7. PARTIAL FRACTIONS

Sections	No. of periods (6)	Weightage in IPE [1x4 = 4]
1. Partial Fractions	4	4
2. Coefficient of x^n of a Rational fraction	2	

The fraction, in which the Numerator is less than the denominator, is called a proper fraction. Eg: $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \ldots\ldots\ldots\ldots$

The fraction, in which the Numerator is greater than or equal to the denominator, is called an improperfraction. Eg : $\frac{3}{2}, \frac{22}{7}, \frac{10}{3}, \frac{2}{2}$ etc.,

We know the process of adding or subtracting fractions to get a unique output.

Eg: $\frac{1}{2}+\frac{3}{4}=\frac{2+3}{4}=\frac{5}{4}$

Now, we see the converse process i.e., separating a given fraction into a group of simple fractions viz., partial fractions (observe that this process is not unique).

Eg: $\frac{5}{4}=\frac{2+3}{4}=\frac{2}{4}+\frac{3}{4}=\frac{1}{2}+\frac{3}{4}$ *here* $\frac{1}{2}, \frac{3}{4}$ *are the partial fractions of* $\frac{5}{4}$

Again $\frac{5}{4}=\frac{4+1}{4}=\frac{4}{4}+\frac{1}{4}=1+\frac{1}{4}$ *here* $1, \frac{1}{4}$ *are the partial fractions of* $\frac{5}{4}$

Also $\frac{5}{4}=\frac{6-1}{4}=\frac{6}{4}-\frac{1}{4}=\frac{3}{2}-\frac{1}{4}=\frac{2+1}{2}-\frac{1}{4}=\frac{2}{2}+\frac{1}{2}-\frac{1}{4}=1+\frac{1}{2}-\frac{1}{4}$ *etc.,*

But we are not interested in splitting fractional numbers as above. Infact, in this chapter we are interested in the study of splitting (resolving, decomposing) a given Rational fraction of the form $\frac{f(x)}{g(x)}$ into its partial fractions.

Applications of this topic "partial fractions" are seen in 'integration'.

In order to avoid confusion among various rational fractions, these are classified into Type-1.1, Type-1.2, Type-1.3, Type-2.1, Type-2.2.

In section 2, we determine the coefficient of x^n of a given rational fraction using power series expansions.

SYNOPSIS POINTS

1. **Proper Rational fraction:** If the degree of f(x) is less than the degree of g(x) then $\frac{f(x)}{g(x)}$ is called a proper Rational fraction.

2 **Improper Rational Fraction:** If the degree of f(x) is greater than or equal to the degree of g(x) then $\frac{f(x)}{g(x)}$ is called an improper rational fraction.

3. **Type-1.1:** Rational fraction of the form $\frac{f(x)}{g(x)}$ where g(x) contains non repeated linear factors. Here, with respect to every factor of g(x) of the form (ax+b), there will be one partial fraction of the form $\frac{A}{ax+b}$

4. **Type 1.2:** Rational fraction of the form $\frac{f(x)}{g(x)}$ where g(x) contains repeated and non-repeated linear factors.Here with respect to every repeated factor of g(x) of the form $(ax+b)^n$, $n>1 \in N$, there are n partial fractions of the form $\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+......+\frac{A_n}{(ax+b)^n}$

5. **Type-1.3:** Improper Rational fractions of the form $\frac{f(x)}{g(x)}$ with linear factors in the denominator or repeated linear factors in the denominator. Here express the improper rational fraction $\frac{f(x)}{g(x)}$ as $\frac{f(x)}{g(x)}=q(x)+\frac{r(x)}{g(x)}$ and resolve $\frac{r(x)}{g(x)}$ into its partial fractions accordingly.

6. **Type 1.4:** Rational fraction of the form $\frac{f(x)}{g(x)}$ where g(x) is a single repeated linear factor. Here take g(x)=y and find x interms of y. Then change $\frac{f(x)}{g(x)}$ into a rational function of y and simplify accordingly.

7. **Type 2.1:** Rational fraction of the form $\frac{f(x)}{g(x)}$, where g(x) contains a non repeated irreducible factor of the form ax^2+bx+c. Here corresponding to (ax^2+bx+c), there will be one partial fraction of the form $\frac{Ax+B}{ax^2+bx+c}$ where A,B are real constants.

8. **Type-2.2:** Rational fraction of the form $\frac{f(x)}{g(x)}$, where g(x) contains a repeated irreducible factor of the form $(ax^2+bx+c)^2$. Here corresponding to $(ax^2+bx+c)^2$ there will be partial fractions of the form $\frac{Ax+B}{ax^2+bx+c}+\frac{Cx+D}{(ax^2+bx+c)^2}$