358)** $\frac{t(e^t-\sin t)}{1+t\cos t}$

359) $\cot(t)$ **360)** $\tan(t)$ **361)** $2\sqrt{x}.e^x$ **362)** $e^{\sin x}$ **363)** $\frac{1}{x.a^x(\log_e a)^2}$

6) SECOND DERIVATIVES & MISCELLANEOUS

364) If $f(x)=2x^2+3x-5$, then prove that $f'(0)+3f'(-1)=0$. **[AP M 16]**

365) Find derivative of $y= ae^{nx}+be^{-nx}$, then prove that $y''=n^2y$ **[AP M 15]**

366) If $y=ax^{n+1}+bx^{-n}$ then show that $x^2y''=n(n+1)y$ **[AP J 16]**

367) If $y=x^4+\tan x$ then find y''

368) If $\mathbf{y=\frac{2x+3}{4x+5}}$ then find y'' **[AP M 19]**

369) Find the second order derivative of $y=\text{Tan}^{-1}\left(\frac{2x}{1-x^2}\right)$

370) If $y=\sin(\sin x)$ then show that $y''+(\tan x)y'+y\cos^2 x=0$.

371) Show that the derivative of a constant function in an interval is zero.

372) Show that $f(x)=|x|$ $(x\in R)$ is not differentiable at zero

373) Check whether the function $f(x)=\begin{cases}3+x & \text{if } x\geq 0\\ 3-x & \text{if } x<0\end{cases}$ is differentiable at zero or not

374) If the function $\mathbf{f(x)=\begin{cases}x, \text{ if } 0\leq x\leq 2\\ 2, \text{ if } x\geq 2\end{cases}}$ f derivable at 2? Justify.

375) Find the derivative of $x=e^{\sinh y}$ with respect to x.

376) Find the derivative of $x=\sinh^2 y$ w.r.t x.

377) Find the derivative of $x=\tanh^2 y$, w.r.to x.

378) Find derivative of $x = \tan(e^{-y})$ with respect to x. **[TS M 17]**

379) Find the derivative of $x = \log(1+\sqrt{y})$ w.r.to x.

380) If $f(x) = \log(\tan e^x)$, then find $f'(x)$. **[TS M 19]**

ANSWERS

367) $12x^2+2\sec^2 x\tan x$ **368)** $\frac{16}{(4x+5)^3}$ **369)** $\frac{-4x}{(1+x^2)^2}$ **373)** Not differentiable at 0

374) Not derivable at 2 **375)** $\frac{1}{x\cosh y}$ **376)** $\frac{1}{2\sqrt{x+x^2}}$ if $y>0$, $\frac{-1}{2\sqrt{x+x^2}}$ if $y<0$

377) $\frac{1}{2\sqrt{x}(1-x)}$ if $y>0$, $\frac{-1}{2\sqrt{x}(1-x)}$ if $y<0$ **378)** $\frac{-e^y}{1+x^2}$ **379)** $2(\sqrt{y}+y)$ **380)** $e^x\sec e^x$

VI) APPLICATIONS OF DERIVATIVES

1) DIFFERENTIALS

381) Find Δy & dy for function $y=x^2+x$, when $x=10, \Delta x=0.1$ **[APJ 17][AP M 15,17][TS M17]**

382) If $y=x^2+3x+6$ then find Δy, dy when $x=10$, $\Delta x=0.01$. **[AP 15,20]][TS M 15,22]**

383) If $y=5x^2+6x+6$, then find Δy and dy when $x=2$, $\Delta x=0.001$. **[APM16]**

384) Find Δy and dy for the function $y=1/(x+2)$ when $x=8$, $\Delta x=0.02$.

385) Find Δy and dy for the function $y=e^x+x$ when $x=5$, $\Delta x=0.02$ **[IPE 13]**

386) If $y=\cos x$ then find Δy and dy when $x=60^\circ$ and $\Delta x=1^\circ=0.0174$ rad **[TS M 19]**

2) APPROXIMATE VALUES

387) Find the approximate value of $\sqrt{82}$

388) Find the approximate value of $\sqrt{25.001}$

389) Find the approximate value of $\sqrt[4]{17}$ **[TS J 17]**

390) Find the approximate value of $\sqrt[3]{65}$ **[AP J 18][MP]**

391) Find the approximate value of $\sqrt[3]{7.8}$

392) Find the approximate value of $\sqrt[3]{999}$

393) Find the approximate value of $\sin 62^\circ$

394) Find the approximate value of $\cos(60^\circ 5')$

395) If the increase in the side of a square is 2% then find the approximate percentage of increase in the area of the square. **[TS M 18]**

396) If the increase in the side of a square is 4% then find the approximate percentage of increase in the area of the square. **[AP, TS J 16][AP M 18][TS J 18]**

397) The side of a square is increased from 3 cm to 3.01 cm. Find the approximate increase in the area of the square.

398) If the radius of a sphere is increased from 7 cm to 7.02 cm. then find the approximate increase in the volume of the sphere.

399) The radius of a sphere is measured as 14cm. Later it was found that there is an error 0.02cm in measuring the radius. Find the approximate error in surface of the sphere.

400) The diameter of a sphere is measured to be 40 cm. If an error of 0.02cm is made in it, then find approximate errors in volume and surface area of the sphere.

ANSWERS

381) 2.11; 2.1 **382)** 0.2301; 0.23 **383)** 0.026005; 0.026 **384)** –0.0001996, –0.0002

385) $e^5(e^{0.2}-1)+0.02$; $(e^5+1)(0.02)$ **386)** –0.0152; –0.01506 **387)** 9.0555 **388)** 5.0001

389) 2.0312 **390)** 4.0208 **391)** 1.9834 **392)** 9.9967 **393)** 0.8834 **394)** 0.4987

395) 4 **396)** 8 **397)** 0.06 sq.m **398)** 12.32 cm^3 **399)** 7.0336 cm^2

400) 16π cu.cm; 1.6π sq.cm

3) TANGENTS & NORMALS

401) Find the slope of the tangent to the curve $y=5x^2$ at the point $(-1,5)$

402) Find the slope of the tangent to the curve $y=\frac{1}{x-1}$ at the point $\left(3,\frac{1}{2}\right)$

403) Find the slope of the tangent to the curve $y=3x^4-4x$ at $x=4$ **[AP 20]**

404) Find the slope of the tangent to the curve $\mathbf{y=\frac{x-1}{x-2}}$, $x \neq 2$ at $x = 10$.

405) Find the slope of the tangent to the curve $y=x^3-x+1$ at the point whose x co–ordinate is 2.

406) Find the slope of the tangent to the curve,$y = x^3-3x+2$ at the point whose x co–ordinate is 3.

407) Find the slope of the normal to the curve $x=a\cos^3\theta$, $y=a\sin^3\theta$ at $\theta=\pi/4$.**[AP 22]**

408) Find the slope of the tangent to the curve $x=a\sec\theta$, $y=a\tan\theta$ at $\theta=\frac{\pi}{6}$

409) Find the slope of the normal to the curve $x=1-a\sin\theta$, $y=b\cos^2\theta$ at $\theta=\pi/2$

410) Find the equations of the tangent and normal to the curve $y=x^3$ at $(1,1)$

411) Find the equations of the tangent and the normal to the curve $y = 5x^4$ at the point $(1, 5)$**[AP J 15]**

412) Find the equations of the tangent and normal to the curve $y=x^2$ at $(0,0)$

413) Find the equations of tangent and normal to the curve $y=x^3+4x^2$ at $(-1,3)$

414) Find the equation of tangent and normal to the curve $y = x^4-6x^3+13x^2-10x+5$ at $(0,5)$.

415) Find the equations of tangent and normal to the curve $y=x^2-4x+2$ at $(4,2)$

416) Find the equations of the tangent and the normal to the curve $y^4=ax^3$ at (a,a)

417) Find the equation of tangent and normal to the curve $y=\frac{1}{1+x^2}$ at $(0,1)$.

418) Find the equations of tangent and normal to the curve $xy=10$ at $(2,5)$

ANSWERS

401) -10 **402)** $-\frac{1}{4}$ **403)** 764 **404)** $-\frac{1}{64}$ **405)** 11 **406)** 24 **407)** 1

408) 2 **409)** $\frac{-a}{2b}$ **410)** $3x-y-2=0, x+3y-4=0$ **411)** $20x-y-15=0, x+20y-101=0$

412) $y=0$; $x=0$ **413)** Equation of tangent $5x+y+2=0$, Equation of normal $x-5y+16=0$

414) $10x+y-5=0, x-10y+50=0$

415) Equation of tangent $4x-y-14=0$, Equation of normal $x+4y-12=0$

416) Equation of tangent $4y=3x+a$, Equation of normal $3y+4x=7a$ **417)** $y=1$; $x=0$

418) $5x+2y-20=0$; $2x-5y+21=0$

419) Find the equations of the tangent and normal to the curve x=cost, y=sint at t=π/4

420) Find the points at which the tangent to the curve $y = x^3-3x^2-9x+7$ is parallel to the x-axis.

421) Find the point on the curve $y = x^3-11x+5$ at which the tangent is y = x–11

422) On the curve $y=x^2$, find a point at which the tangent is parallel to the chord joining (0,0), (1,1)

423) Find a point on the curve $y=x^3$, where the tangent is parallel to the chord joining (1,1), (3,27)

424) Find a point on the graph of the curve $y=(x-3)^2$, where the tangent is parallel to the chord joining (3,0) and (4,1)

425) Find a point on the curve $y=(x-2)^2$ at which the tangent is parallel to the chord joining the points (2,0) and (4,4).

426) Show that the tangent at any point θ on the curve x=csecθ, y=ctanθ is ysinθ=x–ccosθ.

427) Find the equations of all lines having slope 0 which are tangents to the curve $y = \frac{1}{x^2-2x+3}$

428) Find the points at which the curve y=sinx has horizontal tangents.

4) LENGTH OF SUBTANGENTS & SUB NORMALS

429) Show that the length of the subtangent at any point on the curve $y=a^x$ (a>0) is a constant.

430) Show that the length of the subnormal at any point on $y^2=4ax$ is constant

431) Find the length of subtangent at a point on the curve $\mathbf{y = b\sin\left(\frac{x}{a}\right)}$.

432) Show that at any point (x,y) on the curve $y = be^{\frac{x}{a}}$, the length of the subtangent is a constant and the lengths of the subnormal is $\frac{y^2}{a}$.

433) Show that the length of the subnormal at any point on thecurve $xy=a^2$ varies as the curve of the ordinate of the point.

434) Find the value of k, so that the length of the subnormal at any point on the curve $y=a^{1-k}x^k$ is a constant

435) Find the value of k so that the length of the subnormal at any point on the curve $xy^k=a^{k+1}$ is a constant.

ANSWERS

419) $x+y=\sqrt{2}$, $x-\frac{1}{\sqrt{2}}$ **420)** (3,–20),(–1,12) **421)** (2,–9)

422) $\left(\frac{1}{2},\frac{1}{4}\right)$ **423)** $\left(\frac{\sqrt{39}}{3},\frac{13\sqrt{39}}{9}\right)$ **424)** $\left(\frac{7}{2},\frac{1}{4}\right)$ **425)** (3,1) **427)** y=1/2

428) $[x_0=(2n+1)\frac{\pi}{2}, y_0=(-1)^n$ for $n\in Z]$ **431)** |a tanx/a| **434)** k = 1/2 **435)** k = –2

5) ROLLE'S & LAGRANGE'S THEOREM

436) Verify Rolle's theorem for the function x^2-1 on $[-1,1]$ **[AP 23] [IPE 13]**

437) Verify Rolle's theorem for $y=f(x)=x^2+4$ on $[-3,3]$ **[TS M,J 15] [AP 17] [TS 19]**

438) Verify Rolle's theorem for the function $f(x)=x^2-5x+6$ in the interval $[-3,8]$ **[TS M 17][AP J 16]**

439) Verify Lagrange's mean value theorem for the function $f(x) = x^2$ on $[2,4]$

440) Verify the conditions of Lagrange's mean value theorem for the function x^2-1 on $[2,3]$ **[TS M 18][AP M 16]**

441) Verify Lagrange's mean value theorem for the function $\log x$ on $[1,2]$

442) Verify Rolle's theorem of the function $\log(x^2+2)-\log 3$ on $[-1,1]$ **[AP M 15]**

443) Verify Rolle's theorem for the function $f(x) = x(x+3)\,e^{-x/2}$ on $[-3,0]$ **[TS J 16][AP M 18]**

444) Let $f(x)=(x-1)(x-2)(x-3)$ then prove that there is more thanone 'c' in (1,3) such that $f'(c)=0$

445) Find c so that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ where $f(x) = e^x$, a=0, b=1

6) INCREASING, DECREASING FUNCTIONS

446) Find the intervals on which $f(x)=x^2-3x+8$ is increasing or decreasing. **[AP 22]**

447) Find the interval in which the function x^2+2x-5 is strictly increasing or strictly decreasing.

448) Find the intervals in which the function $f(x)= x^3-3x^2+4$ is strictly increasing for all $x\in R$

449) Find the intervals on which the function $f(x)=x^3+5x^3-8x+1$ is a strictly increasing function.

450) Find the interval in which the function $x^3+3x^2-6x+12$ is strictly increasing or strictly decreasing.

451) Find the intervals on which $f(x)=x^x$ is increasing and decreasing.

452) Find the interval in which the function $\ln(\ln(x))$, $x>1$ is strictly increasing or strictly decreasing.

453) Show that $f(x)=|x|$ is strictly decreasing on $(-\infty,0)$ and strictly increasing on $(0,\infty)$

454) Show that $f(x)=e^x$ is strictly increasing on R (without graph)

455) Without using derivative, show that the function $f(x)=\left(\frac{1}{2}\right)^x$ is strictly decreasing on R.

456) Show that $\tan x>x$ for every $x\in\left(0,\frac{\pi}{2}\right)$

ANSWERS

444) $2\pm\frac{1}{\sqrt{3}}$ **445)** $\log_e(e-1)$

446) f(x) is strictly decreasing on $\left(-\infty,\frac{3}{2}\right)$ and f(x) is strictly increasing on $\left(\frac{3}{2},\infty\right)$

447) $(-\infty,-1)$ strictly decreasing; $(-1,\infty)$ increasing **448)** $(-\infty,0)\cup(2,\infty)$ strictly increasing

449) Strictly decreasing $(-4,2/3)$; strictly decreasing $(-\infty,-4)$ $(2/3,\infty)$

450) $(-1-\sqrt{3},-1+\sqrt{3})$ decreasing, $(-\infty,-1-\sqrt{3})\cup(-1+\sqrt{3},\infty)$ increasing

451) f(x) is strictly decreasing on (0,1/e) and it is strictly increasing on $(1/e,\infty)$

452) $(1,\infty)$ increasing

7) MAXIMA & MINIMA

457) Use the first derivative test to find local extrema of $f(x)=x^2-6x+8$ on R

458) Use the first derivative test to find the local extrema of $f(x)=x^3-12x$ on R

459) Use the second derivative test to find local extrema of $f(x)=x^3-9x^2-48x+72$ on R

460) Use the second derivative test to find local extrema of $f(x)=-x^3+12x^2-5$ on R

461) Find the absolute extremum of $f(x)=x^2$ defined on $[-2,2]$ **[AP M 19]**

462) Find the absolute maximum value and absolute minimum value of $f(x)=x^3$ on $[-2,2]$

463) Find the absolute maximum and absolute minimum $f(x)=2x^3-3x^2-36x+2$ on the interval $[0,5]$

464) Find the absolute extremum of $f(x)=4x-\frac{x^2}{2}$ on $\left[-2,\frac{9}{2}\right]$

465) Find the maxima or minima of $f(x)=x^3-6x^2+9x+15$.

466) Find the maxima or minima of $f(x)=\frac{x}{2}+\frac{2}{x},(x>0)$.

467) Find the points of local extrema of the function $f(x)=x^3-9x^2-48x+6 \ \forall \ x\in R$. Also find its local extrema.

468) Let $f:R\to R$ be defined by $f(x)=4x^2-4x+11$. Find the global minimum value and a point of global minimum.

469) Let $f:[-2,2]\to R$ be defined by $f(x)=|x|$. Find the global maximum of f(x) and a point of global maximum.

470) Find the global maximum and global minimum of the function $f:R\to R$ defined by $f(x)=x^2$.

ANSWERS

457) Point of Local minimum x= –3; local minimum = –1

458) Point of Local minimum x=2, local minimum = –16

Point of Local maximum x= –2, local maximum = 16

459) Local minimum x= –376;Local maximum x= 124 **460)** Local minimum x= –5;Local maximum x= 251

461) absolute maximum = 4;absolute minimum = 0 **462)** absolute minimum = –8;absolute maximum = 8

463) Absolute minimum = –79;Absolute maximum = 2

464) Absolute minimum = –10 **465)** 9, 15 **466)** 2

467) Local extrema at x = –2,8; Local minimum value = – 442 & Local maximum value =58

468) Global minimum value =10; Points of Global minimum x =1/2

469) Global maximum =2; Points of Global maximum =2,–2

470) Global minimum x=0. No global maximum.

8) MISCELLANEOUS

471) If $y = f(x) = k\,x^n$ then show that the approximate relative error (or increase) in y is n times the relative error (or increase) in x where n and k are constants.

472) The time t, of a complete oscillation of a simple pendulum of length *l* is given by $t = 2\pi\sqrt{\frac{l}{g}}$ where g is gravitational constant. Find the approximate percentage of error in t when the percentage of error in *l* is 1%. **[AP J 19]**

473) S.T there is no real number k, for which the equation $x^2-3x+k=0$ has two distinct roots in [0,1].

474) Find the angle between the curves $2y = e^{-x/2}$ and Y axis.

475) Verify whether the curve $y = f(x) = x^{1/3}$ has a vertical tangent at x=0

476) Find whether the curve $y = f(x) = x^{2/3}$ has a vertical tangent at x=0

477) Without using derivative show that the function f(x)=3x+7 is strictly increasing on R

478) Without using derivative show that the function $f(x) = \left(\frac{1}{2}\right)^x$ is strictly decreasing on R

479) Find the average rate of change of $s=f(t)=2t^2+3$ between t=2 and t=4

480) Find the rate of change of area of a circle w.r.t radius when r=5cm.

481) Define relative error and percentage error of the variable 'y' **[AP M 19]**

482) Define angle between two curves.

483) Define the strictly increasing function and strictly decreasing function on an interval. **[IPE 14]**

484) Define stationary point with example. **[AP J 19]**

ANSWERS

472) $\frac{1}{2}$ **474)** $\tan^{-1}4$ **475)** Vertical Tangent at x=0 **476)** No Vertical Tangent

479) 12 **480)** $10\pi cm^2/cm$

481) Relative Error: If an error Δx occurs in x of y=f(x) then relative error in y is $\frac{\Delta y}{y}$

Percentage Error: If an error Δx occurs in x of y=f(x) then percentage error in y is $\frac{\Delta y}{y}\times 100$

482) The angle between two intersecting curves is defined as the angle between the tangents at the points of intersection of the curves

483) A real function f(x) is said to a strictly increasing function in an interval [a,b] if $x_1>x_2$ $\Rightarrow f(x_1)>f(x_2)$ for $x_1,x_2\in[a,b]$.

A real function f(x) is said to be a strictly decreasing function in an interval [a,b] if $x_1>x_2$ $\Rightarrow f(x_1)<f(x_2)$ for $x_1,x_2\in[a,b]$.

484) A point x=c in the domain of the function f is said to be a stationary point of y=f(x) if f'(c)=0

Ex: $y=x^2$ on [–2,2] has stationary point at x=0