WELCOME

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DIGITAL MATERIAL

DIFFERENTIATION - INDEX

9. DIFFERENTIATION

ADDITIONAL QUESTIONS WITH SOLUTIONS

1. Find the derivative of $y=\sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}$

Sol: Given $y=\sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}$

$$\Rightarrow \log y=\log\left(\frac{(x-3)(x^2+4)}{3x^2+4x+5}\right)^{1/2}=\frac{1}{2}\log\left[\frac{(x-3)(x^2+4)}{3x^2+4x+5}\right]=\frac{1}{2}[\log(x-3)+\log(x^2+4)-\log(3x^2+4x+5)]$$

$[\because \sin 3\theta=3\sin\theta-4\sin^3\theta]$

Diff. w.r. to x, $\dfrac{1}{y}\cdot\dfrac{dy}{dx}=\dfrac{1}{2}\left(\dfrac{1}{x-3}+\dfrac{2x}{x^2+4}-\dfrac{6x+4}{3x^2+4x+5}\right)\Rightarrow\dfrac{dy}{dx}=\dfrac{y}{2}\left(\dfrac{1}{x-3}+\dfrac{2x}{x^2+4}-\dfrac{6x+4}{3x^2+4x+5}\right)$

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2. Find the second order derivative of $y=e^{-2x}\sin^3 x$

Sol: Given $y=e^{-2x}\sin^3 x\Rightarrow y=e^{-2x}\left(\dfrac{3\sin x-\sin 3x}{4}\right)$

Diff. w.r. to x, $\Rightarrow y'=\dfrac{1}{4}[e^{-2x}(3\cos x-3\cos 3x)+(3\sin x-\sin 3x)e^{-2x}(-2)]$

$\Rightarrow y'=\dfrac{1}{4}e^{-2x}(3\cos x-3\cos 3x-6\sin x+2\sin 3x)$

Again diff. w.r. to x,

$\Rightarrow y''=\dfrac{1}{4}\left[e^{-2x}(-3\sin x+9\sin 3x-6\cos x+6\cos 3x)+(3\cos x-3\cos 3x-6\sin x+2\sin 3x)(-2)e^{-2x}\right]$

$\Rightarrow y''=\dfrac{1}{4}e^{-2x}(-3\sin x+9\sin 3x-6\cos x+6\cos 3x-6\cos x+6\cos 3x+12\sin x-4\sin 3x)$

$=\dfrac{1}{4}e^{-2x}(5\sin 3x+12\cos 3x+9\sin x-12\cos x)$

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3. Find the second order derivative of $e^x\sin x\cos 2x$

Sol: $y=e^x\sin x\cos 2x=\dfrac{1}{2}e^x(2\sin x\cos 2x)=\dfrac{1}{2}e^x[\sin 3x+\sin(-x)]=\dfrac{1}{2}e^x(\sin 3x-\sin x)$

Diff. w.r. to x, $\Rightarrow y'=\dfrac{1}{2}[e^x(3\cos 3x-\cos x)+(\sin 3x-\sin x)e^x]$

$y'=\dfrac{1}{2}e^x(3\cos 3x-\cos x+\sin 3x-\sin x)$ Again diff. w.r. to x,

$y''=\dfrac{1}{2}[e^x(-9\sin 3x+\sin x+3\cos 3x-\cos x)+(3\cos 3x-\cos x+\sin 3x-\sin x)e^x]$

$=\dfrac{e^x}{2}(-9\sin 3x+\sin x+3\cos 3x-\cos x+3\cos 3x-\cos x+\sin 3x-\sin x)$

$=\dfrac{e^x}{2}(6\cos 3x-8\sin 3x-2\cos x)=e^x(3\cos 3x-4\sin 3x-\cos x)$

4. If $y = 6(x+1) + (a+bx)e^{3x}$ then prove that $y'' - 6y' + 9y = 54x + 18$

Sol: Given $y = 6(x+1) + (a+bx)e^{3x}$............(1) Diff. w.r.t x

$$\Rightarrow y' = 6\frac{d}{dx}(x+1) + (a+bx)\frac{d}{dx}(e^{3x}) + e^{3x}\frac{d}{dx}(a+bx)$$

$$= 6(1+0) + [(a+bx)3e^{3x} + e^{3x}(b)]$$

$$\Rightarrow y' = 6 + 3(a+bx)e^{3x} + be^{3x} = 6 + 3[y - 6(x+1)] + be^{3x} \quad [\because \text{ From(1)}]$$

$$\Rightarrow y' = 6 + 3y - 18(x+1) + be^{3x} \text{.................(2)} \quad \text{Again Diff. w.r.t x}$$

$$\Rightarrow y'' = 0 + 3y' - 18(1) + b3e^{3x} = 3y' - 18 + 3be^{3x}$$

$$= 3y' - 18 + 3[y' - 6 - 3y + 18(x+1)] \quad [\because \text{From(2)}]$$

$$= 3y' - 18 + 3y' - 18 - 9y + 54(x+1) = 6y' - 9y + 54x + 18$$

$$\Rightarrow y'' - 6y' + 9y = 54x + 18$$

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5. If $y = 128\sin^3 x\cos^4 x$, then find y''.

Sol: Given $y = 128\sin^3 x\cos^4 x$ Differentiating w.r.to x,

$$y' = 128\left[\sin^3 x\frac{d}{dx}(\cos^4 x) + \cos^4 x\frac{d}{dx}(\sin^3 x)\right]$$

$$= 128[\sin^3 x[4\cos^3 x(-\sin x)] + \cos^4 x(3\sin^2 x.\cos x)]$$

$$= 128(3\sin^2 x\cos^5 x - 4\sin^4 x\cos^3 x) \quad \text{Again Differentiating w.r.to x}$$

$$y'' = 128\{3(\sin^2 x\, 5\cos^4 x(-\sin x) + \cos^5 x\,.\,2\sin x\cos x)]$$

$$-4[\sin^4 x.\,3\cos^2 x(-\sin x) + \cos^3 x.4\sin^3 x.\cos x]\}$$

$$= 128[-15\sin^3 x\cos^4 x + 6\sin x\cos^6 x + 12\sin^5 x\cos^2 x - 16\sin^3 x\cos^4 x]$$

$$\therefore y'' = 128(6\sin x\cos^6 x + 12\sin^5 x.\cos^2 x - 31\sin^3 x.\cos^4 x)$$

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6. If $y = ae^{-bx}\cos(cx+d)$ then prove that $y'' + 2by' + (b^2+c^2)y = 0$

Sol: Given $y = ae^{-bx}\cos(cx+d)$. Differentiating w.r.to x

$$y' = ae^{-bx}[-c\sin(cx+d)] + \cos(cx+d)(-abe^{-bx})$$

$$= -ace^{-bx}\sin(cx+d) - by$$

$$\Rightarrow y' + by = -ace^{-bx}\sin(cx+d) \text{.......(1)} \quad \text{Again differentiating w.r.to x}$$

$$\Rightarrow y'' + by' = -ace^{-bx}[c\cos(cx+d)] + \sin(cx+d)\,.\,[-ac(-b)e^{-bx}]$$

$$\Rightarrow y'' + by' = -c^2[ae^{-bx}\cos(cx+d)] - b[-ace^{-bx}\sin(cx+d)]$$

$$\Rightarrow y'' + by' = -c^2y - b(y'+by) \quad [\because \text{From (1)}] \quad \Rightarrow y'' + by' = -c^2y - by' - b^2y$$

$$\Rightarrow y'' + by' + c^2y + by' + b^2y = 0 \Rightarrow y'' + 2by' + (b^2+c^2)y = 0$$

7. If $y=e^{\frac{-k}{2}x}$ (a cos nx + b sin nx) prove that then $y'' + ky' + \left(n^2+\frac{k^2}{4}\right)y = 0$

Sol: Given $y=e^{\frac{-k}{2}x}(a\cos nx+b\sin nx)$ .....(1) . Diff. w.r.t x

$$\therefore y'=e^{\frac{-k}{2}x}(-an\sin nx+bn\cos nx)+(a\cos nx+b\sin nx)\left[e^{\frac{-k}{2}x}\left(\frac{-k}{2}\right)\right]$$

$$\Rightarrow y'=e^{\frac{-k}{2}x}(-an\sin nx+bn\cos nx)-\frac{k}{2}y \Rightarrow y'+\frac{k}{2}y=e^{\frac{-k}{2}x}(-an\sin nx+bn\cos nx)..........(2)$$

Again diff. w.r.t. x on both sides

$$\Rightarrow y''+\frac{k}{2}y'=e^{\frac{-kx}{2}}(-an^2\cos nx-bn^2\sin nx)+(-a\,n\sin nx+b\,n\cos nx)\left[e^{\frac{-k}{2}x}\left(\frac{-k}{2}\right)\right].$$

$$\Rightarrow y''+\frac{k}{2}y'=-n^2e^{\frac{-k}{2}x}(a\cos nx+b\sin nx)-\frac{k}{2}e^{\frac{-k}{2}x}(-a\,n\sin nx+b\,n\cos nx)$$

$$\Rightarrow y''+\frac{k}{2}y'=-n^2y-\frac{k}{2}\left[y'+\frac{k}{2}y\right] \qquad [\because \text{From}(1)\&(2)]$$

$$\Rightarrow y''+\frac{k}{2}y+n^2y+\frac{k}{2}\left[y'+\frac{k}{2}y\right]=0 \quad \Rightarrow y''+ky'+\left(n^2+\frac{k^2}{4}\right)y=0$$

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8. If $ay^4=(x+b)^5$ then show that $5yy''=(y')^2$. TS 17

Sol: Given $ay^4=(x+b)^5 \Rightarrow (ay^4)^{1/4}=(x+b)^{5/4} \Rightarrow a^{1/4}y=(x+b)^{5/4}$

$$\Rightarrow y=\frac{1}{a^{1/4}}(x+b)^{5/4}\ ....(1)$$

Differentiating w.r.t 'x' we get $y'=\frac{1}{a^{1/4}}.\frac{5}{4}(x+b)^{1/4}$

Again differentiating w.r.t 'x' we get $y''=\frac{1}{a^{1/4}}.\frac{5}{4}.\frac{1}{4}.(x+b)^{-3/4}$ ....(2)

From (1) & (2), $5yy''=5.\frac{1}{a^{1/4}}.\frac{1}{a^{1/4}}.\frac{5}{16}(x+b)^{-3/4}(x+b)^{5/4}$

$$=\frac{25}{16a^{2/4}}(x+b)^{2/4}=\left[\frac{5}{4a^{1/4}}.(x+b)^{1/4}\right]^2=(y')^2$$

9. Find the derivative of $Tan^{-1}\left(\frac{3a^2x - x^3}{a(a^2 - 3x^2)}\right)$

Sol: Put x=a tanθ then

$$\frac{3a^2x - x^3}{a(a^2 - 3x^2)} = \frac{3a^2x - x^3}{a^3 - 3ax^2} = \frac{3a^2(a\tan\theta) - (a\tan\theta)^3}{a^3 - 3a(a\tan\theta)^2}$$

$$= \frac{3a^3\tan\theta - a^3\tan^3\theta}{a^3 - 3a(a^2\tan^2\theta)} = \frac{\cancel{a^3}(3\tan\theta - \tan^3\theta)}{\cancel{a^3}(1 - 3\tan^2\theta)} = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \tan 3\theta$$

Now $y = Tan^{-1}(\tan 3\theta) = 3\theta \Rightarrow y = 3Tan^{-1}\left(\frac{x}{a}\right)$ $\left[\because x = a\tan\theta \Rightarrow \tan\theta = \frac{x}{a} \Rightarrow \theta = Tan^{-1}\left(\frac{x}{a}\right)\right]$

$$\therefore 3\frac{d}{dx}Tan^{-1}\left(\frac{x}{a}\right) = 3.\frac{1}{1+\left(\frac{x}{a}\right)^2}\frac{d}{dx}\left(\frac{x}{a}\right) = 3\left(\frac{\cancel{a^2}}{a^2 + x^2}\right)\frac{1}{\cancel{a}} = \frac{3a}{x^2 + a^2}$$