9.PROBABILITY

Sections	No. of periods (18)	Weightage in IPE [2x4+1x7=15]
1. Basic Terminology, Definitions of Probability	6	4 marks
2. Addition Theorem	6	4 or 7 marks
3. Conditional Probability	6	4 or 7 marks

In our every day life, we often come across the phrases such as 'Probably it will rain today', 'most probably, I will stand first in exam', 'odds in favour of India winning the match against Australia are 2:1 ' etc., All such phrases are involved with certain uncertainity. How to measure such uncertainity? A Mathematical measure of such uncertainity is provided in the 'theory of probability'. The 'theory of probability' is a branch of 'applied Mathematics' and it is the back bone of Mathematical statitics. It is widely used in various branches of Engineering, Physics, Chemistry, Medicine, Economics, Information theory, etc.,

The theory of probability has its origin in the 16th century, as a result of investigations of certain games of chance, like; throwing of coins/dice, playing cards in gambling. French Mathematician Fierre Simon de Laplace's work "Theorie analytique des probabilities" (Theory of analytical probability) published in 1812, gave the classical definition of probability of an event. ' The theory of Probability, as we know it today, is axiomatized by Andrei Nikolaevich Kolmogorov (1903-1987) in his work 'Foundations of the theory of Probability' and that axiomatic development is called 'Modern Probaility theory'.

Basic terminology with sufficient illustrations are given in each section. Classical definition with its limitations, axiomatic approach to probability, addition theorem, Conditional Probability, Multiplication theorem, Baye's theorem are discussed .

- By a coin, we mean it as an unbiased fair coin with possible outcomes head(H) , tail(T).
- By a die, we mean it as an unbiased fair cubical die with outcomes 1,2,3,4,5,6.
- The description of pack of 52 playing cards is as follows:

52 cards

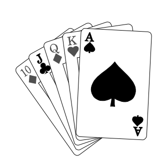
2 colours : *26 Red cards* *26 Black cards*

4 suits : *13 hearts (♥)* *13 diamonds(♦)* *13 clubs (♣)* *13 spades(♠)*

(A,2,3,4,5,6,7,8,9,10, J,Q,K) *(A,2......Q,K)* *(A,2,.....Q,K)* *(A,2,......Q,K)*

number cards

A,Q,K are honours and J,Q,K are face cards or count cards.



SYNOPSIS POINTS

1. If there are n mutually exclusive, equally likely, exhaustive events of a Random experiment, and m of them are favourable to an event E, then the probability of event E is $P(E) = \frac{m}{n}$

2. $P(E) + P(\overline{E}) = 1 \Rightarrow P(\overline{E}) = 1 - P(E)$

3.1. If $P(E) = \frac{m}{n}$ then odds in favour of event $E = \frac{m}{n-m} = \frac{P(E)}{P(\overline{E})}$

3.2. If $P(E) = \frac{m}{n}$ then odds against to event $E = \frac{n-m}{m} = \frac{P(\overline{E})}{P(E)}$

4.1. If odds in favour of event $E = \frac{m}{n}$ then $P(E) = \frac{m}{m+n}$ and $P(\overline{E}) = \frac{n}{m+n}$

4.2. If odds against to event $E = \frac{m}{n}$ then $P(E) = \frac{n}{m+n}$ and $P(\overline{E}) = \frac{m}{m+n}$

5. Probability function: Suppose S is the sample space of a random experiment E and S is finite, then a function $P:\mathcal{P}(S)\to R$ satisfying the following axioms is called Probability function.

(i) Axiom of non-negativity : $P(E)\geq 0,\ \forall\ E\in \mathbf{P}(S)$

(ii) Axiom of certainity : $P(S)=1$

(iii) Axiom of Union : If E_1, E_2 are exclusive events of S so that $E_1\cap E_2=\phi$, then $P(E_1\cup E_2)=P(E_1)+P(E_2)$

6.1. Addition Theorem on probability: If A, B are any two events of a random experiment then $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

6.2. If A, B are mutually exclusive events then $P(A\cup B)=P(A)+P(B)$.

6.3. $P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - P(\overline{A} \cap \overline{B})$

6.4. $P(A \cup B \cup C) = 1 - P(\overline{A \cup B \cup C}) = 1 - P(\overline{A} \cap \overline{B} \cap \overline{C})$

7. The conditional probability of B given A, is $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{n(A \cap B)}{n(A)}$

8. Multiplication Theorem on Probability: If A and B are two events of a random experiment then $P(A \cap B) = P(A)P\left(\frac{B}{A}\right) = P(B)P\left(\frac{A}{B}\right)$, $P(A)\neq 0$, $P(B)\neq 0$

9. If A,B are two mutually exclusive and exhaustive events and E is any event that happens from either A or B then $P(E)=P(A)P(E|A)+P(B)P(E|B)$

10.1. Two events A,B in a sample space S are said to be independent if $P(B|A)=P(B)$

10.2. Two events A,B in a sample space S are independent iff $P(A\cap B)=P(A)P(B)$

11. BAYE'S THEOREM: If $A_1, A_2, \ldots A_n$ are mutually exclusive and exhaustive events in a sample space S and E is any event intersecting with every A_i such that $P(E)\neq 0$ then

$$P(A_k \mid E) = \frac{P(A_k)P(E \mid A_k)}{\sum_{i=1}^{n} P(A_i)P(E \mid A_i)} \quad \text{for } k = 1,2,\ldots..n.$$

ADDITIONAL QUESTIONS WITH SOLUTIONS

1 **A number x is drawn arbitrary from the set {1,2,3,...,100}. Find the probability that $x+\frac{100}{x}>29$.** EAM Q

Sol: The sample space is S={1,2,3,....,100} $\Rightarrow$ n(S)=100

Now, $x+\frac{100}{x}>29 \Leftrightarrow x^2-29x+100>0 \Leftrightarrow (x-4)(x-25)>0 \Leftrightarrow x<4 \text{ or } x>25$

$\Leftrightarrow x \in \{1,2,3,26,27,...100\}$

$\Rightarrow$ the number of favourable cases satisfying the given inequality is 78.

$\therefore$ the required probability is $\frac{78}{100}=0.78$

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2 **Two unit squares are chosen at random on a chess board. Then show that the probability that they have a side in common is 1/18.** EAM Q

Sol: Total number of unit squares in the chess Board is 64

The number of ways of selecting 2 unit squares out of 64 is n(S)=$^{64}C_2$

The number of 2 squares with a side in common in the first row is 7

Since there are 8 rows, the total number of such squares is 7×8=56

Similarly, the number of 2 squares with a side in common along the 8 columns is 7×8=56

$\therefore$ the total number of ways of selecting 2 squares with a side in common is n(E)=56+56=112

$\therefore P(E)=\frac{112}{^{64}C_2}=\frac{112}{\frac{64\times 63}{2}}=\frac{1}{18}$

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3 **On a Festival day, a man plans to visit 4 holy temples A, B, C, D in a random order. Find the probability that he visits (i) A before B (ii) A before B and B before C.**

Sol: **i) Visiting A before B:**

Number of ways of visiting A,B,C,D = 4!=24

Among these 24 ways, A comes before B in 12 ways and A comes after B in 12 ways.

$\therefore$ The probability of visiting 4 temples, so that A comes before $B=\frac{12\times 1}{24}=\frac{1}{2}$

ii) Visiting A before B and B before C (A,B,C occur in a specific order):

Treating A,B,C as alike, the four can be arranged in $\frac{4!}{3!}=\frac{4\times \cancel{3\times 2\times 1}}{\cancel{3\times 2\times 1}}=4$

$\therefore$ The probability of visiting 4 temples, so that A comes before B and B before $C=\frac{4}{24}=\frac{1}{6}$

4 **Form the employees of a company. 5 persons are selected to represent them in the managing committee of the company. The particulars of 5 persons are as follows.**

Sl.No	Name	Sex	Age in years
1	Harish	M	30
2	Rohan	M	33
3	Sheetal	F	46
4	Alia	F	28
5	Salim	M	41

A person is selected at random from this group to act as a spokes person. Find the probability that the spokes person will be either male or above 35 years.

Sol: When a person is selected at random from the group of 5 persons to act as a spokesperson and S be the sample space, then $n(S) = {}^5C_1 = 5$

Let A be the event that selected person is male, then $n(A) = {}^3C_1 = 3$

Let B be the event that the selected person above 35 years, then $n(B) = {}^2C_1 = 2$,

Also $n(A \cap B) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{5},\ P(B) = \frac{n(B)}{n(S)} = \frac{2}{5} \Rightarrow P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{5}$$

The required probability $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{5} + \frac{2}{5} - \frac{1}{5} = \frac{4}{5}$

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5 **Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, find the probability that (i) you both enter the same section (ii) you both enter the different sections.**

Sol: Let S be the sample space of dividing 100 students into 2 sections of 40 & 60.

Then $n(S) = \frac{100!}{40!60!}$

Let us name section of 40 students as section I & section of 60 students as section II.

Also we name 'you' as A and 'your friend' as B.

i) A,B entering the same section: First A and B both enter the section I .

Then the remaining 98 students can be **divided** in two sections with first section 38 and second section 60 in $= \frac{98!}{38!60!}$ ways. Let A & B both enter the section II.

Then the remaining 98 students can be **divided** in two sections with First section 40 and second section 58 in $= \frac{98!}{40!58!}$ ways. $\therefore n(E) = \frac{98!}{38!60!} + \frac{98!}{40!58!}$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{\frac{98!}{38!60!} + \frac{98!}{40!58!}}{\frac{100!}{40!60!}} = \frac{98!}{38!60!} \times \frac{40!60!}{100!} + \frac{98!}{40!58!} \times \frac{40!60!}{100!}$$

$$= \frac{\cancel{(98!)} \times 40 \times 39 \times \cancel{38!}}{\cancel{38!} \times 100 \times 99 \times \cancel{98!}} + \frac{\cancel{98!} \times 60 \times 59 \times \cancel{58!}}{\cancel{58!} \times 100 \times 99 \times \cancel{98!}} = \frac{40 \times 39}{100 \times 99} + \frac{59 \times 60}{100 \times 99} = \frac{26}{165} + \frac{59}{165} = \frac{85}{165} = \frac{17}{33}$$

(ii) A & B enter the different sections:

Probability that the both A,B in different sections $P(\bar{E}) = 1 - P(E) = 1 - \frac{17}{33} = \frac{33-17}{33} = \frac{16}{33}$

6 **Prove that A and B are independent events if and only if $P\left(\frac{A}{B}\right)=P\left(\frac{A}{B^C}\right)$**

Sol: **Part I:** Let A and B are independent .Then $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{P(A).\cancel{P(B)}}{\cancel{P(B)}}=P(A)$ ....(1)

Also $P\left(\frac{A}{B^c}\right)=\frac{P(A\cap B^c)}{P(B^c)}=\frac{P(A).P(B^c)}{P(B^c)}=P(A)$ ....(2)

From (1) & (2), $P\left(\frac{A}{B}\right)=P\left(\frac{A}{B^c}\right)$

Part II: Let $P\left(\frac{A}{B}\right)=P\left(\frac{A}{B^c}\right)\Rightarrow\frac{P(A\cap B)}{P(B)}=\frac{P(A\cap B^c)}{P(B^c)}\Rightarrow\frac{P(A\cap B)}{P(B)}=\frac{P(A)-P(A\cap B)}{1-P(B)}$

By cross multiplying we get $P(A\cap B) - P(B)\,P(A\cap B) = P(A)\,P(B) - P(B)\,P(A\cap B)$

$\Rightarrow P(A\cap B) = P(A).P(B)$ $\therefore$ A,B are independent.

From Part I and Part II we conclude that A, B are independent iff $P\left(\frac{A}{B}\right)=P\left(\frac{A}{B^c}\right)$

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7 **For any two events A, B show that**

$P(A\cap B) - P(A)\,P(B) = P(A^c)P(B) - P(A^c\cap B) = P(A)P(B^c) - P(A\cap B^c)$

Sol: **(i)** Consider $P(A^c)P(B) - P(A^c\cap B)$

$= [1-P(A)]P(B) - [P(B) - P(A\cap B)] = P(B) - P(A)\,P(B) - P(B) + P(A\cap B)$

$= P(A\cap B) - P(A)\,P(B)$ ....(1)

(ii) Consider $P(A)P(B^c) - P(A\cap B^c)$

$= P(A)\,[1 - P(B)] - [P(A) - P(A\cap B)] = P(A) - P(A)\,P(B) - P(A) + P(A\cap B)$

$= P(A\cap B) - P(A)\,P(B)$ ....(2)

From (1), (2)

$P(A\cap B) - P(A)\,P(B) = P(A^c)P(B) - P(A^c\cap B) = P(A)P(B^c) - P(A\cap B^c)$

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8 **If A, B,C are three events in a random experiment, prove the following:**

i) $P\left(\frac{A}{A}\right)=1$ **ii)** $P\left(\frac{\phi}{A}\right)=0$ **iii)** $A\subseteq B\Rightarrow P\left(\frac{A}{C}\right)\le P\left(\frac{B}{C}\right)$

iv) $P(A - B) = P(A) - P(A\cap B)$

v) If A, B are mutually exclusive and $P(B) > 0$ then $P\left(\frac{A}{B}\right)=0$

vi) If A, B are mutually exclusive then $P(A/B^C)=\frac{P(A)}{1-P(B)}$, When $P(B)\neq 1$

vii) If A, B are mutually exclusive and $P(A\cup B)\neq 0$ then $P\left(\frac{A}{A\cup B}\right)=\frac{P(A)}{P(A)+P(B)}$

Sol: **i)** L.H.S= $P\left(\frac{A}{A}\right)=\frac{P(A\cap A)}{P(A)}=\frac{P(A)}{P(A)}=1$ = R.H.S

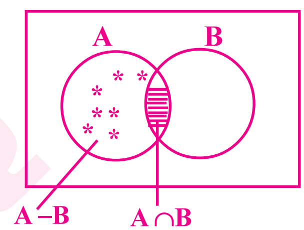
ii) L.H.S = $P\left(\frac{\phi}{A}\right)=\frac{P(\phi\cap A)}{P(A)}=\frac{0}{P(A)}=0$ = R.H.S

iii) We know that $P\left(\frac{A}{C}\right)=\frac{P(A\cap C)}{P(C)},P\left(\frac{B}{C}\right)=\frac{P(B\cap C)}{P(C)}$ ....(1)

Given $A\subseteq B\Rightarrow A\cap C\subseteq B\cap C\Rightarrow P(A\cap C)\le P(B\cap C)$

$$\Rightarrow\frac{P(A\cap C)}{P(C)}\le\frac{P(B\cap C)}{P(C)}\Rightarrow P\left(\frac{A}{C}\right)\le P\left(\frac{B}{C}\right)$$ [∵ from (1)]

iv) To prove P(A – B) = P(A) – P(A ∩B)



$A=(A-B)\cup(A\cap B)$

$\Rightarrow P(A) = P[(A-B)\cup(A\cap B)]$ [∵ $(A-B)\cap(A\cap B)=\phi$]

$\Rightarrow P(A) = P(A-B)+P(A\cap B)$ [∵ Union axiom]

$\Rightarrow P(A-B) = P(A)-P(A\cap B)$

v) If A, B are mutually exclusive and P(B) > 0 then $P\left(\frac{A}{B}\right)=0$

Given A, B are mutually exclusive events , $A\cap B=\phi\Rightarrow P(A\cap B)=0$

L.H.S= $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{0}{P(B)}=0$ =R.H.S

vi) If A, B are mutually exclusive then $P(A/B^C)=\frac{P(A)}{1-P(B)}$ **,When P(B) ≠ 1**

Given A, B are mutually exclusive events , $A\cap B=\phi\Rightarrow P(A\cap B)=0$

Also $(A\cap\bar{B})\cup(A\cap B)=A\Rightarrow P\left[(A\cap\bar{B})\cup(A\cap B)\right]=P(A)$

$\Rightarrow P(A\cap\bar{B})+P(A\cap B)=P(A)\Rightarrow P(A\cap\bar{B})=P(A)-P(A\cap B)$

L.H.S= $P(A/B^C)=\frac{P(A\cap B^C)}{P(B^C)}=\frac{P(A)-P(A\cap B)}{1-P(B)}=\frac{P(A)-0}{1-P(B)}=\frac{P(A)}{1-P(B)}$ = R.H.S

vii) If A, B are mutually exclusive and P(A ∪ B) ≠ 0 then $P\left(\frac{A}{A\cup B}\right)=\frac{P(A)}{P(A)+P(B)}$

Given A, B are mutually exclusive events , $A\cap B=\phi\Rightarrow P(A\cap B)=0$

L.H.S= $P\left(\frac{A}{A\cup B}\right)=\frac{P\left[A\cap(A\cup B)\right]}{P(A\cup B)}=\frac{P(A)}{P(A)+P(B)}$ = R.H.S [∵ $A\cap(A\cup B)=A$]

9 **A bag contains 10 balls; 5 of which are red and the remaining blue. Two balls are drawn at random from the bag one after the other with replacement. Let A be the event that the first ball drawn is red B be the event that the second ball is red. Find whether these events are independent or not.**

Sol: Since the first ball drawn is replaced before drawing the second ball, there are $10 \times 10 = 100$ ways of picking the two balls.

Out of these 100, $5 \times 10 = 50$ draws have the property that the first ball is red.

Thus . $P(A) = \frac{50}{100} = \frac{1}{2}$ Similarly $P(B) = \frac{1}{2}$.

Also, there are $5 \times 5 = 25$ ways of drawing two balls such that the first and second are both red balls.

$\therefore P(A \cap B) = \frac{25}{100} = \frac{1}{4}$

But $P(A \cap B) = \frac{1}{4} = \frac{1}{2} \times \frac{1}{2} = P(A).P(B)$

Hence event A and B are independent. This outcome makes sense because the colour of the ball we observe on the second draw does not in any way depend on the colour of the first ball.

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10 **A person secures a job in a construction company in which the probability that the workers go on strike is 0.65 and the probability that the construction job will be completed on time if there is no strike is 0.80. If the probability that the construction job will be completed on time even if there is a strike is 0.32, determine the probability that the constructed job will be completed on time.**

Sol: Let P(S) = Probability of the workers go on strike = 0.65

$P(\bar{S})$ = Probability on the workers go no strike = 1 – P(S) = 1 –0.65 = 0.35

$P\left(\frac{E}{S}\right)$ =Probability that the job completed if there is a no strike = 0.32.

$P\left(\frac{E}{\bar{S}}\right)$ = Probability that the job completed if there is a strike = 0.80.

p(E) = Probability that the construction job will be completed on time.

$= P(S)\ P\left(\frac{E}{S}\right) + P(\bar{S})\ P\left(\frac{E}{\bar{S}}\right) = (0.65)\,(0.32) + (0.35)\,(0.08) = 0.2080 + 0\ .2800 = 0.4880$

MODEL PROBLEMS IN PROBABILITY

Concept

1. Classical Definition

$P(E) = \frac{n(E)}{n(S)} = \frac{m}{n}$

Illustrations

1. Find the probability of getting 2 heads when 5 coins are tossed.

✍ $n(S) = 2^5 = 32$; The number of heads when 2 coins are tossed is $n(E) = {}^5C_2 = \frac{5.4}{2.1} = 10 \therefore P(E) = \frac{n(E)}{n(S)} = \frac{10}{32} = \frac{5}{16}$

2. Find the probability of getting equal numbers when 2 dice are thrown.

✍ $n(S)=6^2=36$; The event of getting the same number is $E=\{(1,1), (2,2), ...(6,6)\} \Rightarrow n(E)=6 \therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

3. Two fair dice are rolled. What is the probability that the sum on the faces of 2 dice is 10?

✍ $n(S)=6^2=36$; The event of getting sum 10 is $E=\{(4,6), (5,5), (6,4)\} \Rightarrow n(E)=3$ $\therefore P(E) = \frac{3}{36} = \frac{1}{12}$

4. What is the probability of drawing a king or a queen from a well shuffled pack of 52 cards.

✍ $n(S)=52$; The pack of cards contains 4 king cards, 4 queen cards $\Rightarrow n(E)=4+4=8 \therefore P(E) = \frac{8}{52} = \frac{2}{13}$

5. Find the probability that a non-leap year contains 53 Sundays.

✍ The number of days in a non-leap year is 365; 365=(52x7)+1=52 weeks + 1 day $\Rightarrow$ S={Sun, Mon, Tue, Wed, Thu, Fri, Sat}
$\Rightarrow n(S)=7$, E={Sun} $\Rightarrow n(E)=1$ $\therefore P(E)=1/7$

6. Find the probability of getting 52 Sundays in a leap year.

✍ The number of days in a leap year is 366. Hence 366=53 weeks+2days
$\therefore$ S={Sun & Mon, Mon & Tue, Tue&Wed, Wed&Thu, Thu&Fri, Fri&Sat, Sat & Sun } $\Rightarrow n(S)=7$.
There are 5 favourable cases in S for a non Sunday. Hence $n(E)=5$ $\therefore P(E)=5/7$

7. A page is opened arbitrarily from a book of 200 pages. What is the probability that the number on the page is a perfect square.

✍ $S=\{1,2,....200\} \Rightarrow n(S)=200$; $E=\{1^2, 2^2,....14^2\} \Rightarrow n(E)=14 \therefore P(E) = \frac{14}{200} = \frac{7}{100}$

8. A number is picked from 1 to 20, both inclusive. Find the probability that it is a prime.

✍ $S=\{1,2,...20\} \Rightarrow n(S)=20$; $E=\{2,3,5,7,11,13,17,19\} \Rightarrow n(E)=8 \therefore P(E) = \frac{8}{20} = \frac{2}{5}$

Concept	Illustrations
2.Addition theorem $P(A\cup B)=P(A)+P(B)-P(A\cap B)$	See below

Illustrations

1. If $P(A)=\frac{1}{2}, P(B)=\frac{1}{4}, P(A\cap B)=\frac{1}{8}$ find $P(A\cap B)$

✍ $P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{1}{2}+\frac{1}{4}-\frac{1}{8}=\frac{4+2-1}{8}=\frac{5}{8}$

2. If P(A)=0.25, P(B)=0.5, P(A∩B)=0.14 then find $P(\overline{A}\cap\overline{B})$

✍ $P(\overline{A}\cap\overline{B})=P(\overline{A\cup B})=1-P(A\cup B)=1-(P(A)+P(B)-P(A\cap B))=1-(0.2+0.5-0.14)=1-0.61=0.39$

3. If P(A∪B)=0.65, P(A∩B)=0.15 then find $P(\overline{A})+P(\overline{B})$

✍ $P(A\cup B)=P(A)+P(B)-P(A\cap B)=1-P(\overline{A})+1-P(\overline{B})-P(A\cap B)$

$\Rightarrow P(\overline{A})+P(\overline{B})=2-(P(A\cup B)+P(A\cap B)=2-(0.65+0.15))=1.2$

4. When two dice are thrown, find the probability of getting the sums of 10 or 11.

✍ Let S be the sample space associated with the throwing of two dice. $\Rightarrow n(S)=6X6=36$

Let A denote the event of getting the sum 10. $\Rightarrow A=\{(4,6),(5,5),(6,4)\}\Rightarrow n(A)=3$

Let B denote the event of getting the sum 11. $\Rightarrow B=\{(5,6),(6,5)\}\Rightarrow n(B)=2$

$\therefore$ the event of getting a sum 10 or 11 = $A\cup B$ and $A\cap B=\phi$.

$\therefore P(A\cup B)=P(A)+P(B)-0=\frac{3}{36}+\frac{2}{36}=\frac{5}{36}$

5. If one ticket is randomly selected from tickets numbered 1 to 30, then find the probabilty that the number on the ticket is a multiple of 3 or 5

✍ n(S)=30, The event of multiple of 3 is $A=\{3,6,...27,30\}\Rightarrow n(A)=10$; The event of multiple of 5 is $B=\{5,10,...25,30\}\Rightarrow n(B)=6$

The event of multiple of 3, 5 is $A\cap B=\{15,30\}\Rightarrow n(A\cap B)=2$

$\therefore P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{10}{30}+\frac{6}{30}-\frac{2}{30}=\frac{14}{30}=\frac{7}{15}$

6. Find the probability of drawing an ace or a spade from a pack of well shuffled cards.

✍ n(S)=52, Let A be the event of getting an Ace, B be the event of getting a spade. n(A)=4, n(B)=13; Also $n(A\cap B)=1$

$\therefore P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{4}{52}+\frac{13}{52}-\frac{1}{52}=\frac{16}{52}=\frac{4}{13}$

Concept	Illustrations
3.**Conditional probability** $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{n(A\cap B)}{n(B)}$	**1.** If $P(A)=\frac{1}{2}, P(A\cap B)=\frac{1}{6}$ then find $P\left(\frac{B}{A}\right)$ ✍ $P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}=\frac{1/6}{1/2}=\frac{1}{3}$ **2. If P(A)>0, P(B)≠1, then show that** $P\left(\frac{\overline{A}}{\overline{B}}\right)=\frac{1-P(A\cup B)}{1-P(B)}$ ✍ $P\left(\frac{\overline{A}}{\overline{B}}\right)=\frac{P(\overline{A}\cap\overline{B})}{P(\overline{B})}=\frac{P(\overline{A\cup B})}{1-P(B)}=\frac{1-P(A\cup B)}{1-P(B)}$ **3. What is the probability that a 6 is obtained, on one of the dice in a throw of two dice, given that the sum is 7?** ✍ Let 'A' be the event of getting sum is 7 $\Rightarrow$ A={(2,5), (5,2), (1,6), (6,1), (3,4), (4,3)}. $\therefore$ n(A)=6 Let B be the event of getting 6 on any one of the dice $\Rightarrow A\cap B=\{(6,1),(1,6)\}\Rightarrow n(A\cap B)=2$ $\therefore P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}=\frac{n(A\cap B)}{n(A)}=\frac{2}{6}=\frac{1}{3}$ **4. An urn contains 7 red and 3 black balls. Two balls are drawn without replacement. Find the probability that the second ball is red if it is known that the first is red.** ✍ Let R_1 be the event that the first ball drawn is red and R_2 be the event that the second ball drawn is red again. Then $P(R_1)=\frac{7}{10}$. But, after one red ball is chosen, in the remaining there are 6 red balls and 3 black balls only. Therefore, the required probability is $P\left(\frac{R_2}{R_1}\right)=\frac{6}{9}=\frac{2}{3}$
4.**Independent, dependent events.** A,B are independent $\Leftrightarrow P(A\cap B)=P(A).P(B)$ (or) $P\left(\frac{B}{A}\right)=P(B)$	**1. If A,B are 2 independent events in a random experiment such that P(A)=2/5, P(B)=1/2 find P(A∪B)** ✍ $P(A\cup B)=P(A)+P(B)-P(A\cap B)=P(A)+P(B)-P(A).P(B)$ ($\because$ A,B are independent) $=\frac{2}{5}+\frac{1}{2}-\frac{2}{5}.\frac{1}{2}=\frac{7}{10}$ **2. If P(A)=0.4, P(A∪B)=0.7 then find P(B) when A,B are independent events.** ✍ $P(A\cup B)=P(A)+P(B)-P(A\cap B)=P(A)+P(B)-P(A).P(B)$. ($\because$ A,B are independent)$=P(A)+P(B)(1-P(A))$ $\therefore 0.7=0.4+P(B)(1-0.4)=0.4+P(B)(0.6)\Rightarrow(0.6)P(B)=0.7-0.4=0.3\Rightarrow P(B)=\frac{0.3}{0.6}=\frac{1}{2}=0.5$ **3. If A,B are two independent events in a sample space S, then prove that $\overline{A}$,B are independent** ✍ $B=(B-A)\cup(A\cap B)$ also $(B-A)\cap(A\cap B)=\phi$. $\therefore P(B)=P[(B-A)\cup(A\cap B)]=P(B-A)+P(A\cap B)$ $\Rightarrow P(B-A)=P(B)-P(A\cap B)\Rightarrow P(B\cap\overline{A})=P(B)-P(A)P(B)\Rightarrow P(\overline{A}\cap B)=[1-P(A)]P(B)=P(\overline{A})P(B)$ $\therefore \overline{A}$,B are independent

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4. Two cards are drawn in succession (without replacement) froma pack of cards. Find the probability that the first card is king and the second card is queen.

✍ Let A be the event of drawing a king in the first draw and the B be the event of drawing a queen in the second draw. Here A,B are dependent events.

$\therefore$ The required probability is $P(A\cap B) = P(A)P\left(\frac{B}{A}\right) = \left(\frac{4}{52}\right)\left(\frac{4}{51}\right)$

5. A problem in Calculus is given to two students A and B whose chances of solving it are 1/3, 1/4. What is probability that the problem will be solved if both of them try independently?

✍ Let A, B denote two independent events that the problem is solved by A, B respectively.

Given that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$ $\therefore$ the required probability $P(A\cup B) = P(A)+P(B)-P(A\cap B)$

$=P(A)+P(B)-P(A)P(B)$ ($\because$ A, B are indepedent) $= \frac{1}{3}+\frac{1}{4}-\frac{1}{12} = \frac{1}{2}$

6. A talks truth in 75% of the cases; B in 80% cases. What is the probability that their statements about an incident do not match?

✍ Let A, B be two independent events that A and B respectively speak truth about an incident.

Then $P(A) = \frac{75}{100} = \frac{3}{4} \Rightarrow P(\overline{A}) = 1-\frac{3}{4} = \frac{1}{4} =$, $P(B) = \frac{80}{100} = \frac{4}{5} \Rightarrow P(\overline{B}) = 1-\frac{4}{5} = \frac{1}{5}$

Let E be the event that their statements do not match about an incident

This happens in two mutually exclusive ways.

(i) A speaks truth and B tells lie; $A\cap\overline{B}$ (ii) A tells lie and B speaks truth; $\overline{A}\cap B$

$\therefore P(E) = P(A\cap\overline{B})\cup P(\overline{A}\cap B) = P(A\cap\overline{B}) + P(\overline{A}\cap B)$ [$\because A\cap\overline{B}, \overline{A}\cap B$ are exclusive]

$= P(A)P(\overline{B}) + P(\overline{A})P(\overline{B})$ [$\because$ A, B are indepedent]

$= \frac{3}{4}\times\frac{1}{5}+\frac{1}{4}\times\frac{4}{5} = \frac{7}{20}$

Concept	Illustrations
5.Multiplication th. on compound probability $P(A\cap B)=P(A)P\left(\frac{B}{A}\right)$ $=P(B).P\left(\frac{E}{B}\right)$ *Cor*: $P(E)=$ $P(A).P\left(\frac{E}{A}\right)+P(B).P\left(\frac{E}{B}\right)$	**1. A bag contains 3 white and 4 red balls. 1 ball is drawn from it and it is not replaced. Now one more ball is drawn from it. Find the probability that first ball is white and second ball is red.** ✍ $P(A)=\frac{3}{7},\ P\left(\frac{B}{A}\right)=\frac{4}{6}\ \therefore P(A\cap B)=P(A).P\left(\frac{B}{A}\right)=\frac{3}{7}.\frac{4}{6}=\frac{2}{7}$ **2. A bag B_1 contains 4 white and 2 black balls. Bag B_2 contains 3 white and 4 black balls. A bag is drawn at random and a ball is chosen at random from it. Then what is the probability that the ball drawn is white?** ✍ Let E_1,E_2 be the events of choosing bag B_1 and bag B_2 respectively, then $P(E_1)=P(E_2)=1/2$ Let W be the event that the ball chosen from the bag selected is white. $\Rightarrow P\left(\frac{W}{E_1}\right)=\frac{4}{6}=\frac{2}{3},\quad P\left(\frac{W}{E_2}\right)=\frac{3}{7}$ Also, E_1,E_2 are mutually exclusive and exhaustive events. $\therefore P(W)=P(E_1).P\left(\frac{W}{E_1}\right)+P(E_2)P\left(\frac{W}{E_2}\right)=\frac{1}{2}.\frac{2}{3}+\frac{1}{2}.\frac{3}{7}=\frac{1}{3}+\frac{3}{14}=\frac{14+9}{3.14}=\frac{23}{42}$
6. Baye's Theorem $P\left(\frac{A_k}{E}\right)=\frac{P(A_k)P(E/A_k)}{\sum_{i=1}^{n}P(Ai).P(E/A_i)}$	**1. A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that the red ball drawn is from bag B.** ✍ Let A, B be the events of choosing bag A, bag B E be the event of drawing red ball $\Rightarrow P(A)=\frac{1}{2},\ P(B)=\frac{1}{2},\ P\left(\frac{E}{A}\right)=\frac{3}{5},\ P\left(\frac{E}{B}\right)=\frac{5}{9}$ $\therefore P\left(\frac{B}{E}\right)=\frac{P(B)P\left(\frac{E}{B}\right)}{P(A)P\left(\frac{E}{A}\right)+P(B)P\left(\frac{E}{B}\right)}=\frac{(1/2)(5/9)}{(1/2)(3/5)+(1/2)(5/9)}=\frac{25}{52}$