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DIGITAL MATERIAL

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6.1.TRIGONOMETRIC RATIOS

1. INTRODUCTION PAGE

Sections	No. of periods (6)	Weightage in IPE [1x2 = 2]
1. Trigonometric Ratios	6	2 Marks

The name 'Trigonometry' is derived from three Greek words viz., 'tri', 'gon', 'metry' . 'Tri' means 'three', 'gon' means 'sides' and 'metry' means 'measure'. Thus, trigonometry is a branch of Mathematics which deals with the measurement of sides and angles of a triangle.

Trigonometry has a wide scope of applications in various branches of science and engineering. Engineers use trigonometry for (i) finding the heights and distances of inaccessible objects (ii) surveying of land scapes. Astronomers and Navigators use it for the computations of Latitude, Longitude. It has got an important role, in the study of all sorts of ibrating phenomina-sound, light, flow of electricity. Geometry, Calculus, Vector Algebra are also mingled with the concepts of Trigonometry to some extent.

The six trigonometric ratios (functions of θ) $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, $\cot\theta$ connect the sides of a right angled triangle with the angle θ. These trigonometric functions are interrelated in a variety of important ways. The part of the trigonometry which deals with various types of relations, formulae,problems of these trigonometic ratios is called analytical trigonometry (goniometry).

The basic concepts of trigonometry are already introduced in the lower classes. Hence, we made a move in the first section to recapitulate the basic concepts.

2. ADDITIONAL Q'S ON T'RATIOS

1. Prove that $\dfrac{\sin(3\pi - A)\cos\left(A-\frac{\pi}{2}\right)\tan\left(\frac{3\pi}{2}-A\right)}{\operatorname{cosec}\left(\frac{13\pi}{2}+A\right)\sec(3\pi+A)\cot\left(A-\frac{\pi}{2}\right)} = \cos^4 A$

Sol: $\sin(3\pi - A) = \sin(2\pi+\pi-A) = \sin(\pi - A) = \sin A$;

$$\cos\left(A-\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}-A\right) = \sin A;\quad \tan\left(\frac{3\pi}{2}-A\right) = \cot A$$

$$\operatorname{cosec}\left(\frac{13\pi}{2}+A\right) = \operatorname{cosec}\left(\frac{12\pi+\pi}{2}+A\right) = \csc\left(6\pi+\frac{\pi}{2}+A\right) = \csc\left(\frac{\pi}{2}+A\right) = -\sec A$$

$\sec(3\pi+A) = \sec(2\pi+\pi+A) = \sec(\pi+A) = -\sec A$

$$\cot\left(A-\frac{\pi}{2}\right) = -\cot\left(\frac{\pi}{2}-A\right) = -\tan A$$

$$\therefore G.E = \frac{(\sin A)(\sin A)(\cot A)}{(\sec A)(-\sec A)(-\tan A)} = \frac{\sin^2 A\cot A}{(\sec^2 A)(\tan A)} = \frac{\sin^2 A\left(\frac{\cos A}{\sin A}\right)}{\left(\frac{1}{\cos^2 A}\right)\left(\frac{\sin A}{\cos A}\right)} = \frac{\cos A}{\left(\frac{1}{\cos^3 A}\right)} = \cos^4 A$$

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2. Simplify $\dfrac{\sin\left(-\frac{11\pi}{3}\right)\tan\left(\frac{35\pi}{6}\right)\sec\left(-\frac{7\pi}{3}\right)}{\cos\left(\frac{5\pi}{4}\right)\operatorname{cosec}\left(\frac{7\pi}{4}\right)\cos\left(\frac{17\pi}{6}\right)}$

Sol: $$\sin\left(-\frac{11\pi}{3}\right) = -\sin\frac{11\pi}{3} = -\sin\left[\frac{12\pi-\pi}{3}\right] = -\sin\left[4\pi-\frac{\pi}{3}\right] = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{35\pi}{6}\right) = \tan\left[\frac{36\pi-\pi}{6}\right] = \tan\left[6\pi-\frac{\pi}{6}\right] = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$$

$$\sec\left(\frac{-7\pi}{3}\right) = \sec\left(\frac{7\pi}{3}\right) = \sec\left[\frac{6\pi+\pi}{3}\right] = \sec\left[2\pi+\frac{\pi}{3}\right] = \sec\frac{\pi}{3} = 2$$

$$\cos\left(\frac{5\pi}{4}\right) = \cos\left[\frac{4\pi+\pi}{4}\right] = \cos\left[\pi+\frac{\pi}{4}\right] = -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

$$\operatorname{cosec}\left(\frac{7\pi}{4}\right) = \operatorname{cosec}\left[\frac{8\pi-\pi}{4}\right] = \operatorname{cosec}\left[2\pi-\frac{\pi}{4}\right] = -\operatorname{cosec}\frac{\pi}{4} = -\sqrt{2}$$

$$\cos\left(\frac{17\pi}{6}\right) = \cos\left[\frac{18\pi-\pi}{6}\right] = \cos\left[3\pi-\frac{\pi}{6}\right] = \cos\left(2\pi+\pi-\frac{\pi}{6}\right) == \cos\left(\pi-\frac{\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\therefore G.E = \frac{\sin\left(-\frac{11\pi}{3}\right)\tan\left(\frac{35\pi}{6}\right)\sec\left(-\frac{7\pi}{3}\right)}{\cos\left(\frac{5\pi}{4}\right)\operatorname{cosec}\left(\frac{7\pi}{4}\right)\cos\left(\frac{17\pi}{6}\right)} = \frac{\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{3}}\right)(2)}{\left(-\frac{1}{\sqrt{2}}\right)(\sqrt{2})\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

3. If $\cos A = \cos B = -\frac{1}{2}$ and A does not lie in the second quadrant and B does not lie in the third quadrant, then find the value of $\frac{4\sin B - 3\tan A}{\tan B + \sin A}$

Sol: Given cosA is negative and A does not lie in $Q_2 \Rightarrow$ A lies in Q_3

Given cosB is negative and B does not lie in $Q_3 \Rightarrow$ B lies in Q_2

Now, $\cos A = -\frac{1}{2} \Rightarrow A = 120^o, 240^o$. But A lies in Q_3 $\therefore A = 240^o$

Also $\cos B = -\frac{1}{2} \Rightarrow B = 120^o, 240^o$. But B lies in Q_2 $\therefore B = 120^o$

Now $\frac{4\sin B - 3\tan A}{\tan B + \sin A} = \frac{4\sin 120^o - 3\tan 240^o}{\tan 120^o + \sin 240^o} = \frac{4\sin(180^o - 60^o) - 3\tan(180^o + 60^o)}{\tan(180^o - 60^o) + \sin(180^o + 60^o)}$

$$= \frac{4\sin 60^o - 3\tan 60^o}{-\tan 60^o + (-\sin 60^o)} = \frac{4\left(\frac{\sqrt{3}}{2}\right) - 3\left(\sqrt{3}\right)}{-\sqrt{3} - \frac{\sqrt{3}}{2}} = \frac{\frac{4\sqrt{3} - 6\sqrt{3}}{2}}{\frac{-2\sqrt{3} - \sqrt{3}}{2}} = \frac{4\sqrt{3} - 6\sqrt{3}}{-2\sqrt{3} - \sqrt{3}} = \frac{-2\sqrt{3}}{-3\sqrt{3}} = \frac{2}{3}$$

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4. If 8 tanA= –15 and 25 sinB= –7 and neither A nor B is in the fourth quadrant, then show that $\sin A\cos B + \cos A \sin B = \frac{-304}{425}$

Sol: Given 8 tanA = –15 $\Rightarrow$ tanA is negative . But A is not in Q_4. $\therefore A \in Q_2$

Given 25 sinB = –7 $\Rightarrow$ sinB is negative . But B is not in Q_4. $\therefore B \in Q_3$

Now, 8 tan A = –15 $\Rightarrow \tan A = \frac{-15}{8} \Rightarrow \sin A = \frac{15}{17}, \cos A = \frac{-8}{17}$

Also, 25 sin B = –7 $\Rightarrow \sin B = -\frac{7}{25} \Rightarrow \cos B = \frac{-24}{25}$

$\therefore$ sinA cosB+ cosA sinB $= \left(\frac{15}{17}\right)\left(\frac{-24}{25}\right) + \left(\frac{-8}{17}\right)\left(\frac{-7}{25}\right) = \frac{-360}{425} + \frac{56}{425} = -\frac{304}{425}$

6.2 TRIGONOMETRIC RATIOS OF COMPOUND ANGLES

3. INTRODUCTION PAGE

Sections	No. of periods (6)	Weightage in IPE [1x2 or 1x4]
1. Trigonometric Ratios of Compound Angles	6	2 or 4 Marks

The algebraic sum of two or more angles is called a compound angle. If A,B,C are angles, then the angles $A+B$, $A-B$, $A+B+C$, $A-B+C$ etc., are all compound angles. We have to note that, in general, $\sin(A+B) \neq \sin A + \sin B$. Hence, certain standard formulae are derived in order to find the Trigonometric ratios of compound angles like $\sin(A \pm B)$, $\cos(A \pm B)$, $\tan(A \pm B)$, $\cot(A \pm B)$. We know the values of Trigonometric ratios for 45^0 and 30^0 and the algebraic sum of these angles give 75^0 and 15^0 and the values of Trigonometric ratios for these angles can be determined using $\sin(A+B)$, $\cos(A-B)$ etc., Certain useful conditional identities can be derived using the formulae on compound angles.

eg: In ΔABC, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$.

4. ADDITIONAL Q'S ON COMPOUND ANGLES

1. **Prove that** $\cos^2\theta + \cos^2\left(\frac{2\pi}{3}+\theta\right) + \cos^2\left(\frac{2\pi}{3}-\theta\right) = \frac{3}{2}$

Sol: $$\text{L.H.S.} = \cos^2\theta + \cos^2\left(\frac{2\pi}{3}+\theta\right) + \cos^2\left(\frac{2\pi}{3}-\theta\right)$$

$$= \cos^2\theta + \cos^2\left(\frac{2\pi}{3}+\theta\right) + \left[1-\sin^2\left(\frac{2\pi}{3}-\theta\right)\right]$$

$$= 1 + \cos^2\theta + \cos^2\left(\frac{2\pi}{3}+\theta\right) - \sin^2\left(\frac{2\pi}{3}-\theta\right)$$

$$= 1 + \cos^2\theta + \cos\left(\frac{2\pi}{3}+\cancel{\theta}+\frac{2\pi}{3}-\cancel{\theta}\right).\cos\left(\cancel{\frac{2\pi}{3}}+\theta-\cancel{\frac{2\pi}{3}}+\theta\right) \quad \left[\because \cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)\right]$$

$$= 1 + \cos^2\theta + \cos\frac{4\pi}{3}.\cos 2\theta = 1 + \cos^2\theta - \frac{1}{2}\cos 2\theta$$

$$= 1 + \left(\frac{1+\cos 2\theta}{2}\right) - \frac{1}{2}\cos 2\theta = 1 + \frac{1}{2} + \cancel{\frac{1}{2}\cos 2\theta} - \cancel{\frac{1}{2}\cos 2\theta} = 1 + \frac{1}{2} = \frac{3}{2} = \textbf{R.H.S.}$$

NOTE: This can be done 'same as' in the class work model problem.

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2. **Express $\tan\theta$ in terms of $\tan\alpha$, if $\sin(\theta+\alpha) = \cos(\theta+\alpha)$.**

Sol: Given $\sin(\theta+\alpha) = \cos(\theta+\alpha) \Rightarrow \sin\theta\cos\alpha + \cos\theta\sin\alpha = \cos\theta\cos\alpha - \sin\theta\sin\alpha$

Dividing by $\cos\theta\cos\alpha$ we have, $\frac{\sin\theta\cos\alpha}{\cos\theta\cos\alpha} + \frac{\cos\theta\sin\alpha}{\cos\theta\cos\alpha} = \frac{\cos\theta\cos\alpha}{\cos\theta\cos\alpha} - \frac{\sin\theta\sin\alpha}{\cos\theta\cos\alpha}$

$$\Rightarrow \tan\theta + \tan\alpha = 1 - \tan\theta\tan\alpha \Rightarrow \tan\theta + \tan\theta\tan\alpha = 1 - \tan\alpha$$

$$\Rightarrow \tan\theta(1+\tan\alpha) = 1 - \tan\alpha \Rightarrow \tan\theta = \frac{1-\tan\alpha}{1+\tan\alpha}$$

3. If $\sin A = \frac{12}{13}, \cos B = \frac{3}{5}$ and neither A nor B is in the first quadrant, then find the quadrant in which A+B lies.

Sol: Given sin A is positive. But A doesnot belongs to Q_1. $\therefore A \in Q_2$

Also, cos B is positive. But B doesnot belongs to Q_1. $\therefore B \in Q_4$

Now $\sin A = \frac{12}{13}$ and A belongs to $Q_2 \Rightarrow \cos A = \frac{-5}{13}$

Also, $\cos B = \frac{3}{5}$ and B belongs to $Q_4 \Rightarrow \sin B = \frac{-4}{5}$

Now consider $\sin(A+B) = \sin A \cos B + \cos A \sin B = \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) + \left(\frac{-5}{13}\right)\left(\frac{-4}{5}\right) = \frac{36+20}{65} = \frac{56}{65}$

Also $\cos(A+B) = \cos A \cos B - \sin A \sin B = \left(\frac{-5}{13}\right)\left(\frac{3}{5}\right) - \left(\frac{12}{13}\right)\left(\frac{-4}{5}\right) = \frac{-15+48}{65} = \frac{33}{65}$

Thus sin(A+B), cos(A+B) both are positive. Hence we conclude that A+B lies in Q_1.

Here it is interesting to observe that neither A nor B lies in Q_1 but A+B lies in Q_1.

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4. If $\tan(\alpha-\beta) = \frac{7}{24}$ and $\tan\alpha = \frac{4}{3}$, where α and β are in the first quadrant prove that α+β=π/2

Sol : **Claim:** $\alpha+\beta = \frac{\pi}{2} \Rightarrow \beta = \frac{\pi}{2} - \alpha \Rightarrow \tan\beta = \tan\left(\frac{\pi}{2}-\alpha\right) = \cot\alpha = \frac{3}{4} \quad \left(\because \tan\alpha = \frac{4}{3}\right)$

We have to prove that $\tan\beta = \frac{3}{4}$

Now, $\tan\beta = \tan(\alpha-(\alpha-\beta)) = \frac{\tan\alpha - \tan(\alpha-\beta)}{1+\tan\alpha.\tan(\alpha-\beta)} = \frac{\frac{4}{3}-\frac{7}{24}}{1+\frac{4}{3}\left(\frac{7}{24}\right)} = \frac{4(24)-7(3)}{3(24)+4(7)} = \frac{96-21}{72+28} = \frac{75}{100} = \frac{3}{4}$

6.3 TRIGONOMETRIC RATIOS OF MULTIPLE ANGLES AND SUBMULTIPLE ANGLES

5. INTRODUCTION PAGE

Sections	No. of periods (6)	Weightage in IPE [1x2 or 1x4]
1. Trigonometric Ratios of Multiple Angles and Submultiple Angles.	6	2 or 4 Marks

If A is any angle, then 2A, 3A etc., are called Multiple Angles of A and A/2, A/3 etc., are called Submultiple Angles of A. The formulae on multiple angles like sin2A, cos2A are very much useful in various parts of Trigonometry.

This chapter contains some power reduction formulae like $\sin^2 A = \frac{1-\cos 2A}{2}, \cos^2 A = \frac{1+\cos 2A}{2}$, *which are useful to determine the periodicity, Maximum and Minimum values of certain Trigonometric functions. In this chapter, we determine the value of sin18⁰ and hence followingly the values of sin36⁰, sin54⁰, sin72⁰, cos18⁰, cos36⁰ etc., be determined.*

The inter relations such as

$$\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x} = \frac{\cos x+\sin x}{\cos x-\sin x} = \frac{1+\sin 2x}{\cos 2x} = \sec 2x + \tan 2x = \sqrt{\frac{1+\sin 2x}{1-\sin 2x}}$$

are useful in the chapter "derivatives" of calculus.

6.4.TRANSFORMATIONS

6. INTRODUCTION PAGE

Sections	No. of periods (6)	Weightage in IPE [1x7=7]
1. **Transformations**	6	7 Marks

Some times, we need to transform the sum of two Trigonometric functions of certain combinations into corresponding single product and vice versa. Then, we use the formulae which transform 'sum to product' and 'product to sum'.

Various types of conditional identities are established using the formulae on Transformations.

7. ADDITIONAL Q'S ON TRANSFORMATIONS

1. If tan(A+B)=λtan(A–B), then show that (λ+1)sin2B=(λ–1)sin2A

Sol: Given that tan(A+B)=λtan(A–B) $\Rightarrow \frac{\lambda}{1}=\frac{\tan(A+B)}{\tan(A-B)}$ Apply Componendo & Dividendo rule

$$\Rightarrow \frac{\lambda+1}{\lambda-1}=\frac{\tan(A+B)+\tan(A-B)}{\tan(A+B)-\tan(A-B)} \Rightarrow \frac{\lambda+1}{\lambda-1}=\frac{\frac{\sin(A+B)}{\cos(A+B)}+\frac{\sin(A-B)}{\cos(A-B)}}{\frac{\sin(A+B)}{\cos(A+B)}-\frac{\sin(A-B)}{\cos(A-B)}}$$

$$\Rightarrow \frac{\lambda+1}{\lambda-1}=\frac{\sin(A+B)\cos(A-B)+\cos(A+B)\sin(A-B)}{\sin(A+B)\cos(A-B)-\cos(A+B)\sin(A-B)}$$

$$\Rightarrow \frac{\lambda+1}{\lambda-1}=\frac{\sin[(A+B)+(A-B)]}{\sin[(A+B)-(A-B)]} \Rightarrow \frac{\lambda+1}{\lambda-1}=\frac{\sin(2A)}{\sin(2B)}$$

$$\Rightarrow (\lambda+1)\sin 2B=(\lambda-1)\sin(2A)$$

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2. If x,y,z are non zero real numbers and if $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right)=z\cos\left(\theta+\frac{4\pi}{3}\right)$ for same θ∈R, then show that xy + yz + zx=0

Sol: Given $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right)=z\cos\left(\theta+\frac{4\pi}{3}\right)$

$$\Rightarrow \frac{\cos\theta}{1/x}=\frac{\cos\left(\theta+\frac{2\pi}{3}\right)}{1/y}=\frac{\cos\left(\theta+\frac{4\pi}{3}\right)}{1/z}=\frac{1}{k}(\text{say}), k\neq 0$$

$$\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=k\left[\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{4\pi}{3}\right)\right]$$

$$\left[\because \cos\left(\frac{4\pi}{3}+\theta\right)=\cos\left(\frac{6\pi-2\pi}{3}+\theta\right)=\cos\left(2\pi+\theta-\frac{2\pi}{3}\right)=\cos\left(\theta-\frac{2\pi}{3}\right)\right]$$

$$=k\left[\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta-\frac{2\pi}{3}\right)\right] \qquad [\because \cos(A+B)+\cos(A-B) \text{ formula}]$$

$$=k\left[\cos\theta+2\cos\theta\cos\frac{2\pi}{3}\right]=k[\cos\theta+\not{2}\cos\theta\left(\frac{-1}{\not{2}}\right)]=k[\cos\theta-\cos\theta]=0 \qquad \left[\because \cos\frac{2\pi}{3}=\cos 120^\circ=\frac{-1}{2}\right]$$

$$\therefore \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0 \Rightarrow \frac{yz+xz+xy}{xyz}=0 \Rightarrow xy+yz+zx=0$$

3. If sinA = sinB and cosA = cosB, then prove that A=2nπ+B for some integer n.

Sol: Given sinA = sinB and cosA = cos B $\Rightarrow \sin A - \sin B = 0$ and $\cos A - \cos B = 0$

$$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) = 0 \ldots\ldots\ldots\ldots(1) \qquad \text{[Use sinC–sinD formula]}$$

$\because \cos A - \cos B = 0$

$$\Rightarrow -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) = 0 \ldots\ldots\ldots\ldots(2) \qquad \text{[Use cosC–cosD formula]}$$

From (1) and (2) $\Rightarrow \sin\left(\frac{A-B}{2}\right) = 0 \Rightarrow \frac{A-B}{2} = n\pi$ $\quad [\because \sin\theta = 0 \Rightarrow \theta = n\pi, n \in Z]$

$\Rightarrow A - B = 2n\pi \Rightarrow A = 2n\pi + B, (n \in Z)$

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4. Prove that $\cot\left(\frac{\pi}{2} - A\right) + \tan\left(\frac{\pi}{12} + A\right) = \frac{4\cos 2A}{1 - 2\sin 2A}$.

When neither $\left[A - \frac{\pi}{12}\right]$ **nor** $\left[A - \frac{5\pi}{12}\right]$ **is an integral multiple of π.**

Sol: L.H.S=cot(15°–A)+tan(15°+A)

$$= \frac{\cos(15^\circ - A)}{\sin(15^\circ - A)} + \frac{\sin(15^\circ + A)}{\cos(15^\circ + A)} = \frac{\cos(15^\circ - A)\cos(15^\circ + A) + \sin(15^\circ - A)\sin(15^\circ + A)}{\sin(15^\circ - A)\cos(15^\circ + A)}$$

$$= \frac{\cos[(15^\circ - A) - (15^\circ + A)]}{\frac{1}{2}[2\sin(15^\circ - A)\cos(15^\circ + A)]} = \frac{2\cos(-2A)}{\sin(15^\circ - A + 15^\circ + A) + \sin(15^\circ - A - 15^\circ - A)}$$

$$= \frac{2\cos(2A)}{\sin 30^\circ + \sin(-2A)} = \frac{2\cos 2A}{\frac{1}{2} - \sin 2A} = \frac{4\cos 2A}{1 - 2\sin 2A} = \text{R.H.S}$$

6.5.MAXIMA & MINIMA, PERIODICITY & GRAPHS

Sections	No. of periods (6)	Weightage in IPE [1x2=2]
1. Maxima & Minima, Periodicity & Graphs	6	2 Marks

8. INTRODUCTION PAGE

For any real value of x, the maximum value of sinx or cosx is 1 and their minimum value is –1. Such values are also known as extreme values. When the expression is of the form acosx+bsinx, then its maximum value is proved to be $\sqrt{a^2+b^2}$ *and minimum value is* $-\sqrt{a^2+b^2}$. *We can find the extreme values of Trigonometric expressions which can be reduced into the form acosx+bsinx. While determining the values of maxima / minima, occasionally, we use the power reducing formulae like* $\sin^2 x=\frac{1-\cos 2x}{2}, \cos^2 x=\frac{1+\cos 2x}{2}$, $\sin^3 x=\frac{1}{4}(3\sin x-\sin 3x)$, $\cos^3 x=\frac{1}{4}(3\cos x+\cos 3x)$.

Any phenomenon, that occurs in regular intervals is said to exhibit the periodic property. Periodicity is one of the characteristic properties of trigonometric functions. The period of sinx is 2π. *That means, the value of sinx does not change for any x with the addition of* 2π *to that x. i.e,* $\sin(x+2\pi)=\sin x$. *Similarly, the remaining Trigonometric functions also exhibit this nature. The period of sinkx, coskx is* $\frac{2\pi}{|k|}$. *The period of tankx is* $\frac{\pi}{|k|}$. *We can find the period of any trigonometric function that can be reduced to the form sinkx, coskx, tankx etc.,*

Every trigonometric ratio is basically a relation between an angle and a corresponding real value. Hence all the trigonometric ratios are all real functions. Therefore each trigonometric function can be expressed by means of Graphs. The graph of trigonometric functions are useful to understand and study various properties like domain, range, "one one ness", "onto ness", periodicity, maxima and minima, continuity etc.,