WELCOME

STAR 'QR CODE'

DIGITAL MATERIAL

THE PLANE - INDEX

7. THE PLANE

ADDITIONAL QUESTIONS WITH SOLUTIONS

1. Show that the plane through (1,1,1), (1,–1,1) and (–7,–3,–5) is parallel to the Y-axis.

Sol: The equation of the plane passing through A (1,1,1) be taken as

$a(x-1)+b(y-1)+c(z-1)=0$.......(1)

If (1) passes through (1,–1,1) then $a(1-1)+b(-1-1)+c(1-1)=0 \Rightarrow 2b=0 \Rightarrow b=0$.

If (1) passes through (–7,–3,–5) then $a(-7-1)+b(-3-1)+c(-5-1) = 0$

$\Rightarrow a(-8)+0(-4)+c(-6)=0 \Rightarrow 8a= -6c \Rightarrow a=\frac{-3}{4}c$

$\therefore$ (1) $\Rightarrow \frac{-3}{4}c(x-1)+0(y-1)+c(z-1)=0 \Rightarrow c\left[\frac{-3}{4}x+\frac{3}{4}+z-1\right]=0$

$\Rightarrow \frac{-3}{4}x+z-\frac{1}{4}=0 \Rightarrow 3x-4z+1 = 0$. Thisa plane is parallel to the Y -axis.

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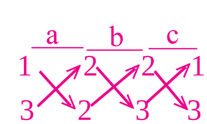
2. Find the equation of the plane through (4, 4, 0) and perpendicular to the planes $2x + y + 2z + 3 = 0$ and $3x + 3y + 2z - 8 = 0$

Sol: Equation of the plane passing through P(4, 4, 0) is $a(x-4) + b(y-4) + c(z-0) = 0$......(1)

Given planes are $2x + y + 2z + 3 = 0$............(2), $3x + 3y + 2z - 8 = 0$.........(3)

Since the planes (1) & (2) perpendicular to each other

$\Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0 \Rightarrow 2a + b + 2c = 0$...........(4)



Since the planes (1) & (3) perpendicular to each other

$\Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0 \Rightarrow 3a + 3b + 2c = 0$..........(5)

Solving (4) & (5)

$\frac{a}{2-6}=\frac{b}{6-4}=\frac{c}{6-3} \Rightarrow \frac{a}{-4}=\frac{b}{2}=\frac{c}{3} \Rightarrow (a, b, c) = (-4, 2, 3)$

From equation (1), $-4(x-4)+2(y-4)+3(z-0)=0 \Rightarrow -4x + 16 + 2y - 8 + 3z = 0$

$\Rightarrow -4x + 2y + 3z + 8 = 0$ be the required equation of the plane

3. Find the equation of the plane through (6, –4, 3), (0, 4, –3) and cutting of intercepts whose sum is zero.

Sol: The equation of the plane with intercepts a, b, c is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

If the plane passes through (6, –4, 3) $\Rightarrow \frac{6}{a}-\frac{4}{b}+\frac{3}{c}=1$.........(1)

If the plane passes through (0, 4, –3) $\Rightarrow \frac{0}{a}+\frac{4}{b}-\frac{3}{c}=1$.........(2)

Adding (1) and (2) $\Rightarrow \frac{6}{a}-\frac{4}{b}+\frac{3}{c}+\frac{0}{a}+\frac{4}{b}-\frac{3}{c}=1+1 \Rightarrow \frac{6}{a}=2 \Rightarrow a=3$

Given that the sum of the intercepts a + b + c = 0 $\Rightarrow$ 3 + b + c = 0 $\Rightarrow$ c = – 3 – b...........(3)

From (2) and (3) $\Rightarrow \frac{4}{b}-\frac{3}{-3-b}=1$

$\Rightarrow \frac{4}{b}+\frac{3}{3+b}=1 \Rightarrow 4(3+b)+3b=b(3+b) \Rightarrow b^2-4b-12=0 \Rightarrow b^2-6b+2b-12=0$

$\Rightarrow b(b-6)+2(b-6)=0 \Rightarrow (b-6)(b+2)=0 \Rightarrow b=6 \text{(or)} b=-2$

Case (i): Put b = 6 in (3) $\Rightarrow$ c = – 3 – 6 $\Rightarrow$ c = – 9

From (1), $\frac{x}{3}+\frac{y}{6}+\frac{z}{-9}=1 \Rightarrow \frac{6x+3y-2z}{18}=1 \Rightarrow 6x+3y-2z-18=0$

Case (ii): Put b = –2 in (3) $\Rightarrow$ c = –3 + 2 $\Rightarrow$ c = –1

From (1), $\frac{x}{3}+\frac{y}{-2}+\frac{z}{-1}=1 \Rightarrow \frac{2x-3y-6z}{6}=1 \Rightarrow 2x-3y-6z-6=0$

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4. Show that the equations ax + by + r = 0, by + cz + p = 0, cz + ax + q = 0 represent planes perpendicular to xy, yz, zx planes respectively.

Sol: Equation of the given plane is ax + by + r = 0...........(1)

Equation of the XY – plane is z = 0. Now, $a_1a_2+b_1b_2+c_1c_2 = a(0)+b(0)+0(1)=0$

∴ The given plane (1) is perpendicular to the XY–plane

Equation of the given plane is by + cz + p = 0...............(2)

Equation of the YZ – plane is x = 0. Now, $a_1a_2+b_1b_2+c_1c_2 = 0(1)+b(0)+c(0)=0$

∴ The given plane (2) is perpendicular to the YZ–plane

Equation of the given plane is cz + ax + q = 0...............(3)

Equation of the ZX – plane is y = 0 . Now, $a_1a_2+b_1b_2+c_1c_2 = a(0)+0(1)+c(0)=0$

∴ The given plane (3) is perpendicular to the ZX–plane