WELCOME

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DIGITAL MATERIAL

LIMITS & CONTINUITY - INDEX

8. LIMITS & CONTINUITY

ADDITIONAL QUESTIONS WITH SOLUTIONS

1. Show that the function $f(x)=[\cos(x^{10}+1)]^{1/3}$, $x\in R$ is a continuous function.

Sol: Define 3 real functions f_1, f_2 and f_3 as follows:

$f_1(x)=x^{10}+1$, $f_2(x)=\cos x$, $f_3(x)=x^{1/3}$

Then $(f_3 o f_2 o f_1)(x)=f_3[f_2(f_1(x))]$

$=[\cos(x^{10}+1)]^{1/3}=f(x)$

Since f_1, f_2, f_3 are continuous on their respective domains it is obvious that f(x) is continuous on R.

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2. If f is a function defined by $f(x)=\begin{cases}\frac{x-1}{\sqrt{x}-1} & \text{if } x>1\\ 5-3x & \text{if } -2\le x\le 1\\ \frac{6}{x-10} & \text{if } x<-2\end{cases}$ then discuss the continuity of f.

Sol: We check the continuity of f(x) at the split points x = 1,–2

(a) Continuity at x =1

(i) At x =1, f(1) =5–3(1)= 5–3 =2

$[\because \text{At } x=1, f(x)=5-3x]$

(ii) L.H.L $= \underset{x\to1-}{\text{Lt}} f(x) = \underset{x\to1-}{\text{Lt}} (5-3x) = 5-3(1) = 5-3 = 2$ $[\because x\to1- \Rightarrow x<1 \Rightarrow f(x)=5-3x]$

(iii) R.H.L $= \underset{x\to1+}{\text{Lt}} f(x) = \underset{x\to1+}{\text{Lt}} \frac{x-1}{\sqrt{x}-1} = \underset{x\to1+}{\text{Lt}} \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{(\sqrt{x}-1)} = \underset{x\to1+}{\text{Lt}} \sqrt{x}+1 = \sqrt{1}+1 = 2$

Thus f(1) = L.H.L = R.H.L. Hence f(x) is continuous at x =1

(b) Continuity at x = –2

$[\because \text{At } x=-2, f(x)=5-3x]$

(i)At x = –2, f(–2) =5–3(–2)= 5+6 =11

$\left[\because x\to-2- \Rightarrow x<-2 \Rightarrow f(x)=\frac{6}{x-10}\right]$

(ii)L.H.L $= \underset{x\to-2-}{\text{Lt}} f(x) = \underset{x\to-2-}{\text{Lt}} \frac{6}{x-10} = \frac{6}{-2-10} = \frac{-6}{12} = -\frac{1}{2}$

Here f(–2) ≠ L.H.L Hence f(x) is discontinuous at x = –2.

3. Check the continuity of 'f' given by $f(x) = \begin{cases} 4-x^2 & \text{If } x \le 0 \\ x-5 & \text{If } 0 < x \le 1 \\ 4x^2-9 & \text{If } 1 < x < 2 \\ 3x+4 & \text{If } x \ge 2 \end{cases}$ **at the points 0, 1 and 2**

Sol: We check the continuity of $f(x)$ at the split points $x = 0,1,2$

(a) Continuity at x =0

(i) At x =0, $f(0) = 4 - 0^2 = 4$ $\left[\because \text{At } x = 0, f(x) = 4 - x^2\right]$

(ii) $\text{R.H.L} = \underset{x \to 0+}{\text{Lt}} f(x) = \underset{x \to 0+}{\text{Lt}} (x-5) = 0 - 5 = -5$ $\left[\because x \to 0+ \Rightarrow x > 0 \Rightarrow f(x) = x - 5\right]$

Here $f(0) \neq$ R.H.L. $\therefore$ f(x) is not continuous at x =0

(b) Continuity at x = 1

(i) At x = 1, $f(1) = 1 - 5 = -4$ $\left[\because \text{At } x = 1, f(x) = x - 5\right]$

(ii) $\text{R.H.L} = \underset{x \to 1+}{\text{Lt}} f(x) = \underset{x \to 1+}{\text{Lt}} (4x^2 - 9) = 4(1)^2 - 9 = 4 - 9 = -5$

Here $f(1) \neq$ R.H.L. $\therefore$ f(x) is not continuous at x =1 $\left[\because \text{At } x = 2, f(x) = 3x + 4\right]$

(c) Continuity at x = 2

(i) At x = 2, $f(2) = 3(2) + 4 = 10$

(ii) $\text{L.H.L} = \underset{x \to 2-}{\text{Lt}} f(x) = \underset{x \to 2-}{\text{Lt}} (4x^2 - 9) = 4(2)^2 - 9 = 16 - 9 = 7$

Here $f(2) \neq$ L.H.L. $\therefore$ f(x) is not continuous at x =2