PRACTICE MAKES A MAN PERFECT

&

PRACTICE KEEPS A STUDENT TOP

IPE SCANNER

Total Textual Bits

- **IPE SCANNER includes All Textual Solved Examples & Exercise Q's.**
- **All VSAQ in each topic are arranged in the order of Simple to Complex.**
- **Practice without peeping for solutions enhances Grip & Confidence levels.**
- **All together this IPE Scanner is a confidence booster.**
- **"Spot- Answers" are provided to avoid 'waste of time' for checking answers.**
- **This IPE Scanner is very useful to EAPCET, IIT-JEE Aspirants, because practising this IPE Scanner strengthens the basic 'funda' of the textual content, which is a strong pre-requisite for entrance exams.**

IPE SCANNER

EXCLUSIVE FOR REAL TOPPERS

VERY SHORT QUESTIONS (2 Marks)

I) FUNCTIONS

1) COMPOSITION OF FUNCTIONS

1) If $f:R\to R$, $g:R\to R$ are defined by $f(x)=3x-1$ and $g(x)=x^2+1$, then find $(fog)(2)$ **[AP M 18]**

2) If $f:R\to R$, $g:R\to R$ are defined by $f(x)=2x^2+3$ and $g(x)=3x-2$, then find (i) $(fog)(x)$ (ii) $(fof)(0)$

3) If $f:R\to R$, $g:R\to R$ are defined by $f(x)=2x^2+3$ and $g(x)=3x-2$, then find (i) $(gof)(x)$ (ii) $go(fof)(3)$.

4) If $f:R\to R$, $g:R\to R$ are defined by $f(x)=4x-1$, $g(x)=x^2+2$ then find (i) $(gof)(x)$ (ii) $fof(x)$

5) If $f:R\to R$, $g:R\to R$ are defined by $f(x)=4x-1$, $g(x)=x^2+2$ then find (i) $(gof)\left(\frac{a+1}{4}\right)$ (ii) $go(fof)(0)$

6) If $f(x)=2x-1$, $g(x)=\frac{x+1}{2}$ for all $x\in R$, find $(gof)(x)$ **[AP M 19,22, 23]**

7) If $f(x)=1/x$, $g(x)=\sqrt{x}$ for all $x\in(0,\infty)$, then find $(gof)(x)$.

8) If $f(x)=2$, $g(x)=x^2$, $h(x)=2x$ then find $(fogoh)(x)$ **[TS M 17]**

9) If $f(x)=\frac{x+1}{x-1}$, $x\neq 1$ then find (i) $(fofof)(x)$ (ii) $(fofofof)(x)$

10) If $f(y)=\frac{y}{\sqrt{1-y^2}}$, $g(y)=\frac{y}{\sqrt{1+y^2}}$ then show that $(fog)(y)=y$ **[TS J 16]**

2) FUNCTIONAL VALUES, ALGEBRAIC FUNCTIONS , FUNCTIONS AS ORDERED PAIRS

11) If the function f is defined by $f(x)=\begin{cases}3x-2, & x>3\\ x^2-2, & -2\le x\le 2\\ 2x+1, & x<-3\end{cases}$ then find the values of (i) $f(4)$, (ii) $f(2.5)$

12) If the function f is defined by $f(x)=\begin{cases}x+2, & x>1\\ 2, & -1\le x\le 1\\ x-1, & -3<x<-1\end{cases}$ then find the values of (i) $f(3)$ (ii) $f(-1.5)$

ANSWERS

1) 14 **2)** (i) $18x^2-24x+11$, (ii) 21 **3)** (i) $6x^2+7$, (ii) 2653 **4)** (i) $16x^2-8x+3$, (ii) $16x-5$

5) (i) a^2+2 (ii) 27 **6)** x **7)** $\frac{1}{\sqrt{x}}$ **8)** 2 **9)** (i) $f(x)$ (ii) x

11) 10 , not defined. **12)** 5, –2.5

13) If the function f is defined by $\mathbf{f(x)} = \begin{cases} \mathbf{x+2,\ x>1} \\ \mathbf{2,\ -1 \le x \le 1} \\ \mathbf{x-1,\ -3<x<-1} \end{cases}$ then find the values of (i) f(0) (ii) f(–5)

14) If f: R - {0}→R is defined by $f(x) = x^3 - \frac{1}{x^3}$, then show that $f(x) + f\left(\frac{1}{x}\right) = 0$ **[TS J 19][AP M22]**

15) f : R-{0} → R is defined as $f(x) = x + \frac{1}{x}$ then show that $(f(x))^2 = f(x^2) + f(1)$

16) If f : R - {± 1} →R is defined by $f(x) = \log\left|\frac{1+x}{1-x}\right|$, show that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$ **[AP J 18]**

17) If f:R→R is defined by $f(x) = \frac{1-x^2}{1+x^2}$ then show that f(tanθ)=cos2θ. **[TS M 22]**

18) If $f(x) = \frac{\cos^2 x + \sin^4 x}{\sin^2 x + \cos^4 x}\ \forall\ x \in R$ then show that f(2012) =1

19) If f(x) = 2x–1 and g(x) = x^2 then find (i) (3f–2g)(x) (ii)(fg)(x)

20) If f(x) = 2x–1 and g(x) = x^2 then find (i) (f+g+2)(x) (ii) $(\sqrt{f}/g)(x)$

21) If f = {(1,2), (2,–3), (3,–1)} then find (i) 2+f (ii) $\sqrt{f}$ **[IPE 08]**

22) If f = {(1,2), (2,–3), (3,–1)} then find (i) 2f (ii)f^2 **[IPE 12][AP M 20]**

23) If f = {(4,5),(5,6),(6,–4)}; g = {(4,–4), (6,5), (8,5)} find (i) f+g (ii) fg

24) If f = {(4,5),(5,6),(6,–4)}; g = {(4,–4), (6,5), (8,5)} find (i) f–g (ii) $\sqrt{f}$

3) ONE ONE, ONTO FUNCTIONS

25) If A={–2,–1,0,1,2} and f:A→B is a surjection defined by f(x)=x^2+x+1 then find B.
[TS 23][AP,TS M 19][AP,TS M 17][AP J 19][AP, TS J 16]

26) If $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ and f:A→B is a surjection defined by f(x)=cosx then find B.
[TS M 17][AP M 16][AP, TS J 15,17][MP]

27) Is g = {(1,1),(2,3),(3,5),(4,7)} is a function from A = {1,2,3,4} to B= {1,3,5,7} ? If this is given by the formula g(x)=ax+b, then find a and b.

28) If the function f:{–1,1}→{0,2}, defined by f(x)=ax+b is a surjection, then find a & b.

29) Determine whether the function f:R→R defined by $f(x) = \frac{2x+1}{3}$ is one one (or)onto(or)bijection.

ANSWERS

13) 2,f(–5) is undefined **19)** (i) $-2x^2+6x-3$ (ii) $2x^3-x^2$ **20)** (i) x^2+2x+1 (ii) $\frac{\sqrt{2x-1}}{x^2}$

21) 2+f={(1,4), (2,–1), (3,1)}, $\sqrt{f} = \{(1,\sqrt{2})\}$]

22) 2f={(1,4), (2,–6), (3,–2)}, $f^2 = \{(1,4), (2,9), (3,1)\}$ **23)**{(4,1), (6,1)};{(4,–20), (6,–9)}

24){(4,9), (6,–9)}; $\{(4,\sqrt{5}), (5,\sqrt{6})\}$ **25)**{3,1,7} **26)**$\{1, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}, 0\}$ **27)**Yes; a=2, b = –1

28) a=1, b=1 **29)** f is both one one and onto, hence bijective

30) If Q is the set of all rationals, $f:Q\to Q$ is defined by $f(x)=5x+4$, $\forall x\in Q$, show that f is a bijection.

31) Determine whether $f:[0,\infty)\to[0,\infty)$ defined by $f(x)=x^2$ is one one (or) onto (or) bijection.

32) Determine whether function $f:R\to[0,\infty)$ defined by $f(x)=x^2$ is one one (or) onto (or) bijection.

33) Determine whether the function $f:R\to R$ defined by $f(x)=x^2$ is one one (or) onto (or) bijection.

34) If $A=\{x: -1\le x\le 1\}$, $f(x)=x^2$ and $g(x)=x^3$, which of the following are onto?(i) $f:A\to A$(ii) $g:A\to A$

35) Determine whether the function $f:R\to(0,\infty)$ defined by $f(x)=2^x$ is one one (or) onto (or) bijection.

36) Determine whether the function $f:(0,\infty)\to R$ defined by $f(x)=\log_e x$ is one one (or)onto(or)bijection.

4) INVERSE FUNCTIONS

37) If $f:Q\to Q$ is defined by $f(x)=5x+4$, find f^{-1}. **[AP M 16]** **[IPE 12]**

38) Find the inverse of the real function of $f(x)=ax+b$, $a\neq 0$.**[TS M 18][IPE 13]**

39) Find the inverse function of $f(x)=5^x$. **[IPE 11][AP M 15,16]**

40) Find the inverse function of $f(x)=\log_2 x$

41) If $f(x)=e^x$ and $g(x)=\log_e x$, then show that fog = gof and find f^{-1} and g^{-1}.

42) If $f(x)=1+x+x^2+.....$ for $|x|<1$ then show that $f^{-1}(x)=\frac{x-1}{x}$

43) If $f(x)=\frac{x-1}{x+1}$, $x\neq\pm 1$, show that $fof^{-1}(x)=x$.

44) If $f: R\to R$, $g:R\to R$ are defined by $f(x)=3x-2$, $g(x)=x^2+1$, then find $(gof^{-1})(2)$

45) If $f:R\to R$, $g:R\to R$ are defined by $f(x)=2x-3$, $g(x)=x^3+5$ then find $(fog)^{-1}(x)$

46) If $f:[1,\infty)\to[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$ then find $f^{-1}(x)$.

ANSWERS

31) f is onto **32)** f is not one one but onto, hence f is not bijective

33) f is not one one & not onto, hence f is not bijective **34)**(i) f is not onto. (ii) g is onto

35) f is both one one and onto, hence bijective **36)** f is both one one and onto, hence bijective

37) $\frac{x-4}{5}$ **38)** $\frac{x-b}{a}$ **39)** $\log_5 x$ **40)** 2^x **41)** g,f

44) 25/9 **45)** $\left(\frac{x-7}{2}\right)^{1/3}$ **46)** $\frac{1+\sqrt{1+4\log_2 x}}{2}$

5) EVEN & ODD FUNCTIONS

47) Determine whether the function $f(x)=\log(x+\sqrt{x^2+1})$ is even or odd.

48) Determine whether the function $f(x)=x\left(\frac{e^x-1}{e^x+1}\right)$ is even or odd

49) Determine whether the function $f(x)=a^x-a^{-x}+\sin x$ is even or odd. **[AP M 22]**

50) Determine whether the function $f(x)=\sin x+\cos x$ is even or odd.

6) DOMAINS

FIND THE DOMAIN OF EACH OF THE FOLLOWING REAL FUNCTIONS

51) $f(x)=\frac{1}{2x+1}$

52) $f(x)=\frac{3^x}{x+1}$

53) $f(x)=\frac{2x^2-5x+7}{(x-1)(x-2)(x-3)}$ **[TS J 19][TS 23]**

54) $f(x)=\frac{1}{(x^2-1)(x+3)}$ **[AP J 17] [IPE 14,14] [TS 18,20,22]**

55) $f(x)=\frac{1}{6x-x^2-5}$

56) $f(x)=\sqrt{(x+2)(x-3)}$

57) $f(x)=\sqrt{x^2-3x+2}$

58) $f(x)=\sqrt{x^2-25}$ **[AP M,J 18][AP J 15]**

59) $f(x)=\sqrt{16-x^2}$

60) $f(x)=\sqrt{a^2-x^2}, a>0$

61) $f(x)=\sqrt{4x-x^2}$ **[TS M 18,19][AP J 19]**

62) $f(x)=\sqrt{(x-\alpha)(\beta-x)}, (0<\alpha<\beta)$

63) $f(x)=\log(x^2-4x+3)$ **[AP TS 16]**

64) $\frac{1}{\sqrt{x^2-a^2}}$, a>0 **[AP M 15]**

65) $f(x)=\frac{1}{\sqrt{1-x^2}}$

66) $f(x)=\frac{1}{\log(2-x)}$

67) $f(x)=\sqrt{2-x}+\sqrt{1+x}$

68) $\frac{\sqrt{2+x}+\sqrt{2-x}}{x}$ **[TS J 15]**

69) $\frac{\sqrt{3+x}+\sqrt{3-x}}{x}$

70) $f(x)=\sqrt{x+2}+\frac{1}{\log_{10}(1-x)}$

71) $f(x)=\sqrt{|x|-x}$

ANSWERS

47) odd **48)** even **49)** odd **50)** neither even nor odd **51)** $R-\{-1/2\}$

52) $R-\{-1\}$ **53)** $R-\{1, 2, 3\}$ **54)** $R-\{1,-1,-3\}$ **55)** $R-\{1,5\}$ **56)** $(-\infty,-2]\cup[3,\infty)\}$

57) $[(-\infty,1]\cup[2,\infty)]$ **58)** $[(-\infty,-5]\cup[5,\infty)]$ **59)** $[-4,4]$ **60)** $[-a,a]$

61) $[0,4]$ **62)** $[\alpha,\beta]$ **63)** $[(-\infty,1)\cup(3,\infty)]$ **64)** $(-\infty,-a)\cup(a,\infty)$

65) $[(-1,1)]$ **66)** $[(-\infty,2)-\{1\}]$ **67)** $[-1,2]$ **68)** $[-2,0)\cup(0,2]$

69) $[-3,0)\cup(0,3]$ **70)** $[-2,1)-\{0\}$ **71)** R

7) RANGES

72) If A={1,2,3,4} and f:A→R is a function defined by $f(x)=\frac{x^2-x+1}{x+1}$ then find the range of f.

73) Find the domain and range of the function $f(x)=\frac{x}{2-3x}$

74) Find the domain and range of the real valued function $f(x)=\frac{2+x}{2-x}$.

75) Find the domain and range of the function $f(x)=\frac{x}{1+x^2}$

76) Find domain and range of the real function $\frac{x^2-4}{x-2}$

77) Find the domain & range of the real function $f(x)=\sqrt{9-x^2}$ **[TS M 15,17]**

78) Find the domain & range of the real function $f(x)=\sqrt{9+x^2}$

8) MISCELLANEOUS

79) On what domain the functions $f(x)=x^2-2x$ and $g(x)=-x+6$ are equal?

80) If f: N→N is defined as f(x) =2x+5, is f onto?

81) If f:R→R is defined as f(x+y)=f(x)+f(y) ∀x,y∈R and f(1) =7, find $\sum_{r=1}^{n} f(r)$

82) Find the domain of definition of the function y(x), given by the equation $2^x+2^y=2$.

83) If f(x+y) = f(xy)∀x,y∈R then prove that f is a constant function.

84) If $f(x)=x^2$ and g(x) = |x|, find the functions. (i) f+g (ii) fg

85) Let A = {1,2,3}, B={a,b,c}, C={p,q,r}. If f:A→B, g:B→C are defined by f ={(1,a), (2,c),(3,b)}, g={(a,q),(b,r),(c,p)} then show that $f^{-1}og^{-1}=(gof)^{-1}$.

86) Let f={(1,a), (2,c), (4,d), (3,d)} and g^{-1}={(2,a),(4,b),(1,c), (3,d)} then show that $(gof)^{-1}=f^{-1}og^{-1}$.

87) If f:R→R is defined by $f(x)=\frac{3^x+3^{-x}}{2}$, then show that f(x+y)+f(x–y)=2f(x)f(y)

88) If f(x)=cos(logx), then show that $f\left(\frac{1}{x}\right).f\left(\frac{1}{y}\right)-\frac{1}{2}\left[f\left(\frac{x}{y}\right)+f(xy)\right]=0$

ANSWERS

72) $\left\{\frac{1}{2},1,\frac{7}{4},\frac{13}{5}\right\}$

73) $[R-\left\{\frac{2}{3}\right\}, R-\left\{-\frac{1}{3}\right\}]$

74) R –{2}, R – {–1}

75) R, $f=\left[\frac{-1}{2},\frac{1}{2}\right]$

76) R–{2},R–{4}

77) [–3,3], [0,3]

78) [3,∞)

79) {–2,3}

80) No

81) $\frac{7n(n+1)}{2}$

82) (–∞, 1)

84) (i) $x^2+|x|=\begin{cases}x^2+x, & x\geq 0\\ x^2-x, & x<0\end{cases}$ (ii) $fg=x^2|x|=\begin{cases}x^3, & x\geq 0\\ -x^3, & x<0\end{cases}$

89) Prove that the real valued function $f(x)=\frac{x}{e^x-1}+\frac{x}{2}+1$ is an even function on R–{0}

90) If f, g:R→R are defined $f(x)=\begin{cases}0 \text{ if } x\in Q\\ 1 \text{ if } x\notin Q\end{cases}$, $g(x)=\begin{cases}-1 \text{ if } x\in Q\\ 1 \text{ if } x\notin Q\end{cases}$ then find (fog)(π)+(gof)(e).

91) Let $f(x) = x^2$, $g(x) = 2^x$. Then solve the equation (fog)(x)=(gof)(x)

92) If $f(x) = x^2$, $g(x) = |x|$, then find (i) f+g (ii) fg

93) Find the domain of the real valued function f(x)=$\sqrt{\log_{0.3}(x-x^2)}$

94) Find the domain of f(x)=log(x–[x])

95) Find the domain of f(x)=$\sqrt{x-[x]}$

96) Find the domain of f(x)=$\sqrt{[x]-x}$

97) Find the domain of $f(x)=\frac{1}{\sqrt{|x|-x}}$

98) Find the domain of $f(x)=\frac{1}{x+|x|}$

99) Find the domain of $f(x)=\sqrt{x^2-1}+\frac{1}{\sqrt{x^2-3x+2}}$

100) Find the domain of $f(x)=\sqrt{\log_{10}\left(\frac{3-x}{x}\right)}$

101) Find the domain of $f(x)=\frac{1}{\sqrt{[x]^2-[x]-2}}$

102) Find the domain of $f(x)=\frac{1}{\sqrt[3]{(x-2)}\,\log_{(4-x)}10}$

103) Find the domain of $f(x)=\sqrt{\frac{4-x^2}{[x]+2}}$

104) Find the domain of $f(x)=\frac{\tan\pi[x]}{1+\sin\pi[x]+[x^2]}$

105) Find the domain and range of the function f(x)=|x|+|1+x|

106) Find the range of the real function $\log|4-x^2|$

107) Find the range of the real function $\sqrt{|x|-x}$

108) Find the range of the real function $\frac{\sin\pi[x]}{1+[x]^2}$

ANSWERS

90) –1 **91)** 0, 2 **92)** (i) $\begin{cases}x^2+x, & x\ge 0\\ x^2-x, & x<0\end{cases}$ (ii) $\begin{cases}x^3, & x\ge 0\\ -x^3, & x<0\end{cases}$ **93)**(0,1) **94)**R-Z **95)** R

96)Z **97)**(–∞,0) **98)**(0,∞) **99)**(–∞,1]∪[2,∞) **100)** $\left(0,\frac{3}{2}\right]$ **101)**(–∞, –1) ∪ [3,∞)

102)(–∞,4)\{2,3} **103)**(–∞,–2)∪[–1,2] **104)**(–∞,–2)∪[–1,2] **105)** R, [1,∞)

106)R **107)**R\{1,2,3} **108)**(–∞,2)\{1}

109) Define a one one function. Give one example.

110) Define an onto function. Give one example.

111) Define an even function, odd function. Give an example to each.

112) Define Constant Function.

113) Define Identity function.

114) Define Inverse function.

115) Define modulus function. What is its domain and range?

116) Define Signum function. What is its domain and range?

117) Define Greatest integer function.

ANSWERS

109) A function $f:A\to B$ is said to be a oneone function if, $a_1, a_2 \in A$ be such that $f(a_1)=f(a_2) \Rightarrow a_1=a_2$
Ex: $f:R\to R$ such that $f(x) = 2x+3$

110) A function $f:A\to B$ is said to be an onto function if $\forall\ b\in B\ \exists\ a\in A$ such that $f(a)=b$. (or)

if range of f = codomain of f then f is an onto function. **Ex:** $f:R\to R$ such that $f(x) = 2x+3$

111) A function f(x) is said to be an (i) even function if $f(-x)=f(x)$ Ex: $f(x) = x^2$.

(ii) odd function if $f(-x)=-f(x)$ Ex: $f(x) = x^3$.

112) A function $f:A\to B$ is said to be a constant function if $f(x)=k, \forall x\in A$, k is a fixed element of B.

113) A function $f:A\to A$ is said to be an identity function on A if $f(x)=x,\ \forall\ x\in A$

114) If $f:A\to B$ is a bijection, then the relation $f^{-1}=\{(b,a)|(a,b)\in f\}$ is a function from B to A and is called the inverse of f.

115) The modulus function $f:R\to R$ is defined by $f(x) = |x| = \begin{cases} x, & x\geq 0 \\ -x, & x<0 \end{cases}$.

The domain is R and range is $[0,\infty)$

116) The Signum function $f:R\to R$ is defined by $\text{sgn}(x) = f(x) = \dfrac{|x|}{x} = \begin{cases} 1, & x>0 \\ 0, & x=0 \\ -1, & x<0 \end{cases}$

The domain is R and range is $\{-1,0,1\}$

117) The function defined by $f(x)=[x]=n$, where n is an integer such that $n\leq x<n+1,\ \forall\ x\in R$, is called Greatest integer function.

II) MATRICES

1) ADDITION, EQUALITY & TRACE OF MATRICES

118) If $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$ then find x, y,z and a. **[AP J 16][TS J 18]**

119) If $\begin{bmatrix} x-1 & 2 & 5-y \\ 0 & z-1 & 7 \\ 1 & 0 & a-5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 1 & 0 & 0 \end{bmatrix}$ then find the values of x,y,z and a. **[TS M 20]**

120) If $\begin{bmatrix} x-1 & 2 & y-5 \\ z & 0 & 2 \\ 1 & -1 & 1+a \end{bmatrix} = \begin{bmatrix} 1-x & 2 & -y \\ 2 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ then find values of x,y,z and a.

121) If $A=\begin{bmatrix} -1 & 3 \\ 4 & 2 \end{bmatrix}, B=\begin{bmatrix} 2 & 1 \\ 3 & -5 \end{bmatrix}, X=\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ and A+B=X, then find the values of x_1, x_2, x_3, x_4. **[TS M 22]**

122) If $A = \begin{bmatrix} 2 & 3 & -1 \\ 7 & 8 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -4 & -1 \end{bmatrix}$ then find A+B

123) Write $\begin{bmatrix} 3 & 9 & 0 \\ 1 & 8 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 2 \\ 7 & 1 & 4 \end{bmatrix}$ as a single matrix **124)** Write $\begin{bmatrix} -1 & 2 \\ 1 & -2 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ -2 & 1 \end{bmatrix}$ as a single matrix

125) If $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 0 \\ 1 & 3 & 1 \end{bmatrix}, B = \begin{bmatrix} -3 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & -1 & 2 \end{bmatrix}$ and X=A+B then find X

126) If $A = \begin{bmatrix} -1 & -2 & 3 \\ 1 & 2 & 4 \\ 2 & -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 & 5 \\ 0 & -2 & 2 \\ 1 & 2 & -3 \end{bmatrix}$ **and** $C = \begin{bmatrix} -2 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$ **find A+B+C.**

127) If $A = \begin{bmatrix} 2 & 3 & 1 \\ 6 & -1 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ then find the matrix X such that A+B–X=0. What is the order of the matrix X?

ANSWERS

118) 8, 5, –4, 10 **119)** 2, 2, 5, 5 **120)** 1, 5/2, 2, 0 **121)** $x_1 = 1, x_2 = 4, x_3 = 7, x_4 = -3$

122) $\begin{bmatrix} 3 & 3 & 0 \\ 9 & 4 & 4 \end{bmatrix}$ **123)** $\begin{bmatrix} 7 & 9 & 2 \\ 8 & 9 & 2 \end{bmatrix}$ **124)** $\begin{bmatrix} -1 & 3 \\ 0 & -2 \\ 1 & 0 \end{bmatrix}$ **125)** $X = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -1 & 3 \\ 5 & 2 & 3 \end{bmatrix}$

126) $\begin{bmatrix} -2 & -3 & 10 \\ 2 & 1 & 8 \\ 5 & 1 & 1 \end{bmatrix}$ **127)** $X=\begin{bmatrix} 3 & 5 & 0 \\ 6 & -2 & 8 \end{bmatrix}$; Order=2x3

128) If $A=\begin{bmatrix}4 & -5\\ -2 & 3\end{bmatrix}$ then find $-5A$

129) If $A=\begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}3 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$, find 3B–2A **[TS M 17, 23]**

130) If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}3 & 8\\ 7 & 2\end{bmatrix}$ and 2X+A=B then find X.**[AP M 15]**

131) Find the additive inverse of A where $A=\begin{bmatrix}i & 0 & 1\\ 0 & -i & 2\\ -1 & 1 & 5\end{bmatrix}$

132) If $A=\begin{bmatrix}0 & 1 & 2\\ 2 & 3 & 4\\ 4 & 5 & -6\end{bmatrix}$, $B=\begin{bmatrix}-1 & 2 & 3\\ 0 & 1 & 0\\ 0 & 0 & -1\end{bmatrix}$ then find B–A and 4A–5B.

133) If $A=\begin{bmatrix}0 & 1 & 2\\ 2 & 3 & 4\\ 4 & 5 & 6\end{bmatrix}$, $B=\begin{bmatrix}1 & -2 & 0\\ 0 & 1 & -1\\ -1 & 0 & 3\end{bmatrix}$ then find (i) A–B (ii) 4B–3A .

134) Find the trace of $\begin{bmatrix}1 & 3 & -5\\ 2 & -1 & 5\\ 2 & 0 & 1\end{bmatrix}$ **[IPE 13][TS J 19][AP M 20]**

135) Find the trace of A, if $A=\begin{bmatrix}1 & 2 & -1/2\\ 0 & -1 & 2\\ -1/2 & 2 & 1\end{bmatrix}$ **[TS J 15]**

136) Construct a 3×2 matrix whose elements are defined by $a_{ij}=\frac{1}{2}|i-3j|$ **[TS M 15]**

ANSWERS

128) $\begin{bmatrix}-20 & 25\\ 10 & -15\end{bmatrix}$ **129)** $\begin{bmatrix}7 & 2 & -3\\ -3 & 2 & 7\end{bmatrix}$ **130)** $\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}$ **131)** $\begin{bmatrix}-i & 0 & -1\\ 0 & i & -2\\ 1 & -1 & -5\end{bmatrix}$

132) $B-A=\begin{bmatrix}-1 & 1 & 1\\ -2 & -2 & -4\\ -4 & -5 & 5\end{bmatrix}$, $4A-5B=\begin{bmatrix}5 & -6 & -7\\ 8 & 7 & 16\\ 16 & 20 & -19\end{bmatrix}$ **133)** $\begin{bmatrix}-1 & 3 & 2\\ 2 & 2 & 5\\ 5 & 5 & 3\end{bmatrix}$, $\begin{bmatrix}4 & -11 & -6\\ -6 & -5 & -16\\ -16 & -15 & -6\end{bmatrix}$

134) 1 **135)** 1 **136)** $A=\begin{bmatrix}1 & \frac{5}{2}\\ \frac{1}{2} & 2\\ 0 & \frac{3}{2}\end{bmatrix}$

2) MULTIPLICATION OF MATRICES

137) Find A^2, where $A=\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$ **[TS M 22]**

138) If $A=\begin{bmatrix}i & 0\\ 0 & i\end{bmatrix}$ then find A^2.

139) If $A=\begin{bmatrix}i & 0\\ 0 & -i\end{bmatrix}$ then show that $A^2=-I$ **[AP M 16,22]**

140) If $A=\begin{bmatrix}2 & 4\\ -1 & k\end{bmatrix}$, $A^2=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ find the value of k **[TS J 16][AP M 17][TS M 18,22]**

141) If $A=\begin{bmatrix}i & 0\\ 0 & -i\end{bmatrix}$, $B=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ and $C=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}$ and I is the unit matrix of order 2, then show that (i) $A^2=B^2=C^2=-I$ (ii) $AB=-BA=-C$

142) Find the product $\begin{bmatrix}-1 & 4 & 2\end{bmatrix}\begin{bmatrix}5\\ 1\\ 3\end{bmatrix}$

143) Find the product $\begin{bmatrix}2 & 1 & 4\\ 6 & -2 & 3\end{bmatrix}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}$

144) If $A=\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & 2 & 0\\ 1 & 0 & 4\end{bmatrix}$, then find AB. Find BA if it exists.

145) If $A=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}$, $B=\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}$ find AB and BA if exist? Do A and B commute with respect to multiplication?

146) If $A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 2 & 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}1 & -2\\ -1 & 0\\ 2 & -1\end{bmatrix}$ then find AB and BA.

147) If $A=\begin{bmatrix}1 & -2 & 3\\ 2 & 3 & -1\\ -3 & 1 & 2\end{bmatrix}$, $B=\begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 2\\ 1 & 2 & 0\end{bmatrix}$ then verify whether A ,B commute w.r.t multiplication.

148) Give examples of two square matrices A and B of the same order for which AB=O, but BA≠O

ANSWERS

137) $\begin{bmatrix}14 & 10\\ -5 & -1\end{bmatrix}$ **138)** $-I$ **140)** -2 **142)** $[5]$ **143)** $\begin{bmatrix}8\\ 5\end{bmatrix}$

144) $\begin{bmatrix}7 & 4 & 4\\ 6 & 2 & 12\end{bmatrix}$, BA does n't exist **145)** No, AB≠BA **146)** $AB=\begin{bmatrix}3 & -2\\ 5 & -5\\ 7 & -8\end{bmatrix}$, BA is not defined

147) AB≠BA **148)** $A=\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}$

149) Find the product $\begin{bmatrix} 3 & 4 & 9 \\ 0 & -1 & 5 \\ 2 & 6 & 12 \end{bmatrix}\begin{bmatrix} 13 & -2 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

150) Find the product $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ 6 & -2 & 3 \end{bmatrix}$

151) Find the product $\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

3) SYMMETRIC MATRICES, TRANSPOSE

152) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$ is symmetric, find value of x. **[TS J 15,18],[AP M 16][TS M 22]**

153) If $\begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ is a skew symmetric matrix then find the value of x. **[IPE 13,14][TS 23]**

154) If $\begin{bmatrix} 0 & 4 & -2 \\ -4 & 0 & 8 \\ 2 & -8 & x \end{bmatrix}$ is a skew symmetric matrix then find the value of x.

155) Is $\begin{bmatrix} 0 & 1 & 4 \\ -1 & 0 & 7 \\ -4 & -7 & 0 \end{bmatrix}$ symmetric or skew symmetric?

156) For any square matrix A, show that AA' is symmetric. **[AP M 15]**

157) If $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ then find AA'. Do A and A' commute with respect to multiplication of matrices?

158) If $A = \begin{bmatrix} -2 & 1 & 0 \\ 3 & 4 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \\ -1 & 5 \end{bmatrix}$ then find A+B'.

159) If $A = \begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$ then find 2A + B' & 3B' – A.

ANSWERS

149) Not possible **150)** Not possible **151)** O_{3x3} **152)** 6 **153)** 2 **154)** 0

155) Skew symmetric **157)** $AA' = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$, No **158)** $\begin{bmatrix} -1 & 5 & -1 \\ 5 & 7 & 0 \end{bmatrix}$ **159)** $\begin{bmatrix} -6 & 6 \\ 13 & 0 \\ -1 & 10 \end{bmatrix}$,

160) If $A=\begin{bmatrix}1 & 5 & 3\\2 & 4 & 0\\3 & -1 & -5\end{bmatrix}, B=\begin{bmatrix}2 & -1 & 0\\0 & -2 & 5\\1 & 2 & 0\end{bmatrix}$ then find 3A–4B'.

161) If $A=\begin{bmatrix}7 & -2\\-1 & 2\\5 & 3\end{bmatrix}, B=\begin{bmatrix}-2 & -1\\4 & 2\\-1 & 0\end{bmatrix}$ then find AB' & BA'

162) If $A=\begin{bmatrix}2 & -4\\-5 & 3\end{bmatrix}$ then find A+A' and AA'. **[TS M 19] [AP J 19]**

163) If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\-\sin\alpha & \cos\alpha\end{bmatrix}$, then show that AA'=A'A

164) If $A=\begin{bmatrix}1 & 4 & 7\\2 & 5 & 8\end{bmatrix}$ and $B=\begin{bmatrix}-3 & 4 & 0\\4 & -2 & -1\end{bmatrix}$ then verify that (i) (A')' = A (ii) (A+B)' = A'+B'

165) If $A=\begin{bmatrix}2 & 0 & 1\\-1 & 1 & 5\end{bmatrix}, B=\begin{bmatrix}-1 & 1 & 0\\0 & 1 & -2\end{bmatrix}$ then find (AB')' **[IPE 12]**

166) If $A=\begin{bmatrix}2 & -1 & 2\\1 & 3 & -4\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2\\-3 & 0\\5 & 4\end{bmatrix}$ then verify that (AB)'=B'A'. **[TS J 19]**

ANSWERS

160) $\begin{bmatrix}-5 & 15 & 5\\10 & 20 & -8\\9 & -23 & -15\end{bmatrix}$

161) $\begin{bmatrix}-12 & 24 & -7\\0 & 0 & 1\\-13 & 26 & -5\end{bmatrix}, \begin{bmatrix}-12 & 0 & -13\\24 & 0 & 26\\-7 & 1 & -5\end{bmatrix}$

162) $\begin{bmatrix}4 & -9\\-9 & 6\end{bmatrix}, \begin{bmatrix}20 & -22\\-22 & 34\end{bmatrix}$

165) $\begin{bmatrix}-2 & 2\\-2 & -9\end{bmatrix}$

4) DETERMINANTS

167) Find the determinant of $A = \begin{bmatrix} 1 & -1 \\ -3 & 1 \end{bmatrix}$

168) Find the determinant of $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

169) Find the determinant of $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 4 \\ -4 & -2 & 5 \end{bmatrix}$

170) Find the determinant of the matrix $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

171) Find the determinant of the matrix $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

172) Find the determinant of the matrix $\begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$ **[IPE 10]**

173) If $A=\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$, det A= 45 then find x. **[AP J 16][TS J 19]**

174) Find the Adjoint and Inverse of the matrix $\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$

175) Find the Adjoint, Inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ **[APM 18]**

176) Find the Adjoint and Inverse of the matrix $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

ANSWERS

167) -2 **168)** 1 **169)** 27 **170)** $abc + 2fgh - af^2 - bg^2 - ch^2$ **171)** $3abc - a^3 - b^3 - c^3$ **172)** -8

173) 7 **174)** $\begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 1/4 & 1/8 \\ -1/6 & 1/12 \end{bmatrix}$ **175)** $\begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}, \begin{bmatrix} \frac{5}{11} & \frac{2}{11} \\ \frac{3}{11} & \frac{-1}{11} \end{bmatrix}$ **176)** $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}, \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

177) If $A=\begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix}$, $a^2+b^2+c^2+d^2=1$, then find the inverse of A

178) Find the minors of –1 and 3 in the matrix $\begin{bmatrix} 2 & -1 & 4 \\ 0 & -2 & 5 \\ -3 & 1 & 3 \end{bmatrix}$

179) Find the cofactors of 2 and –5 in the matrix $\begin{bmatrix} -1 & 0 & 5 \\ 1 & 2 & -2 \\ -4 & -5 & 3 \end{bmatrix}$

180) If ω is complex cube root of 1 then show that $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}=0$ **[IPE 14]**

181) If AB=I or BA=I, then prove that A is invertible and $B=A^{-1}$.

5) RANK OF A MATRIX

182) Find the rank of the matrix $A=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

183) Find the rank of the matrix $A=\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

184) Find the rank of the matrix $A=\begin{bmatrix} 1 & 0 & -4 \\ 2 & -1 & 3 \end{bmatrix}$

185) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 6 \\ 2 & 4 & 3 \end{bmatrix}$

186) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$

ANSWERS

177) $\begin{bmatrix} a-ib & -c-id \\ c-id & a+ib \end{bmatrix}$ **178)** 5, –4 **179)** 17, 3 **182)** 2 **183)** 1

184) 2 **185)** 2 **186)** Rank =3

187) Find the rank of the matrix $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ **[AP J 19] [TS M 19][AP M 22]**

188) Find the rank of the matrix $\begin{bmatrix} -1 & -2 & -3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$

189) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ **[TS M 15] [AP M 19]**

190) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **[AP J 18][TS M 18] [TS J 16]**

191) Find the rank of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

192) Find the rank of $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ using elementary transformations. **[AP M 20]**

193) Find the rank of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

194) Find the rank of $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$

ANSWERS

187) 3 **188)** Rank =2 **189)** 2 **190)** 1 **191)** Rank =3

192) Rank(A) =2 **193)** Rank =3 **194)** Rank =3

6) MISCELLANEOUS

195) Define symmetric & skew symmetric matrix and give an example to each. [TS J 15,17][AP 18, 23]

196) Define Invertible Matrix. [AP J 15]

197) Define Triangular Matrix. [TS M 16]

198) Prove that the determinant of skew symmetric matrix of order 3 is zero.

199) Define Trace of a matrix.

200) What is transpose of a matrix?

201) Define a diagonal matrix. Give one example

202) Define a scalar matrix. Give one example.

203) Define Rank of a matrix. [TS M 20]

204) Give example to show product of non-zero matrices can be a zero matrix.

205) Solve the following system of homogeneous equations $x-y+z=0$, $x+2y-z=0$, $2x+y+3z=0$.

206) A certain book shop has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs.80, Rs.60, Rs.40 each respectively. Using matrix algebra, find the total value of the books in the shop.

ANSWERS

195) ***Symmetric matrix***: A square matrix A is said to be symmetric if $A^T=A$ Ex: $I=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Skew symmetric matrix: A square matrix A is said to be skew symmetric if $A^T=-A$. **Ex:** $A=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$

196) A square matrix A is said to be an invertible matrix if there exists a square matrix B such that AB = BA = I. The matrix B is called inverse of A and we write $A^{-1}=B$

197) **Triangular Matrix:** In the square matrix $A=(a_{ij})_{n\times n}$ if $a_{ij}=0$ for $i>j$ or $i<j$ then A is called a triangular Matrix.

199) If A is a square matrix then the sum of the elements in the principal diagonal of A is called trace of A and it is denoted by Tr (A)

200) The matrix obtained by interchanging the rows and columns of a given matrix is called the Transpose of the given matrix. The transpose of the matrix A is denoted by A']

201) If each non-diagonal element of a square matrix is zero then it is called a diagonal matrix.Ex: $\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$

202) If each non-diagonal element of a square matrix is zero and each diagonal element is same, then the matrix is called a scalar matrix

203) The rank of a non-zero matrix A is defined as maximum of the orders of the non-singular square submatrices of A and it is denoted by rank (A). The rank of a null matrix is defined as zero.

204) $A=\begin{bmatrix}0 & 1\\0 & 2\end{bmatrix}$, $B=\begin{bmatrix}3 & 4\\0 & 0\end{bmatrix}$ **205)** $x=0, y=0, z=0$ **206)** Rs. 20160

III) ADDITION OF VECTORS

1) UNIT VECTORS, COLLINEAR VECTORS, POSITION VECTORS, DC'S

207) Find unit vector in the direction of vector $\bar{a}=(2\bar{i}+3\bar{j}+\bar{k})$ **[IPE14][TS J 17]**

208) Let $\bar{a}=\bar{i}+2\bar{j}+3\bar{k}$, $\bar{b}=3\bar{i}+\bar{j}$. Find a unit vector in the direction of $\bar{a}+\bar{b}$ **[TS J 19]**

209) Find the unit vector in the direction of the sum of the vectors $\bar{a}=2\bar{i}+2\bar{j}-5\bar{k}$ and $\bar{b}=2\bar{i}+\bar{j}+3\bar{k}$

210) Let $\bar{a}=2\bar{i}+4\bar{j}-5\bar{k}, \bar{b}=\bar{i}+\bar{j}+\bar{k}, \bar{c}=\bar{j}+2\bar{k}$ .Find the unit vector in opposite direction of $\bar{a}+\bar{b}+\bar{c}$ **[AP M 19][AP M, J 15][TS 23]**

211) Find a vector in the direction of vector $\bar{a}=\bar{i}-2\bar{j}$ has magnitude 7 units

212) If $\bar{a}=2\bar{i}+5\bar{j}+\bar{k}$ and $\bar{b}=4\bar{i}+m\bar{j}+n\bar{k}$ are collinear vectors then find m,n
[AP M 20,22[TS M 17,18] [TS M, J 15] [AP TS 16]

213) If the vectors $-3\bar{i}+4\bar{j}+\lambda\bar{k}, \mu\bar{i}+8\bar{j}+6\bar{k}$ are collinear vectors then find λ & μ.
[TS 18,20,22][AP M 18]

214) Let $\bar{a}, \bar{b}$ be non-collinear vectors. If $\bar{\alpha}=(x+4y)\bar{a}+(2x+y+1)\bar{b}$, $\bar{\beta}=(y-2x+2)\bar{a}+(2x-3y-1)\bar{b}$ are such that $3\bar{\alpha}=2\bar{\beta}$ then find x,y.

215) If the position vectors of the points A,B,C are $-2\bar{i}+\bar{j}-\bar{k}, -4\bar{i}+2\bar{j}+2\bar{k}, 6\bar{i}-3\bar{j}-13\bar{k}$ respectively and $\overline{AB}=\lambda\overline{AC}$ then find the value of λ. **[TS M 19][AP J 17,19]**

216) Show that the points whose P.V are $-2\bar{a}+3\bar{b}+5\bar{c}, \bar{a}+2\bar{b}+3\bar{c}, 7\bar{a}-\bar{c}$ are collinear,where $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors. **[TS J 17]**

217) Show that the points whose P.V are $\bar{a}-2\bar{b}+3\bar{c}, 2\bar{a}+3\bar{b}-4\bar{c}, -7\bar{b}+10\bar{c}$ are collinear.

218) If $\overline{OA}=\bar{i}+\bar{j}+\bar{k}, \overline{AB}=3\bar{i}-2\bar{j}+\bar{k}, \overline{BC}=\bar{i}+2\bar{j}-2\bar{k}, \overline{CD}=2\bar{i}+\bar{j}+3\bar{k}$ $7\bar{i}+2\bar{j}+3\bar{k}$ then find the vector $\overline{OD}$ **[AP 23] [TS M 22]**

219) ABCDE is a pentagon. If the sum of the vectors $\overline{AB}, \overline{AE}, \overline{BC}, \overline{DC}, \overline{ED}, \overline{AC}$ is $\lambda\overline{AC}$ then find the value of λ.

220) Show that the triangle formed by the vectors $3\bar{i}+5\bar{j}+2\bar{k}, 2\bar{i}-3\bar{j}-5\bar{k}, -5\bar{i}-2\bar{j}+3\bar{k}$ is equilateral. **[TS J 19]**

221) Show that the points $A(2\bar{i}-\bar{j}+\bar{k}), B(\bar{i}-3\bar{j}-5\bar{k}), C(3\bar{i}-4\bar{j}-4\bar{k})$ are the vertices of a right angled triangle.

ANSWERS

207) $\frac{2}{\sqrt{14}}\bar{i}+\frac{3}{\sqrt{14}}\bar{j}+\frac{1}{\sqrt{14}}\bar{k}$ **208)** $\frac{1}{\sqrt{34}}(4\bar{i}+3\bar{j}+3\bar{k})$ **209)** $\frac{1}{\sqrt{29}}(4\bar{i}+3\bar{j}-2\bar{k})$

210) $\frac{-(3\bar{i}+6\bar{j}-2\bar{k})}{7}$ **211)** $\frac{7}{\sqrt{5}}\bar{i}-\frac{14}{\sqrt{5}}\bar{j}$ **212)** 10, 2 **213)** 3, –6 **214)** x = 2, y = –1

215) –1/4 **218)** $7\bar{i}+2\bar{j}+3\bar{k}$ **219)** 3

222) Consider two points P ,Q with position vectors $\overline{OP} = 3\bar{a} - 2\bar{b}$ and $\overline{OQ} = \bar{a} + \bar{b}$ .Find the position vector of a point R which divides the line joining P, Q in the ratio 2:1, (i) internally (ii) externally

223) Write direction ratios of the vector $\bar{r} = \bar{i} + \bar{j} - 2\bar{k}$ and hence calculate its direction cosines.

224) If α,β and γ be the angles made by the vector $3\bar{i} - 6\bar{j} + 2\bar{k}$ with the positive directions of the coordinate axes, find $\cos\alpha$, $\cos\beta$, $\cos\gamma$. **[AP M 17]**

225) Find the angles made by the straight line passing through the points (1,–3,2) and (3,–5,1) with the coordinate axes.

226) If $\bar{a} = 2\bar{i} + \bar{j} + \bar{k}, \bar{b} = \bar{i} - 3\bar{j} - 5\bar{k}$. Find the vector $\bar{c}$ such that $\bar{a}$ & $\bar{b}$ form the sides of a triangle. **[AP J 19]**

2) VECTOR EQUATIONS

227) Find the vector equation of the line passing through the point $2\bar{i} + \bar{j} + 3\bar{k}$ parallel to vector $4\bar{i} - 2\bar{j} + 3\bar{k}$. **[TS 23][TS 16,19,22][AP 17,22]**

228) Find the vector equation of the line passing through the points $2\bar{i} + \bar{j} + 3\bar{k}$, $-4\bar{i} + 3\bar{j} - \bar{k}$ **[AP TS 16] [TS M 17,18]**

229) Find the vector equation of the plane passing through the points $\bar{i} - 2\bar{j} + 5\bar{k}, -5\bar{j} - \bar{k}, -3\bar{i} + 5\bar{j}$ **[TS M 19][TS J 18] [AP J 15,19][AP M 17,19, 23]**

230) Find the vector equation of the plane passing through the points (0,0,0), (0,5,0) and (2,0,1) **[AP M 20]**

231) OABC is a parallelogram. If $\overline{OA} = \bar{a}, \overline{OC} = \bar{c}$ find the vector equation of the side $\overline{BC}$.

232) If $\bar{a}, \bar{b}, \bar{c}$ are P.V's of the vertices A,B,C respectively of ΔABC then find the vector equation of the median through the vertex A.

233) Using the vector equation of the straight line passing through two points, prove that the points whose position vectors are $\bar{a}, \bar{b}$ and $(3\bar{a} - 2\bar{b})$ are collinear.

ANSWERS

222) $\frac{5\bar{a}}{3}$, $4\bar{b} - \bar{a}$ 223) (1, 1, –2) $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$ 224) $\cos\alpha = \frac{3}{7}; \cos\beta = -\frac{6}{7}; \cos\gamma = \frac{2}{7}$

225) $\alpha = \cos^{-1}\frac{2}{3}$; $\beta = \cos^{-1}\left(-\frac{2}{3}\right)$; $\gamma = \cos^{-1}\left(-\frac{1}{3}\right)$ 226) $-3\bar{i} + 2\bar{j} + 4\bar{k}$

227) $\bar{r} = (2\bar{i} + \bar{j} + 3\bar{k}) + t(4\bar{i} - 2\bar{j} + 3\bar{k})$ 228) $\bar{r} = (1-t)(2\bar{i} + \bar{j} + 3\bar{k}) + t(-4\bar{i} + 3\bar{j} - \bar{k}), t \in R$

229) $\bar{r} = (1-s-t)(\bar{i} - 2\bar{j} + 5\bar{k}) + s(-5\bar{j} - \bar{k}) + t(-3\bar{i} + 5\bar{j})$ 230) $\bar{r} = s(5\bar{j}) + t(2\bar{i} + \bar{k}), s, t \in R$

231) $\bar{r} = (1-t)(\bar{a} + \bar{c}) + t\bar{c}, t \in R$ 232) $\bar{r} = (1-t)\bar{a} + t\frac{1}{2}(\bar{b} + \bar{c}), t \in R$

IV) PRODUCT OF VECTORS

1) SCALAR PRODUCT, PERPENDICULAR VECTORS, ANGLES

234) If the vectors $2\bar{i}+\lambda\bar{j}-\bar{k}$ and $4\bar{i}-2\bar{j}+2\bar{k}$ are perpendicular to each other find λ. **[TS J 15,16]**

235) For what values of λ the vectors $\bar{i}-\lambda\bar{j}+2\bar{k}, 8\bar{i}+6\bar{j}-\bar{k}$ are at right angles. **[AP M 22]**

236) If vectors $\lambda\bar{i}-3\bar{j}+5\bar{k}, 2\lambda\bar{i}-\lambda\bar{j}-\bar{k}$ are perpendicular to each other find λ **[AP M 19]**

237) If $\bar{a}=\bar{i}+2\bar{j}-3\bar{k}, \bar{b}=3\bar{i}-\bar{j}+2\bar{k}$ then show that $\bar{a}+\bar{b}, \bar{a}-\bar{b}$ are perpendicular. **[AP J 18]**

238) If $\bar{a}=6\bar{i}+2\bar{j}+3\bar{k}$ and $\bar{b}=2\bar{i}-9\bar{j}+6\bar{k}$, then find $\bar{a}.\bar{b}$ and the angle between $\bar{a}$ and $\bar{b}$.

239) Find the angle between $\bar{i}+2\bar{j}+3\bar{k}$ & $3\bar{i}-\bar{j}+2\bar{k}$ **[TS M 17][AP M 18][TS J 19]**

240) If $\bar{a}=2\bar{i}+2\bar{j}-3\bar{k}, \bar{b}=3\bar{i}-\bar{j}+2\bar{k}$ then find the angle between $2\bar{a}+\bar{b}$ and $\bar{a}+2\bar{b}$

241) Find angle between planes $\bar{r}.(2\bar{i}-\bar{j}+2\bar{k})=3$, $\bar{r}.(3\bar{i}+6\bar{j}+\bar{k})=4$ **[AP M 20] [TS M 15,20]**

242) If $|\bar{a}+\bar{b}|=|\bar{a}-\bar{b}|$ then find the angle between $\bar{a}$ and $\bar{b}$

243) If $\bar{a}+\bar{b}+\bar{c}=\bar{0}$, $|\bar{a}|=3, |\bar{b}|=5$, $|\bar{c}|=7$ then find angle between $\bar{a}$, $\bar{b}$ **[AP J 19]**

2) PROJECTIONS, VE OF PLANE IN NORMAL FORM

244) If $\bar{a}=\bar{i}+\bar{j}+\bar{k}$, $\bar{b}=2\bar{i}+3\bar{j}+\bar{k}$ then find the projection vector of $\bar{b}$ on $\bar{a}$ and its magnitude

245) If $\bar{a}=\bar{i}-\bar{j}-\bar{k}, \bar{b}=2\bar{i}-3\bar{j}+\bar{k}$ then find the projection vector of $\bar{b}$ on $\bar{a}$ and its magnitude.

246) If $\bar{P},\bar{Q},\bar{R}$ and $\bar{S}$ are points whose position vectors are $\bar{i}-\bar{k}, -\bar{i}+2\bar{j}, 2\bar{i}-3\bar{k}$ and $3\bar{i}-2\bar{j}-\bar{k}$ then find component of $\overline{RS}$ on $\overline{PQ}$.

247) Find the cartesian equation of the plane through the point A(2,–1,–4) and parallel to the plane 4x–12y–3z–7=0.

248) Find the angle between the planes 2x–3y–6z = 5 and 6x+2y–9z=4

249) Find the Cartesian equation of the plane passing through the point (–2,1,3) and perpendicular to the vector $3\bar{i}+\bar{j}+5\bar{k}$

250) Find the equation of the plane through the point (3,–2,1) and perpendicular to the vector (4,7,–4).

251) Find unit vector parallel to the XOY-plane and perpendicular to the vector $4\bar{i}-3\bar{j}+\bar{k}$

ANSWERS

234) 3 **235)** 1 **236)** –5/2 (or) 1 **238)** $12, \cos^{-1}\left(\frac{12}{77}\right)$ **239)** 60^0

240) $\theta=\cos^{-1}\left(\frac{52}{\sqrt{74\times 65}}\right)$ **241)** $\cos^{-1}\left(\frac{2}{3\sqrt{46}}\right)$ **242)** 90^o **243)** 60^o

244) $2(\bar{i}+\bar{j}+\bar{k}); 2\sqrt{3}$ **245)** $\frac{4}{\sqrt{3}}$ **246)** –4/3 **247)** 4x–12y–3z–32=0

248) $\theta=\cos^{-1}\frac{60}{77}$ **249)** $3x+y+5z-10=0$ **250)** $\bar{r}.(-4\bar{i}-7\bar{j}+4\bar{k})=6$ **251)** $\pm\frac{(3\bar{i}+4\bar{j})}{5}$

4) VECTOR PRODUCT, UNIT NORMAL VECTOR, SINq, AREAS

252) If $\bar{a}=2\bar{i}-\bar{j}+\bar{k}$ and $\bar{b}=\bar{i}-3\bar{j}-5\bar{k}$, then find $|\bar{a}\times\bar{b}|$

253) If $\bar{a}=2\bar{i}-3\bar{j}+\bar{k}$ and $\bar{b}=\bar{i}+4\bar{j}-2\bar{k}$, then find $(\bar{a}+\bar{b})\times(\bar{a}-\bar{b})$.

254) If $|\bar{p}|=2, |\bar{q}|=3$ and $(\bar{p},\bar{q})=\frac{\pi}{6}$, then find $|\bar{p}\times\bar{q}|^2$.

255) If $\bar{p}=x\bar{i}+y\bar{j}+z\bar{k}$, find the value of $|\bar{p}\times\bar{k}|^2$

256) Compute $2\bar{j}\times(3\bar{i}-4\bar{k})+(\bar{i}+2\bar{j})\times\bar{k}$.

257) Compute $\bar{a}\times(\bar{b}+\bar{c})+\bar{b}\times(\bar{c}+\bar{a})+\bar{c}\times(\bar{a}+\bar{b})$

258) Find a unit vector perpendicular to the plane containing the vectors $\bar{a}=4\bar{i}+3\bar{j}-\bar{k}$, $\bar{b}=2\bar{i}-6\bar{j}-3\bar{k}$

259) Find unit vector perpendicular to both $\bar{i}+\bar{j}+\bar{k}$ and $2\bar{i}+\bar{j}+3\bar{k}$

260) If $\bar{a}=2\bar{i}-3\bar{j}+5\bar{k}$, $\bar{b}=-\bar{i}+4\bar{j}+2\bar{k}$ then find $\bar{a}\times\bar{b}$ and unit vector perpendicular to both $\bar{a}, \bar{b}$

261) If $\bar{a}=2\bar{i}-3\bar{j}+5\bar{k}$, $\bar{b}=-\bar{i}+4\bar{j}+2\bar{k}$, then find $(\bar{a}+\bar{b})\times(\bar{a}-\bar{b})$ and unit vector perpendicular to both $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$ **[TS M 18]**

262) If θ is the angle between the vectors $\bar{i}+\bar{j}, \bar{j}+\bar{k}$ then find $\sin\theta$. **[TS J 19]**

263) Let $\bar{a}=2\bar{i}-\bar{j}+\bar{k}, \bar{b}=3\bar{i}+4\bar{j}-\bar{k}$ and if θ is the angle between $\mathbf{\bar{a}, \bar{b}}$ then find $\sin\theta$.

264) If $4\bar{i}+\frac{2p}{3}\bar{j}+p\bar{k}$ is parallel to the vector $\bar{i}+2\bar{j}+3\bar{k}$, find p.

265) If $|\bar{a}|=13, |\bar{b}|=5$, and $\bar{a}.\bar{b}=60$, then find $|\bar{a}\times\bar{b}|$.

266) For any two vectors $\bar{a}, \bar{b}$ prove that $|\bar{a}\times\bar{b}|^2=|\bar{a}|^2|\bar{b}|^2-(\bar{a}.\bar{b})^2$

267) If $\bar{a}=\bar{i}+2\bar{j}+3\bar{k}, \bar{b}=3\bar{i}+5\bar{j}-\bar{k}$ are 2 sides of a triangle, find its area.

268) Find the area of the triangle having $(3\bar{i}+4\bar{j}), (-5\bar{i}+7\bar{j})$ as adjacent sides.

269) Find the area of the parallelogram whose adjacent sides are $\bar{a}=2\bar{j}-\bar{k}, \bar{b}=-\bar{i}+\bar{k}$ **[AP M 16]**

ANSWERS

252) $\sqrt{210}$ 253) $-2(2\bar{i}+5\bar{j}+11\bar{k})$ 254) 9 255) x^2+y^2

256) $-6\bar{i}-\bar{j}-6\bar{k}$ 257) $\bar{0}$ 258) $\pm\left(\frac{3\bar{i}-2\bar{j}+6\bar{k}}{7}\right)$ 259) $\pm\frac{1}{\sqrt{6}}(2\bar{i}-\bar{j}-\bar{k})$

260) $-26\bar{i}-9\bar{j}+5\bar{k}, \pm\frac{1}{\sqrt{782}}(26\bar{i}+9\bar{j}-5\bar{k})$ 261) $\pm\frac{1}{\sqrt{782}}(26\bar{i}+9\bar{j}-5\bar{k})$ 262) $\frac{\sqrt{3}}{2}$

263) $\sqrt{\frac{155}{156}}$ 264) 12 265) 25 267) $\frac{\sqrt{390}}{2}$ 268) 20.5 sq.units 269) 3 sq.units

270) Find the area of the parallelogram whose adjacent sides are $\bar{a} = 2\bar{i} - 3\bar{j}$ and $\bar{b} = 2\bar{i} - 3\bar{k}$. **[IPE 07,08][AP J 16]**

271) Find the vector area and area of the parallelogram having $\bar{a} = \bar{i} + 2\bar{j} - \bar{k}$, $\bar{b} = 2\bar{i} - \bar{j} + 2\bar{k}$ as adjacent sides.

272) Find the area of the parallelogram whose diagonals are $3\bar{i} + \bar{j} - 2\bar{k}$, $\bar{i} - 3\bar{j} + 4\bar{k}$. **[TS J 17]**

273) If $\bar{a} \times \bar{b} = \bar{b} \times \bar{c} \neq \bar{0}$, then show that $\bar{a} + \bar{c} = p\bar{b}$, where p is some scalar.

274) If $\bar{a}.\bar{b} = \bar{a}.\bar{c}$ and $\bar{a} \times \bar{b} = \bar{a} \times \bar{c}$, $\bar{a} \neq 0$, then show that $\bar{b} = \bar{c}$.

275) Find the distance of a point (2,5,–3) from plane $\bar{r}.(6\bar{i} - 3\bar{j} + 2\bar{k}) = 4$.

276) Find the angle between line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane 10x+2y–11z=3.

6) SCALAR TRIPLE PRODUCT

277) If $\bar{a} = \bar{i} - 2\bar{j} - 3\bar{k}, \bar{b} = 2\bar{i} + \bar{j} - \bar{k}$, $\bar{c} = \bar{i} + 3\bar{j} - 2\bar{k}$, compute $\bar{a}.(\bar{b} \times \bar{c})$

278) Simplify $(\bar{i} - 2\bar{j} + 3\bar{k}) \times (2\bar{i} + \bar{j} - \bar{k}).(\bar{j} + \bar{k})$

279) Compute $[\bar{i} - \bar{j} \;\; \bar{j} - \bar{k} \;\; \bar{k} - \bar{i}]$.

280) Find the value of $\left[\bar{i} + \bar{j} + \bar{k}, \bar{i} - \bar{j}, \bar{i} + 2\bar{j} - \bar{k}\right]$.

281) Let $\bar{a}$ $\bar{b}$ and $\bar{c}$ be non-coplanar vectors, if $\left[\bar{a} + 2\bar{b} \;\; 2\bar{b} + \bar{c} \;\; 5\bar{c} + \bar{a}\right] = \lambda[\bar{a} \; \bar{b} \; \bar{c}]$, then find λ.

282) For any three vectors $\bar{a}, \bar{b}, \bar{c}$ prove that $\left[\bar{b} + \bar{c} \;\; \bar{c} + \bar{a} \;\; \bar{a} + \bar{b}\right] = 2[\bar{a} \; \bar{b} \; \bar{c}]$

283) Show that $\left(\bar{a} + \bar{b}\right).\left(\bar{b} + \bar{c}\right) \times \left(\bar{c} + \bar{a}\right) = 2\left[\bar{a}\bar{b}\bar{c}\right]$.

284) If $\bar{a}, \bar{b}, \bar{c}$ are unit coplanar vectors, then find $[2\bar{a} - \bar{b}, 2\bar{b} - \bar{c}, 2\bar{c} - \bar{a}]$

7) VOLUMES, COPLANAR& EQUATION OF PLANE

285) Find the volume of the parallelopiped having co-terminus edges $\bar{i} + \bar{j} + \bar{k}, \bar{i} - \bar{j}, \bar{i} + 2\bar{j} - \bar{k}$

286) Find the volume of the parallelopiped whose coterminus edges are represented by the vectors $2\bar{i} - 3\bar{j} + \bar{k}$, $\bar{i} - \bar{j} + 2\bar{k}, 2\bar{i} + \bar{j} - \bar{k}$

287) Find the volume of the tetrahedron having the edges $\bar{i} + \bar{j} + \bar{k}$, $\bar{i} - \bar{j}$, $\bar{i} + 2\bar{j} + \bar{k}$

288) Find the volume of the tetrahedron whose vertices are (1,2,1) , (3,2,5), (2,–1,0) and (–1,0,1)

289) If $\bar{a}, \bar{b}, \bar{c}$ are non coplanar, then show that the vectors $\bar{a} - \bar{b}$, $\bar{b} + \bar{c}$, $\bar{c} + \bar{a}$ are coplanar.

ANSWERS

270) $\sqrt{94}$ **271)** $3\bar{i} - 4\bar{j} - 5\bar{k}$, $5\sqrt{2}$ sq.units **272)** $5\sqrt{3}$ **275)** 13/7 **276)** $\theta = \sin^{-1}\left(\frac{8}{21}\right)$

277)–20 **278)**12 **279)** 0 **280)** 5 **281)**12 **284)**0

285) 5 cu.units **286)** 14 cu.units **287)**1/6 cu.units **288)** 6

290) Prove that vectors $\bar{a}=2\bar{i}-\bar{j}+\bar{k},\ \bar{b}=\bar{i}-3\bar{j}-5\bar{k}$ and $\bar{c}=3\bar{i}-4\bar{j}-4\bar{k}$ are coplanar.

291) If the vectors $\bar{a}=2\bar{i}-\bar{j}+\bar{k},\ \bar{b}=\bar{i}+2\bar{j}-3\bar{k},\ \bar{c}=3\bar{i}+p\bar{j}+5\bar{k}$ are coplanar then find p.

292) Find t, for which the vectors $2\bar{i}-3\bar{j}+\bar{k},\ \bar{i}+2\bar{j}-3\bar{k},\ \bar{j}-t\bar{k}$ are coplanar.

293) For non coplanar vectors, $\bar{a},\bar{b}$ and $\bar{c}$ determine p for which the vector $\bar{a}+\bar{b}+\bar{c}$, $\bar{a}+p\bar{b}+2\bar{c}$ and $-\bar{a}+\bar{b}+\bar{c}$ are coplanar.

294) Determine λ, for which the volume of the parallelopiped having coterminus edges $\bar{i}+\bar{j},\ 3\bar{i}-\bar{j}$ and $3\bar{j}+\lambda\bar{k}$ is 16 cubic units.

295) Find the equation of the plane passing through the points A=(2,3–1),B=(4, 5, 2) and C= (3,6,5).

296) Find the equation of the plane passing through the point A= (3,–2, –1) and parallel to the vectors $\bar{b}=\bar{i}-2\bar{j}+4\bar{k}$ and $\bar{c}=3\bar{i}+2\bar{j}-5\bar{k}$.

297) Find the distance of the point (2,5,–3) from plane $\bar{r}.(6\bar{i}-3\bar{j}+2\bar{k})=4$.

298) Find the angle between line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane 10x+2y–11z=3.

8) MISCELLANEOUS

299) In ΔABC, if $\bar{a},\bar{b},\bar{c}$ are position vectors of the vertices A,B and C respectively, then prove that the position vector of the centroid G is $1/3(\bar{a}+\bar{b}+\bar{c})$

300) The vector equation of the striaght line passing through the point A($\bar{a}$) and parallel to the vector $\bar{b}$ is $\bar{r}=\bar{a}+t\bar{b},\ t\in IR$

301) The vector equation of the line through the pionts A($\bar{a}$) and B($\bar{b}$) is $\bar{r}=(1-t)\bar{a}+t\bar{b},\ t\in IR$

302) In two dimensional plane, prove by using vector methods, the equation of the line whose intercepts on the axes are 'a' and 'b' is $\frac{x}{a}+\frac{y}{b}=1$

303) If $|\bar{a}|=2,\ |\bar{b}|=3,\ |\bar{c}|=4$ and each of $\bar{a},\bar{b},\bar{c}$ is perpendicular to the sum of the other two vectors, then find the magnitude of $\bar{a}+\bar{b}+\bar{c}$.

304) If $|\bar{a}|=11, |\bar{b}|=23, |\bar{a}-\bar{b}|=30$ then find the angle between the vectors $\bar{a},\bar{b}$ and also find $|\bar{a}+\bar{b}|$

305) If $\bar{a}=2\bar{i}+3\bar{j}+\bar{k}, \bar{b}=4\bar{i}+\bar{j}, \bar{c}=\bar{i}-3\bar{j}-7\bar{k}$ find the vector $\bar{r}$ such that $\bar{r}.\bar{a}=9,\ \bar{r}.\bar{b}=7,\ \bar{r}.\bar{c}=6$

306) Let $\bar{e}_1$ and $\bar{e}_2$ be unit vectors making angle θ. If $\frac{1}{2}|\bar{e}_1-\bar{e}_2|=\sin\lambda\theta$, then find λ.

307) Find the equation of plane passing through point $\bar{a}=2\bar{i}+3\bar{j}-\bar{k}$ and perpendicular to the vector $3\bar{i}-2\bar{j}-2\bar{k}$ and the distance of this plane from the origin.

ANSWERS

291) –4 **292)** 1 **293)** 2 **294)** ±4 **295)** 3x–9y+4z+25 =0 **296)** 2x+17y+8z+36=0

297) 13 **298)** $\theta=\sin^{-1}\left(\frac{8}{21}\right)$ **303)** $\sqrt{29}$ **304)** $\theta=\pi-\cos^{-1}\left(\frac{125}{253}\right)$, 20

305) $\bar{r}=\bar{i}+3\bar{j}-2\bar{k}$ **306)** 1/2 **307)** $\bar{r}.(3\bar{i}-2\bar{j}-2\bar{k})=2$ $\frac{2}{\sqrt{17}}$ units

308) If $\bar{a}\ \bar{b}\ \bar{c}$ and $\bar{d}$ are vectors such that $\bar{a}\times\bar{b}=\bar{c}\times\bar{d}$ and $\bar{a}\times\bar{c}=\bar{b}\times\bar{d}$, then show that the vectors $\bar{a}-\bar{d}$ and $\bar{b}-\bar{c}$ are parallel.

309) Let $\bar{a},\bar{b},\ \bar{c}$ be such that $\bar{c}\neq 0$, $\bar{a}\times\bar{b}=\bar{c}$, $\bar{b}\times\bar{c}=\bar{a}$, then show that $\bar{a},\bar{b},\bar{c}$ are pair orthogonal vectors and $|\bar{b}|=1, |\bar{c}|=|\bar{a}|$.

310) Let $\mathbf{a},\mathbf{b}$ be two non-collinear unit vectors. If $\bar{\alpha}=\bar{a}-(\bar{a}.\bar{b})\bar{b}$ and $\bar{\boldsymbol{\beta}}=\bar{\mathbf{a}}\times\bar{\mathbf{b}}$, then show that $|\bar{\boldsymbol{\beta}}|=|\bar{\boldsymbol{\alpha}}|$.

311) If $\bar{a}=(1,-1,-6)$, $\bar{b}=(1,-3,4)$ and $\bar{c}=(2,-5,3)$, then compute $\bar{a}\times\left(\bar{b}\times\bar{c}\right)$. **[AP J 15]**

312) Prove that $\bar{a}\times\left[\bar{a}\times\left(\bar{a}\times\bar{b}\right)\right]=\left(\bar{a}.\bar{a}\right)\left(\bar{b}\times\bar{a}\right)$.

313) If $\bar{a},\bar{b},\bar{c}$ and $\bar{d}$ are coplanar vectors, then show that $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=\bar{0}$

314) In ABC, if $\overline{BC}=\bar{a},\overline{CA}=\bar{b}$ and $\overline{AB}=\bar{c}$ then show that $\bar{a}\times\bar{b}=\bar{b}\times\bar{c}=\bar{c}\times\bar{a}$

315) If $\bar{a}+\bar{b}+\bar{c}=\bar{0}$ then prove that $\bar{a}\times\bar{b}=\bar{b}\times\bar{c}=\bar{c}\times\bar{a}$

316) If $\bar{a}=2\bar{i}+\bar{j}-\bar{k}$, $\bar{b}=-\bar{i}+2\bar{j}-4\bar{k}$ and $c=\bar{i}+\bar{j}+\bar{k}$ then find $(\bar{a}\times\bar{b}).(\bar{b}\times\bar{c})$ **[AP M 19]**

317) Find the area of the triangle whose vertices are A(1,2,3), B(2,3,1) and C(3,1,2)

318) Show that $\bar{i}\times(\bar{a}\times\bar{i})+\bar{j}\times(\bar{a}\times\bar{j})+\bar{k}\times(\bar{k}\times\bar{a})=2\bar{a}$ for any vector $\bar{a}$

319) For any three vectors $\bar{a},\bar{b},\bar{c}$ prove that $[\bar{b}\times\bar{c}\ \ \bar{c}\times\bar{a}\ \ \bar{a}\times\bar{b}]=\left[\bar{a}\ \bar{b}\ \bar{c}\right]^2$

320) Find the equation of the plane passing through (a,b,c) and parallel to the plane $\bar{r}.(\bar{i}+\bar{j}+\bar{k})=2$

321) Define Linear combination of vectors.

ANSWERS

311) $11\bar{i}+5\bar{j}+\bar{k}$ **316)** -54 **317)** $\frac{3\sqrt{3}}{2}$ **320)** x+y+z=a+b+c

321) Let $\bar{a}_1,\bar{a}_2,\bar{a}_3,.....\bar{a}_n$ be vectors and $x_1,x_2,x_3,...x_n$ be scalars. Then the vector $x_1\bar{a}_1+x_2\bar{a}_2+x_3\bar{a}_3+.....x_n\bar{a}_n$ is called a linear combination of the vectors $\bar{a}_1,\bar{a}_2,\bar{a}_3,.....\bar{a}_n$

V) TRIGONOMETRIC RATIOS, TRANSFORMATIONS

1) T - RATIOS -VALUES

322) Find $\cos 225^\circ - \sin 225^\circ + \tan 495^\circ - \cot 495^\circ$

323) Evaluate $\cos^2 45^\circ + \cos^2 135^\circ + \cos^2 225^\circ + \cos^2 315^\circ$

324) Evaluate $\sin^2 \frac{2\pi}{3} + \cos^2 \frac{5\pi}{6} - \tan^2 \frac{3\pi}{4}$

325) Show that $\sin 1140^\circ \cos 390^\circ - \cos 780^\circ \sin 750^\circ = 1/2$

326) Show that $\frac{\sin 150^\circ - 5\cos 300^\circ + 7\tan 225^\circ}{\tan 135^\circ + 3\sin 210^\circ} = -2$

327) If $\tan 20^\circ = p$ then show that $\frac{\tan 610^\circ + \tan 700^\circ}{\tan 560^\circ - \tan 470^\circ} = \frac{1-p^2}{1+p^2}$

328) If $\tan 20^\circ = \lambda$ then show that $\frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ . \tan 110^\circ} = \frac{1-\lambda^2}{2\lambda}$ **[AP M 16,20]**

329) Show that $\cot\frac{\pi}{20}.\cot\frac{3\pi}{20}.\cot\frac{5\pi}{20}.\cot\frac{7\pi}{20}.\cot\frac{9\pi}{20} = 1$

330) Show that $\cot\frac{\pi}{16}.\cot\frac{2\pi}{16}.\cot\frac{3\pi}{16}.........\cot\frac{7\pi}{16} = 1$ **[IPE 05]**

331) If A,B,C,D are angles of a cyclic quadrilateral prove that cosA+cosB+cosC+cosD=0

332) If A,B,C,D are angles of a cyclic quadrilateral then prove that sinA–sinC =sinD–sinB

333) Find the value of $\tan(855^\circ)$

334) Find the value of $\cot 765^\circ$

335) Find the value of $\sec(2100^\circ)$

336) Find the value of $\sec\left(13\frac{\pi}{3}\right)$

337) Simplify $\cot\left(\theta - \frac{13\pi}{2}\right)$

338) Simplify $\tan\left(-23\frac{\pi}{3}\right)$

ANSWERS

322) 0 **323)** 2 **324)** 1/2 **333)** –1 **334)** 1

335) 2 **336)** 2 **337)** $-\tan\theta$ **338)** $\sqrt{3}$

2) T'C IDENTITIES, T.R-APPLICATIONS & ELIMINANTS

339) Prove that $(\tan\theta+\cot\theta)^2 = \sec^2\theta+\operatorname{cosec}^2\theta=\sec^2\theta.\operatorname{cosec}^2\theta$

340) Prove that (1+cotθ–cosecθ) (1+tanθ+secθ)=2.

341) If 3sinA+5cosA=5, then show that 5sinA–3cosA=±3

342) If 3sinθ+4cosθ=5, then find the value of 4sinθ–3cosθ.

343) If acosθ–bsinθ=c, then show that asinθ+bcosθ $=\pm\sqrt{a^2+b^2-c^2}$

344) If $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$ then show that $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$ **[TS J 15]**

345) Prove that $\sin^2\frac{\pi}{10}+\sin^2\frac{4\pi}{10}+\sin^2\frac{6\pi}{10}+\sin^2\frac{9\pi}{10}=2$

346) Prove that $\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}=2$

347) Eliminate 'θ' from $x = a\cos^3\theta$, $y = b\sin^3\theta$. **[AP M 16]**

348) Eliminate 'θ' from $x = a\cos^4\theta$, $y = b\sin^4\theta$.

349) Eliminate 'θ' from x = a(secθ+tanθ), y =b(secθ–tanθ)

3) ANGLES IN VARIOUS QUADRANTS

350) If $\sin\theta=\frac{4}{5}$ and θ is not in the first quadrant, find the value of cosθ. **[TS M 17][AP M 19]**

351) If $\sin\theta=\frac{-1}{3}$ and θ does not lie in third quadrant, find the values of cosθ **[IPE 13]**

352) If cosθ =t (0<t<1) and θ does not lie in the first quadrant, find the values of (a) sinθ (b) tanθ.

353) If secθ+tanθ =2/3, then find the value of sinθ and determine the quadrant in which θ lies. **[AP J 18]**

354) If secθ+tanθ=5, then find sinθ and determine the quadrant in which θ lies.

355) If cosecθ+cotθ=1/3,then find cosθ and determinequadrant in which θ lies.

356) If cos A = cosB = –1/2 and A does not lie in the second quadrant and B does not lie in the third quadrant, then find the value of $\frac{4\sin B-3\tan A}{\tan B+\sin A}$

357) If 8 tanA= –15 and 25 sinB= –7 and neither A nor B is in the fourth quadrant, then show that sinAcosB+cosA sinB $=\frac{-304}{425}$

ANSWERS

342) 4sinθ–3cosθ=0

347) $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$

348) $\left(\frac{x}{a}\right)^{1/2}+\left(\frac{y}{b}\right)^{1/2}=1$

349) xy=ab

350) –3/5

351) $\frac{2\sqrt{2}}{3}$

352) (a) $-\sqrt{1-t^2}$ (b) $\frac{-\sqrt{1-t^2}}{t}$

353) –5/13, Q_4

354) 12/13, Q_1

355) –4/5, Q_2

356) 2/3

4) COMPOUND ANGLES

357) Find the value of $\sin 330^\circ.\cos 120^\circ + \cos 210^\circ.\sin 300^\circ$ **[AP M 18, 23]**

358) Show that $\sin 750^\circ \cos 480^\circ + \cos 120^\circ \cos 60^\circ = -1/2$

359) Simplify $\cos 100^\circ \cos 40^\circ + \sin 100^\circ \sin 40^\circ$

360) Prove that $\sin 780^\circ \sin 480^\circ + \cos 240^\circ.\cos 300^\circ = 1/2$

361) Simplify $\tan\left(\frac{\pi}{4}+\theta\right).\tan\left(\frac{\pi}{4}-\theta\right)$

362) Simplify $\frac{\cot 55^\circ \cot 35^\circ - 1}{\cot 55^\circ + \cot 35^\circ}$

363) Prove that $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \cot 36^\circ$ **[AP J 17][AP M 15] [AP, TS J 15][TS M 18]**

364) If $\tan\theta = \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$ and θ is in the third quadrant, find θ.

365) Prove that $\tan 70^\circ - \tan 20^\circ = 2\tan 50^\circ$ **[AP J 16]**

366) Prove that $\tan 72^\circ = \tan 18^\circ + 2\tan 54^\circ$

367) Find the value of $\tan 56^\circ - \tan 11^\circ - \tan 56^\circ.\tan 11^\circ$

368) Find $\tan 20^\circ + \tan 40^\circ + \tan 20^\circ \tan 40^\circ$

369) Prove that $\tan 3A.\tan 2A.\tan A = \tan 3A - \tan 2A - \tan A$

370) If $A+B=45^\circ$, then prove that $(1+\tan A)(1+\tan B)=2$ **[TS M 16]**

371) Express $\tan\theta$ in terms of $\tan\alpha$, if $\sin(\theta+\alpha) = \cos(\theta+\alpha)$.

372) Find the value of $\sin^2 42^\circ - \sin^2 12^\circ$

373) Find the value of $\sin^2 82\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$

374) Prove that $\sin^2 52\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$ **[AP J 19]**

375) Find the value of $\cos^2 52\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$ **[AP J 19][TS M 19]**

376) Find the value of $\cos^2 112\frac{1}{2}^\circ - \sin^2 52\frac{1}{2}^\circ$

377) Find the value of $\cos^2 72^\circ - \sin^2 54^\circ$

378) Express $(\cos\theta - \sin\theta)$ as a cosine of an angle.

379) Express $\sqrt{3}\sin\theta + \cos\theta$ as a sine of an angle.

380) Express $\frac{\left(\sqrt{3}\cos 25^\circ + \sin 25^\circ\right)}{2}$ as a sine of an angle. **[TS J 19]**

381) Find the value of $\sin 75^\circ$

382) Find the value of $\tan 75^\circ$.

383) Find the value of $\tan 75^\circ + \cot 75^\circ$

ANSWERS

357) 1

359) 1/2

361) 1

362) 0

364) $\theta = 236^\circ$

367) 1

368) $\sqrt{3}$

371) $\tan\theta = \frac{1-\tan\alpha}{1+\tan\alpha}$

372) $\frac{\sqrt{5}+1}{8}$

373) $\frac{\sqrt{3}(\sqrt{3}+1)}{4\sqrt{2}}$

374) $\frac{\sqrt{3}+1}{4\sqrt{2}}$

375) $\frac{3-\sqrt{3}}{4\sqrt{2}}$

376) $\frac{-(\sqrt{3}+1)}{4\sqrt{2}}$

377) $\frac{-\sqrt{5}}{4}$

378) $\sqrt{2}\cos\left[\frac{\pi}{4}+\theta\right]$

379) $2.\sin\left(\theta+\frac{\pi}{6}\right)$

380) $\sin 85^\circ$

381) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

382) $2+\sqrt{3}$

383) 4

5) T'RATIOS OF ANGLES IN VARIOUS QUADRANTS

384) If $0 < A, B < 90^\circ$, $\cos A = \frac{5}{13}$ and $\sin B = \frac{4}{5}$ then find $\sin(A+B)$.

385) If $0^\circ < A, B < 90^\circ$, such that $\cos A = \frac{5}{13}, \sin B = \frac{4}{5}$, find $\sin(A-B)$

386) If $\sin\alpha = \frac{1}{\sqrt{10}}, \sin\beta = \frac{1}{\sqrt{5}}$ and α, β are acute, show that $\alpha + \beta = \pi/4$ **[TS J 17]**

387) If $\cos\alpha = \frac{-3}{5}$ and $\sin\beta = \frac{7}{25}$ where $\frac{\pi}{2} < \alpha < \pi$ and $0 < \beta < \frac{\pi}{2}$ then find the values of $\tan(\alpha+\beta)$ and $\sin(\alpha+\beta)$.

388) If $\sin A = \frac{12}{13}, \cos B = \frac{3}{5}$ and neither A nor B is in the first quadrant, then find quadrant in which A + B lies.

389) In a Δ ABC, A is obtuse. If $\sin A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$, then show that $\sin C = \frac{16}{65}$

6) MULTIPLE ANGLES

390) Simplify $\frac{1-\cos 2\theta}{\sin 2\theta}$ **391)** Simplify $\frac{\sin 2\theta}{1+\cos 2\theta}$ **392)** Simplify $\frac{3\cos\theta + \cos 3\theta}{3\sin\theta - \sin 3\theta}$

393) Prove that $\frac{\sin\theta + \sin 2\theta}{1+\cos\theta+\cos 2\theta} = \tan\theta$ **394)** Simplify $\frac{1-\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta}$ **[TS J 19]**

395) Prove that $\frac{\cos 3A + \sin 3A}{\cos A - \sin A} = 1 + 2\sin 2A$ **396)** Express $\frac{\sin 4\theta}{\sin\theta}$ in terms of $\cos^3\theta, \cos\theta$.

397) Evaluate $6\sin 20^\circ - 8\sin^3 20^\circ$ **398)** Show that $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$ **[TS M 16]**

399) If $\cos\theta = \frac{-5}{13}$ and $\frac{\pi}{2} < \theta < \pi$, find the value of $\sin 2\theta$.

400) If $0<A<\pi/4$ and $\cos A=4/5$, then find the values of $\sin 2A$ and $\cos 2A$

401) If $\cos\theta = \frac{5}{13}$ and $270^\circ<\theta<360^\circ$, evaluate $\sin(\theta/2)$ and $\cos(\theta/2)$

402) If $\cos\theta = \frac{-3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, find the value of $\tan(\theta/2)$

ANSWERS

384) 56/65 **385)** 16/65 **387)** −3/4, 3/5 **388)** Q_1 or Q_2 **390)** $\tan\theta$ **391)** $\tan\theta$

392) $\cot\theta$ **394)** $\tan(\theta/2)$ **396)** $8\cos^3\theta - 4\cos\theta$ **397)** $\sqrt{3}$ **399)** $\frac{-120}{169}$

400) $\frac{24}{25}, \frac{7}{25}$ **401)** $\frac{2}{\sqrt{13}}, \frac{-3}{\sqrt{13}}$ **402)** −2

403) If $\cos A=\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$, find the value of cos2A.

404) Find the value of $\sin 22\frac{1}{2}^{o}$.

405) Find the values of $\cos 22\frac{1^{o}}{2}$

406) Find the values of $\tan 22\frac{1^{o}}{2}$

407) Find the values of $\cos 18^{o}$

408) Find the values of $\sin 36^{o}$

7) TRANSFORMATIONS

409) Show that $\cos35^{o}+\cos85^{o}+\cos155^{o}=0$

410) Show that $\cos42^{o}+\cos78^{o}+\cos162^{o}=0$

411) Show that $\sin34^{o}+\cos64^{o}-\cos4^{o}=0$

412) Prove that $\sin50^{o}-\sin70^{o}+\sin10^{o}=0$ **[TS M 18]**

413) Prove that $\cos12^{o}+\cos84^{o}+\cos132^{o}+\cos156^{o}=-1/2$

414) Prove that $\frac{\sin 70^{o}-\cos 40^{o}}{\cos 50^{o}-\sin 20^{o}}=\frac{1}{\sqrt{3}}$

415) Prove that $\sin 78^{o}+\cos 132^{o}=\frac{\sqrt{5}-1}{4}$

416) Prove that $4(\cos66^{o}+\sin84^{o})=\sqrt{3}+\sqrt{15}$

417) Prove that $\sin 21^{o}\cos 9^{o}-\cos 84^{o}\cos 6^{o}=\frac{1}{4}$

418) Prove that $\cos48^{o}.\cos12^{o}=\frac{3+\sqrt{5}}{8}$ **[TS M 17]**

419) Prove that $\cos20^{o}\cos40^{o}-\sin5^{o}\sin25^{o}=\frac{\sqrt{3}+1}{4}$

420) Prove that $\cos A+\cos\left(\frac{4\pi}{3}-A\right)+\cos\left(\frac{4\pi}{3}+A\right)=0$

421) Prove that $\cos\theta+\cos\left[\frac{2\pi}{3}+\theta\right]+\cos\left[\frac{4\pi}{3}+\theta\right]=0$

ANSWERS

403 $\frac{1}{\sqrt{2}}$

404) $\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$

405) $\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$

406) $\sqrt{2}-1$

407) $\frac{\sqrt{10+2\sqrt{5}}}{4}$

408) $\frac{\sqrt{10-2\sqrt{5}}}{4}$

8) MAXIMA, MINIMA & PERIODICITY, GRAPHS

422) Find the maximum and minimum values of f(x)=3cosx+4sinx **[TS 18,22]**

423) Find the maximum and minimum values of f(x)=3sinx– 4cosx **[IPE14]**

424) Find the minimum and maximum values of sin2x–cos2x.

425) Find the range of 7cosx–24sinx+5 **[AP M 20][AP J 15]**

426) Find the range of $13\cos x + 3\sqrt{3}\sin x - 4$ **[AP M 17]**

427) Find the extreme values of cos2x+cos^{2}x **[AP M 18]**

428) Find the extreme values of 3sin^{2}x+5cos^{2}x

429) Find the peroid of f(x)=sin(5x+3)∀x∈R

430) Find the period of f(x)=cos(3x+5)+7 **[TS J 17]**

431) Find the period of $f(x) = \cos\left(\frac{4x+9}{5}\right)$ **[TS 23]**

432) Find the period of tan5x. **[TS M 22]**

433) Find the period of tan(x+4x+9x+....+n^{2}x) **[AP J 17][AP,TS M 15,16]**

434) Find a cosine function whose period is 7. **[IPE 13] [TS M 20]**

435) Find a sine function whose period is 2/3 **[IPE 13] [AP M 16,22]**

436) Find the period of $f(x) = 2\sin\frac{\pi x}{4} + 3\cos\frac{\pi x}{3}$

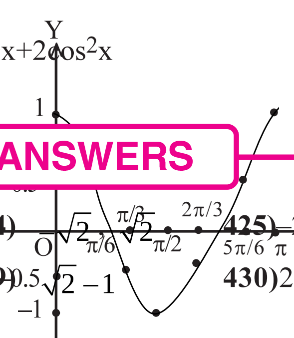
437) Find the period of $\frac{5\sin x + 3\cos x}{4\sin 2x + 5\cos x}$

438) Find the period of cos^{4}x.

439) Find the period of the function sin^{2}x+2cos^{2}x

ANSWERS

422) 5, –5	**423)** 5, –5	**424)** $-\sqrt{2}, \sqrt{2}$	**425)** –20, 30	**426)** –18, 10
427) –1, 2	**428)** 3,5	**429)** $\sqrt{2}-1$	**430)** $2\pi/5$	**431)** $5\pi/2$
432) $\pi/5$	**433)** $\frac{6\pi}{n(n+1)(2n+1)}$		**434)** $\cos\frac{2\pi}{7}x$	**435)** $\sin 3\pi x$
436) 24	**437)** 2π	**438)** π	**439)** π	



440) Sketch the graph of sinx in the intervals $[-\pi, \pi]$

441) Sketch the graph of sin2x in the intervals $(0, \pi)$ **[TS J 19] [AP J 19]**

442) Sketch the graph of cos2x in the intervals $[0, \pi]$

443) Sketch the graph of tanx between 0 and $\pi/4$

9) MISCELLANEOUS

444) If $\dfrac{2\sin\theta}{1+\cos\theta+\sin\theta} = x$, find the value of $\dfrac{1-\cos\theta+\sin\theta}{1+\sin\theta}$

445) If $\sin\alpha + \operatorname{cosec}\alpha = 2$, find value of $\sin^n\alpha + \operatorname{cosec}^n\alpha, n \in Z$ **[IPE 13]**

446) Prove that $\cos^2\dfrac{\pi}{10} + \cos^2\dfrac{2\pi}{5} + \cos^2\dfrac{3\pi}{5} + \cos^2\dfrac{9\pi}{10} = 2$

447) Show that $\cos^4\alpha + 2\cos^2\alpha\left(1 - \dfrac{1}{\sec^2\alpha}\right) = (1-\sin^4\alpha)$

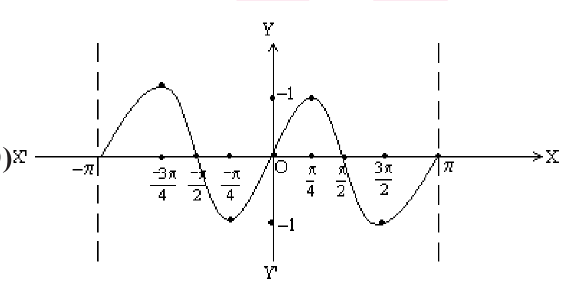
448) Find the value of $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$.

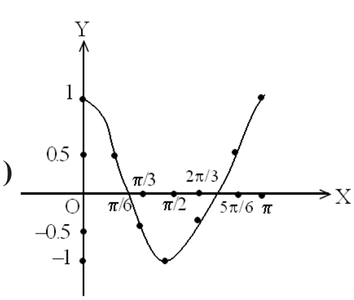
449) Prove that $\sin^2\alpha+\cos^2(\alpha+\beta)+2\sin\alpha\sin\beta\cos(\alpha+\beta)$ is independent of α.

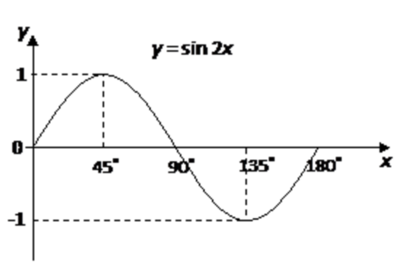
450) If $\cos\theta > 0$, $\tan\theta + \sin\theta = m$, $\tan\theta - \sin\theta = n$, then show that $m^2-n^2 = \mathbf{4\sqrt{mn}}$

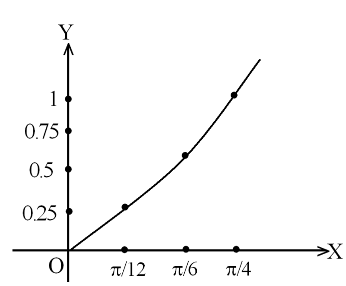
451) If $\tan^2\theta=(1-e^2)$ show that $\sec\theta + \tan^3\theta.\operatorname{cosec}\theta = (2-e^2)^{3/2}$

ANSWERS

440) 

441) 

442) 

443) 

444) x **445)** 2 **448)** –1

452) For what values of x in the first quadrant $\frac{2\tan x}{1-\tan^2 x}$ is positive ?

453) For what values of A in first quadrant the expression $\frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}$ is positive?

454) Find $\tan\left(\frac{\pi}{4}+A\right)$ interms of tanA

455) Evaluate $\sum \frac{\sin(A+B)\sin(A-B)}{\cos^2 A\cos^2 B}$; if none of cosA, cosB, cos C is zero.

456) If α, β are complementary angles such that bsinα =a, then find the value of (sinαcosβ–cosαsinβ).

457) Find the expansion of sin(A+B–C).

458) Prove that $\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right)=\frac{1}{\sqrt{2}}\sin A$.

459) If $0<A<B<\frac{\pi}{4}$ and $\sin(A+B)=\frac{24}{25}$ and $\cos(A-B)=\frac{4}{5}$, then find value of tan 2A

460) If A+B, A are acute angles such that $\sin(A+B)=\frac{24}{25}$, $\tan A=\frac{3}{4}$, then find the value of cos B.

461) If $\tan(\alpha-\beta)=\frac{7}{24}$ and $\tan\alpha=\frac{4}{3}$, where α, β are in first quadrant prove that α+β=π/2

462) Express $\cos^6 A+\sin^6 A$ interms of sin2A.

463) If $\frac{\sin\alpha}{a}=\frac{\cos\alpha}{b}$, then prove that asin2α + b cos2α =b

464) If θ is not an integral multiple of π/2 , prove that
tan θ + 2 tan 2θ + 4 tan 4θ + 8cot 8θ = cotθ **[AP M 19]**

465) If $\sin\alpha=\frac{3}{5}$, where $\frac{\pi}{2}<\alpha<\pi$, evaluate cos3α and tan2α.**[TS M 15]**

466) If $\cos A=\frac{7}{25}$ and $\frac{3\pi}{2}<A<2\pi$, then find the value of $\cot\frac{A}{2}$

467) If $0<\theta<\frac{\pi}{8}$, show that $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4\theta}}}=2\cos\left(\frac{\theta}{2}\right)$

468) Find the maximum and minimum values of $\cos\left(x+\frac{\pi}{3}\right)+2\sqrt{2}\sin\left(x+\frac{\pi}{3}\right)-3$

469) If $a\le\cos\theta+3\sqrt{2}\sin\left[\theta+\frac{\pi}{4}\right]+6\le b$, find largest value of a and smallest value of b.

470) Find the period of the function $2\sin\left(\frac{\pi}{4}+x\right)\cos x$

ANSWERS

452) $0<x<\frac{\pi}{4}$ **453)** $0<A<\frac{\pi}{6}(\text{or})\frac{\pi}{3}<A<\frac{\pi}{2}$ **454)** $\frac{1+\tan A}{1-\tan A}$ **455)** 0 **456)** $\frac{2a^2-b^2}{b^2}$

457) sin A cos B cos C + cos A sin B cos C– cos A cos B sin C + sin A sin B sin C

459) 3/4 **460)** 4/5 **462)** $1-\frac{3}{4}\sin^2 2A$ **465)** 44/125, –24/7 **466)** –4/5

468) 0,–6 **469)** 11, 1 **470)** π

VI) HYPERBOLIC FUNCTIONS

471) If sinhx=3/4 then find cosh2x and sinh2x. **[IPE 10,14,14][TS M 22]**

472) If coshx=5/2, then find the values of (i) cosh(2x) (ii) sinh(2x). **[AP J 18][TS J 15,17][TS M 17]**

473) If sinhx=3 then show that $x=\log(3+\sqrt{10})$ **[MP][TS M 16,19, 23][AP M 19,22]**

474) If sinhx=5, then show that $x=\log_e\left(5+\sqrt{26}\right)$ **[AP J 16]**

475) Show that $\text{Tanh}^{-1}\frac{1}{2}=\frac{1}{2}\log_e 3$ **[AP M 19,23][TS J 18] [AP M,J 17][AP M, J 15]**

476) Prove that $(\cosh x-\sinh x)^n=\cosh(nx)-\sinh(nx)$ **[TS M 15,18][AP M 20]**

477) Prove that $(\cosh x+\sinh x)^n=\cosh(nx)+\sinh(nx)$

478) Prove that $\cosh^2 x-\sinh^2 x=1$

479) Prove that $\cosh^2 x+\sinh^2 x=\cosh 2x$

480) Prove that $\cosh^4 x-\sinh^4 x=\cosh 2x$ **[TS M 22]**

481) Prove that $\sinh(3x)=3\sinh x+4\sinh^3 x$

482) Prove that $\cosh 3x=4\cosh^3 x-3\cosh x$

483) Prove that $\tanh 3x=\dfrac{3\tanh x+\tanh^3 x}{1+3\tanh^2 x}$, $\forall\ x\in R$

484) Prove that $\sinh(x-y)=\sinh x\cosh y-\cosh x\sinh y$.

485) Prove that $\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$

486) Prove that $\tanh(x-y)=\dfrac{\tanh x-\tanh y}{1-\tanh x\tanh y}$

487) Prove that $\coth(x-y)=\dfrac{\coth x.\coth y-1}{\coth y-\coth x}$

488) Prove that $\dfrac{\cosh x}{1-\tanh x}+\dfrac{\sinh x}{1-\coth x}=\sinh x+\cosh x$

489) Prove that $\dfrac{\tanh x}{\text{sech}x-1}+\dfrac{\tanh x}{\text{sech}x+1}=-2\,\text{cosech}x$, for $x\neq 0$

490) If $\cosh x=\sec\theta$ then prove that $\tanh^2\frac{x}{2}=\tan^2\frac{\theta}{2}$ **[IPE 13]**

491) If $u=\log_e\left(\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right)$ and if $\cos\theta>0$, then prove that $\cosh u=\sec\theta$.

ANSWERS

471) 17/8, 15/8 **472)** 23/2, $\pm\dfrac{5\sqrt{21}}{2}$