WELCOME

STAR 'QR CODE'

DIGITAL CONTENT MATERIAL

INDEFINITE INTEGRALS - INDEX

6.INDEFINITE INTEGRALS

SECTIONS	No. of periods (25 to 33)	Weightage in IPE $2\times2+2\times7=18$
1. Integral as anti-derivative	3 to 4	2
2. Integration by substitution	4 to 5	2 or 4
3. Ten more standard forms	2	2
4. Integration of Rational functions that are resolvable into partial fractions	2 to 3	2
5. Integration by parts	3 to 4	2 or 4
6. Integration by Successive Reduction: Reduction formulae	2 to 3	7
7. Integration of Trigonometric Rational functions	3	7
8. Integration of Algebraic Rational & Irrational functions	5	2 or 7

In the 1st year of Intermediate course, we have dealt with differential calculus and the essence of differential calculus is best observed in dealing with the problem of errors & approximations, problem of rate of change, problem of tangents, normals and the problem of maxima & minima.

The study of integration as such and its application in finding Areas, Solutions of Differential Equations etc., will be dealt with in Integral Calculus. Its applications are prominently used in Vector Calculus, Laplace Transformations, Fourier series etc.,

In the chapter 'Indefinite Integrals' various methods and techniques of finding integrals of various given functions are discussed.

In the first section, integration is treated as an Inverse process of differentiation and a list of some standard forms is given.

In the subsequent sections various methods, techniques and rules labelled Type-1, Type-2, Type-2.1 etc., of various kinds of functions are discussed.

It is to be noted that the symbol $\frac{dy}{dx}$ can be used in the sense that, it is a ratio of two differentials dy & dx and so we use the symbol $\frac{dy}{dx}$ in accordance with the necessity.

TABLE

The INTEGRATION TABLE

☞ The Integral of f(x) is F(x)

No.	Type	Result / Rule
1.	T-1	$\int f(x)dx = F(x)$
2.	T-2.1	$\int f(ax+b)dx = \frac{1}{a}F(ax+b)$
3.	T-2.2	$\int \frac{f'(x)}{f(x)}dx = \log\lvert f(x)\rvert$
4.	T-2.3.1	$\int f(x)^n f'(x)dx = \frac{(f(x))^{n+1}}{n+1}$
5.	T-2.3.2	$\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)}$
6.	T-2.4.1	$\int f(g(x))g'(x)dx = F(g(x))$
7.	T-2.4.2	$\int f(x^n)x^{n-1}dx = \frac{1}{n}F(x^n)$
8.	T-3	$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}$ $\int \frac{1}{\sqrt{a^2+x^2}}dx = \sinh^{-1}\frac{x}{a}$ $\int \frac{1}{\sqrt{x^2-a^2}}dx = \cosh^{-1}\frac{x}{a}$ $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ $\int \sqrt{a^2+x^2}dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a}$ $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\frac{x}{a}$ $\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\left(\sec^{-1}\frac{x}{a}\right)$ $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\left(\tan^{-1}\frac{x}{a}\right)$ $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left\lvert\frac{a+x}{a-x}\right\rvert$ $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left\lvert\frac{x-a}{x+a}\right\rvert$
9.	T-4.1	$\int \frac{px+q}{(x+a)(x+b)}dx = \int\left(\frac{A}{x+a}+\frac{B}{x+b}\right)dx$ $= A\log(x+a) + B\log(x+b)$
10.	T-4.2	$\int \frac{dx}{x(1+x^n)} = \frac{1}{n}\log\left(\frac{x^n}{1+x^n}\right)$ $\int \frac{dx}{x(1-x^n)} = \frac{1}{n}\log\left(\frac{x^n}{1-x^n}\right)$ $\int \frac{dx}{x(x^n-1)} = \frac{1}{n}\log\left(\frac{x^n-1}{x^n}\right)$
11.	T-5.1	**Integration by parts** $\int uvdx = u\int vdx - \int\left[\frac{du}{dx}(\int vdx)\right]dx$ $\int udv = uv - \int(du)v$
12.	T-5.2	$\int e^x(f(x)+f'(x))dx = e^x f(x)$
13.	T-6	$I_n = \int \sin^n xdx = \frac{-\sin^{n-1}x\cos x}{n} + \left(\frac{n-1}{n}\right)I_{n-2}$ $I_n = \int \tan^n xdx = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$ $I_{m,n} = \int \sin^m x\cos^n x\,dx = \frac{\sin^{m+1}x\cos^{n-1}x}{m+n} + \frac{n-1}{m+n}I_{m,n-2}$
14.	T-7.1	$\int \frac{dx}{a+b\sin x}$, $\int \frac{dx}{a+b\cos x}$ put $\tan\frac{x}{2} = t \Rightarrow dx = \frac{2dt}{1+t^2}$ $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$
15.	T-7.2	$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$ multiply the Nr & Dr by $\sec^2 x$ and put tanx=t
16.	T-7.3	$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}dx$ express the Nr. as follows: acosx+bsinx= $A(c\cos x + d\sin x) + B\frac{d}{dx}(c\cos x + d\sin x)$
17.	T-8.1	$\int \frac{(px+q)dx}{ax^2+bx+c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$ $\int (px+q)\sqrt{ax^2+bx+c}\;dx$ express px+q as follows: $px+q = A + B\frac{d}{dx}(ax^2+bx+c)$
18.	T-8.2.1	$\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$ put $px+q = \frac{1}{t}$
19.	T-8.2.2	$\int \frac{dx}{(px^2+q)\sqrt{ax^2+b}}$ put $x = \frac{1}{t}$ and in the resulting integrand put $\sqrt{quadratic} = y$
20.	T-8.3	$\int \frac{ax+b}{\sqrt{px+q}}dx$, $\int \frac{dx}{(ax+b)\sqrt{px+q}}$ $\int (ax+b)\sqrt{px+q}\;dx$ put $\sqrt{px+q} = t$ i.e, $px+q = t^2$
21.	T-8.4	$\int \frac{(x^2+1)dx}{x^4+kx^2+1}$, $\int \frac{(x^2-1)dx}{x^4+kx^2+1}$ divide the Nr.& Dr. by x^2 then Put $x - \frac{1}{x} = t$ if the Nr. is $1+\frac{1}{x^2}$ Here $x^2 + \frac{1}{x^2} = \left(x-\frac{1}{x}\right)^2 + 2$ Put $x + \frac{1}{x} = t$ if the Nr. is $1-\frac{1}{x^2}$ Here $x^2 + \frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$
22.	T-8.5	$\int \frac{ax+b}{\sqrt[n]{px+q}}dx$ put $\sqrt[n]{px+q} = t$ i.e, $px+q = t^n$

ADDITIONAL QUESTIONS WITH SOLUTIONS

1 **Evaluate** $\int \frac{dx}{\sin(x-a)\sin(x-b)}$

Sol: Multiplying the Nr & Dr by sin(b–a) we have

$$I = \frac{1}{\sin(b-a)}\int \frac{\sin(b-a)}{\sin(x-a)\sin(x-b)}dx = \frac{1}{\sin(b-a)}\int \frac{\sin[(x-a)-(x-b)]}{\sin(x-a)\sin(x-b)}dx$$

$$= \frac{1}{\sin(b-a)}\int \left(\frac{\sin(x-a)\cos(x-b)-\cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)}\right)dx$$

$$= \frac{1}{\sin(b-a)}\int \left[\frac{\cos(x-b)}{\sin(x-b)} - \frac{\cos(x-a)}{\sin(x-a)}\right]dx$$

$$= \frac{1}{\sin(b-a)}[\log|\sin(x-b)| - \log|\sin(x-a)|] + c \qquad \left[\because \int \frac{f'(x)}{f(x)}dx = \log|f(x)| + c\right]$$

$$= \frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right| + c \qquad \left[\because \log a - \log b = \log\frac{a}{b}\right]$$

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2 **Evaluate** $\int \frac{\sin 2x}{(a+b\cos x)^2}$ **dx**

Sol: Put a + b cos x = t ⇒ cos x = $\frac{t-a}{b}$ ⇒ –sin x dx = $\frac{1}{b}$ dt ⇒ sin x dx = $\frac{-1}{b}$ dt

$$\therefore I = \int \frac{\sin 2x}{(a+b\cos x)^2}dx = \int \frac{2\sin x\cos x}{(a+b\cos x)^2}dx = 2\int \frac{\left(\frac{t-a}{b}\right)\left(\frac{-1}{b}\right)}{t^2}dt = \frac{-2}{b^2}\int \frac{t-a}{t^2}dt$$

$$= \frac{-2}{b^2}\int\left(\frac{t}{t^2} - \frac{a}{t^2}\right)dt = \frac{-2}{b^2}\int\left(\frac{1}{t} - \frac{a}{t^2}\right)dt = \frac{-2}{b^2}\left[\log|t| + \frac{a}{t}\right] + c$$

$$= \frac{-2}{b^2}\left[\log|a+b\cos x| + \frac{a}{(a+b\cos x)}\right] + c$$

3 **Evaluate** $\int \sqrt{1+\sec x}\, dx$

Sol: $I = \int \sqrt{1+\sec x}\, dx = \int \sqrt{1+\frac{1}{\cos x}}\, dx = \int \sqrt{\frac{\cos x+1}{\cos x}} dx$

$$= \int \sqrt{\frac{(1+\cos x)(1-\cos x)}{\cos x(1-\cos x)}} dx = \int \frac{\sqrt{1-\cos^2 x}}{\sqrt{\cos x}\sqrt{1-\cos x}} dx = \int \frac{\sin x\, dx}{\sqrt{\cos x - \cos^2 x}}$$

Put $\cos x = t \Rightarrow -\sin x\, dx = dt \Rightarrow \sin x\, dx = -dt$

Also $\cos x - \cos^2 x = t - t^2 = -t^2 + t = -(t^2 - t) = -\left(t^2 - t + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right)$

$$= -\left(\left(t-\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right) = \left(\left(\frac{1}{2}\right)^2 - \left(t-\frac{1}{2}\right)^2\right)$$

$$\therefore I = \int \frac{-dt}{\sqrt{t-t^2}} = -\int \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(t-\frac{1}{2}\right)^2}} dt = -\mathrm{Sin}^{-1}\left(\frac{t-\frac{1}{2}}{\frac{1}{2}}\right) + c$$

$= -\mathrm{Sin}^{-1}(2t-1) + c = -\mathrm{Sin}^{-1}(2\cos x - 1) + c$

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4 **Evaluate** $\int \frac{dx}{(x^2+a^2)^2}$

Sol: Put $x = a\tan\theta$ then $dx = a\sec^2\theta\, d\theta$

$$\int \frac{dx}{(x^2+a^2)^2} = \int \frac{a\sec^2\theta}{(a^2\tan^2\theta + a^2)^2} d\theta = \int \frac{a\sec^2\theta}{a^4.\sec^4\theta} d\theta = \frac{1}{a^3}\int \frac{d\theta}{\sec^2\theta}$$

$$= \frac{1}{a^3}\int \cos^2\theta\, d\theta = \frac{1}{a^3}\int \left(\frac{1+\cos 2\theta}{2}\right) d\theta = \frac{1}{2a^3}\left[\theta + \frac{\sin 2\theta}{2}\right] + c$$

$$= \frac{1}{2a^3}\mathrm{Tan}^{-1}\left(\frac{x}{a}\right) + \frac{1}{4a^3}\sin\left(2\mathrm{Tan}^{-1}\frac{x}{a}\right) + c$$

5 **Evaluate** $\int e^x \log(e^{2x}+5e^x+6)\,dx$.

Sol: $I = \int e^x \log(e^{2x}+5e^x+6)\,dx$

$= \int e^x \log[(e^x+2)(e^x+3)]dx$ $\quad [\because e^{2x}+5e^x+6 = (e^x+2)(e^x+3)]$

$= \int e^x[\log(e^x+2)+\log(e^x+3)]dx = \int e^x \log(e^x+2)dx + \int e^x \log(e^x+3)dx$

Put $e^x = t \Rightarrow e^x\,dx = dt$

$\therefore I = \int \log(t+2)(1)dt + \int \log(t+3)(1)dt$

Now applying 'by parts rule' we have

$$I = \log(t+2)(t) - \int (t)\frac{1}{t+2}dt + \log(t+3)(t) - \int (t)\frac{1}{t+3}dt$$

$$= t[\log(t+2)+\log(t+3)] - \int\left(\frac{t}{t+2}\right)dt - \int\left(\frac{t}{t+3}\right)dt$$

$$= t\log(t+2)(t+3) - \int\left(\frac{t+2-2}{t+2}\right)dt - \int\left(\frac{t+3-3}{t+3}\right)dt \quad [\because \log a + \log b = \log ab]$$

$$= t\log(t^2+5t+6) - \int\left(1-\frac{2}{t+2}\right)dt - \int\left(1-\frac{3}{t+3}\right)dt$$

$= t\log(t^2+5t+6) - t + 2\log|t+2| - t + 3\log|t+3| + c$

$= e^x\log(e^{2x}+5e^x+6) - 2e^x + 2\log(e^x+2) + 3\log(e^x+3) + c$

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6 **Evaluate** $\int a^x \cos 2x\,dx$

Sol: As per ILATE order we take $u = \cos 2x$ and $v = a^x$

Then $v_1 = a^x \log_a e = \frac{a^x}{\log a}$ and $u' = -2\sin 2x$

$$\therefore I = \int \cos 2x\,(a^x)dx = \cos 2x\left(\frac{a^x}{\log a}\right) - \int\left(\frac{a^x}{\log a}\right)(-2\sin 2x)dx$$

$$= \frac{a^x\cos 2x}{\log a} + \frac{2}{\log a}\int a^x \sin 2x\,dx = \frac{a^x}{\log a}\cos 2x + \frac{2}{\log a}\left[\sin 2x\left(\frac{a^x}{\log a}\right) - \int\left(\frac{a^x}{\log a}\right)(2\cos 2x)dx\right]$$

$$= \frac{a^x\cos 2x}{\log a} + \frac{2}{(\log a)^2}\left[a^x\sin 2x. - 2\int a^x\cos 2x\,dx\right]$$

$$I=\frac{a^x\cos 2x}{\log a}+\frac{2}{(\log a)^2}[a^x\sin 2x-2I] = \frac{a^x\cos 2x}{\log a}+\frac{2a^x\sin 2x}{(\log a)^2}-\frac{4I}{(\log a)^2}$$

$$\Rightarrow I+\frac{4I}{(\log a)^2}=\frac{a^x}{(\log a)^2}[\log a\cos 2x+2\sin 2x]$$

$$\Rightarrow I\left[\frac{(\log a)^2+4}{(\log a)^2}\right]=\frac{a^x}{(\log a)^2}[\log a\cos 2x+2\sin 2x]\Rightarrow I=\frac{a^x}{(\log a)^2+4}[\log a\cos 2x+2\sin 2x]$$

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7 **Evaluate** $\int\frac{dx}{e^x+e^{2x}}$

Sol: $$\frac{1}{e^x+e^{2x}}=\frac{1}{e^x(1+e^x)}=\left(\frac{1}{e^x}-\frac{1}{1+e^x}\right)$$

$$\therefore \int\frac{1}{e^x+e^{2x}}dx=\int\frac{1}{e^x}dx-\int\frac{1}{1+e^x}dx$$

$$=\int e^{-x}dx-\int\frac{e^x}{e^x(1+e^x)}dx=\int e^{-x}dx-\int e^x\left(\frac{1}{e^x}-\frac{1}{1+e^x}\right)dx$$

$$=\int e^{-x}dx-\int 1\,dx+\int\frac{e^x}{1+e^x}dx=\frac{e^{-x}}{(-1)}-x+\log|1+e^x|+c$$

$$=-e^{-x}-\log(e^x)+\log|1+e^x|+c \quad [\because x=\log(e^x)] =-e^{-x}+\log\left(\frac{1+e^x}{e^x}\right)+c$$

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8 **Evaluate** $\int\frac{x^2}{(x+1)(x+2)^2}dx$

Sol: Let $\frac{x^2}{(x+1)(x+2)^2}\equiv\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}\Rightarrow x^2=A(x+2)^2+B(x+1)(x+2)+C(x+1)$....(1)

Putting $x=-2$ in (1), we get $(-2)^2=A(0)+B(0)+C(-2+1)\Rightarrow C=-4$

Putting $x=-1$ in (1), we get $(-1)^2=A(-1+2)^2+B(0)+C(0)\Rightarrow A=1$

Equating the coefficient of x^2 in (1), we get $1=A+B\Rightarrow B=1-A=1-1=0$

$$\therefore \frac{x^2}{(x+1)(x+2)^2}=\frac{1}{x+1}+\frac{0}{x+2}+\frac{(-4)}{(x+2)^2}$$

$$\Rightarrow \int\frac{x^2}{(x+1)(x+2)^2}dx=\int\left[\frac{1}{x+1}-\frac{4}{(x+2)^2}\right]dx=\log|x+1|+\frac{4}{x+2}+c$$

9 **Evaluate** $\int \frac{1}{(1-x)(4+x^2)}dx$

Sol: Let $\frac{1}{(1-x)(4+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{4+x^2}$

$\Rightarrow A(4+x^2)+(Bx+C)(1-x)=1$ ....(1)

Putting x = 1 in (1), we get $A(4+1)=1 \Rightarrow 5A=1 \Rightarrow A=\frac{1}{5}$,

Putting x = 0 in (1), we get $\Rightarrow$ A(4) + C(1)=1

$\Rightarrow C = 1 - 4A = 1 - 4\left(\frac{1}{5}\right) = \frac{5-4}{5} = \frac{1}{5}$

Equating coefficients of x^2 in (1), we get $A - B = 0 \Rightarrow B = A = \frac{1}{5}$

$$\therefore \frac{1}{(1-x)(4+x^2)} = \left(\frac{1}{5}\right)\left[\frac{1}{1-x}\right] + \frac{\left(\frac{1}{5}x+\frac{1}{5}\right)}{4+x^2} = \frac{1}{5}\left(\frac{1}{1-x}\right) + \frac{1}{5}\left[\frac{x}{4+x^2}\right] + \frac{1}{5}\left[\frac{1}{4+x^2}\right]$$

$$\therefore \int \frac{1}{(1-x)(4+x^2)}dx = \frac{1}{5}\int \frac{1}{1-x}dx + \frac{1}{5}\int \frac{x}{4+x^2}dx + \frac{1}{5}\int \frac{1}{4+x^2}dx$$

$$= \frac{1}{5}\int \frac{1}{1-x}dx + \frac{1}{10}\int \frac{2x}{4+x^2}dx + \frac{1}{5}\int \frac{1}{4+x^2}dx$$

$$= -\frac{1}{5}\log|1-x| + \frac{1}{10}\log|4+x^2| + \frac{1}{5}.\frac{1}{2}\mathrm{Tan}^{-1}\left(\frac{x}{2}\right) + c$$

$$= -\frac{1}{5}\log|1-x| + \frac{1}{10}\log|4+x^2| + \frac{1}{10}\mathrm{Tan}^{-1}\left(\frac{x}{2}\right) + c$$

10 **Evaluate** $\int \frac{2x+1}{x(x^2+4)^2}dx.$

Sol: Let $\frac{2x+1}{x(x^2+4)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$

$A(x^2+4)^2 + (Bx+C)x(x^2+4) + (Dx+E)x = 2x+1$

Comparing coefficient of x^4 on both sides we get, $A + B = 0$........(1)

Comparing coefficient of x^3 on both sides we get, $C = 0$ ....(2)

Comparing coefficient of x^2 on both sides we get, $8A + 4B + D = 0$ ....(3)

Comparing coefficient of x on both sides we get, $4C + E = 2$........(4)

Comparing constant terms on both sides we get, $16A = 1 \Rightarrow A = \frac{1}{16}$

From (1), $\frac{1}{16} + B = 0 \Rightarrow B = \frac{-1}{16}$

From (3), $8\left[\frac{1}{16}\right] + 4\left[\frac{-1}{16}\right] + D = 0 \Rightarrow \frac{1}{2} - \frac{1}{4} + D = 0 \Rightarrow \frac{1}{4} + D = 0 \Rightarrow D = \frac{-1}{4}$

From (4) we have $4(0) + E = 2 \Rightarrow E = 2$

$$\therefore I = \int \frac{2x+1}{x(x^2+4)^2}dx = \frac{1}{16}\int \frac{1}{x}dx - \frac{1}{16}\int \frac{x\,dx}{x^2+4} + \int \frac{\frac{-1}{4}x+2}{(x^2+4)^2}dx$$

$$\Rightarrow I = \frac{1}{16}\int \frac{1}{x}dx - \frac{1}{16}\frac{1}{2}\int \frac{2x\,dx}{x^2+4} - \frac{1}{4}\frac{1}{2}\int \frac{2x}{(x^2+4)^2}dx + 2\int \frac{dx}{(x^2+4)^2}$$

$$\Rightarrow I = \frac{1}{16}\log|x| - \frac{1}{32}\log|x^2+4| + \frac{1}{8}\frac{1}{(x^2+4)} + 2\int \frac{dx}{(x^2+4)^2} \text{......(5)}$$

Now let us find $I = \int \frac{dx}{(x^2+4)^2}$ Put $x = 2\tan\theta \Rightarrow dx = 2\sec^2\theta\, d\theta$

$$I_1 = \int \frac{dx}{(x^2+4)^2} = \int \frac{2\sec^2\theta\, d\theta}{16(1+\tan^2\theta)^2} = \frac{2}{16}\int \frac{\sec^2\theta\, d\theta}{(\sec^2\theta)^2} = \frac{1}{8}\int \frac{d\theta}{\sec^2\theta} = \frac{1}{8}\int \cos^2\theta\, d\theta$$

$$= \frac{1}{16}\int (1+\cos 2\theta)d\theta = \frac{1}{16}\left[\theta + \frac{\sin 2\theta}{2}\right] + c$$

$$= \frac{1}{16}\left(\theta + \frac{\tan\theta}{1+\tan^2\theta}\right) + c = \frac{1}{16}\left[\text{Tan}^{-1}\left(\frac{x}{2}\right) + \frac{2x}{4+x^2}\right] + c.$$ Now from (5),

$$I = \int \frac{2x+1}{x(x^2+4)^2}dx = \frac{1}{16}\log|x| - \frac{1}{32}\log(x^2+4) + \frac{1}{8(x^2+4)} + \frac{1}{8}\text{Tan}^{-1}\left(\frac{x}{2}\right) + \frac{1}{4}\left(\frac{x}{4+x^2}\right) + c$$

11 **Evaluate** $\int \frac{dx}{1+x^4}$

Sol: $$I=\int \frac{dx}{1+x^4}=\frac{1}{2}\int \frac{2}{1+x^4}dx=\frac{1}{2}\int \frac{(1+x^2)+(1-x^2)}{1+x^4}dx=\frac{1}{2}\left[\int \frac{1+x^2}{1+x^4}dx+\int \frac{1-x^2}{1+x^4}dx\right]$$

$$=\frac{1}{2}\left[\int\left(\frac{\frac{1}{x^2}+1}{\frac{1}{x^2}+x^2}\right)dx+\int\left(\frac{\frac{1}{x^2}-1}{\frac{1}{x^2}+x^2}\right)dx\right]=\frac{1}{2}\left[\int\left(\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\right)dx-\int\left(\frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\right)dx\right]$$

$$=\frac{1}{2}\left[\int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}dx-\int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-2}dx\right]$$

Put $x-\frac{1}{x}=t \Rightarrow \left(1+\frac{1}{x^2}\right)dx=dt$ and $x+\frac{1}{x}=z \Rightarrow \left(1-\frac{1}{x^2}\right)dx=dz$

$$\therefore I=\frac{1}{2}\left[\int \frac{dt}{t^2+2}-\int \frac{dz}{z^2-2}\right]=\frac{1}{2}\left[\int \frac{dt}{(\sqrt{2})^2+t^2}-\int \frac{dz}{z^2-(\sqrt{2})^2}\right]$$

$$=\frac{1}{2}\left(\frac{1}{\sqrt{2}}\mathrm{Tan}^{-1}\left(\frac{t}{\sqrt{2}}\right)-\frac{1}{2\sqrt{2}}\log\left|\frac{z-\sqrt{2}}{z+\sqrt{2}}\right|\right)+c$$

$$=\frac{1}{2\sqrt{2}}\mathrm{Tan}^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right)-\frac{1}{4\sqrt{2}}\log\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right|+c$$

$$=\frac{1}{2\sqrt{2}}\mathrm{Tan}^{-1}\left(\frac{x^2-1}{\sqrt{2}(x)}\right)-\frac{1}{4\sqrt{2}}\log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right|+c$$

12 **Evaluate** $\int \frac{x+1}{x^2+3x+12}dx.$

Sol: $x + 1 = A\frac{d}{dx}(x^2 + 3x + 12) + B = A(2x+3) + B = 2Ax+(3A+B)$

Equating the coefficient of 'x' on both sides we get $2A=1 \Rightarrow A=1/2$

Equating the constant terms we get $3A+B=1 \Rightarrow B=1-3A=1-\frac{3}{2}=-\frac{1}{2}$

$\therefore x+1=\frac{1}{2}(2x+3)-\frac{1}{2}$ $\quad \therefore I=\int\frac{x+1}{x^2+3x+12}dx=\frac{1}{2}\int\frac{2x+3}{x^2+3x+12}dx-\frac{1}{2}\int\frac{dx}{x^2+3x+12}$

$=\frac{1}{2}\log\left|x^2+3x+12\right|-\frac{1}{2}\int\frac{dx}{\left(x-\frac{3}{2}\right)^2+\frac{39}{4}}=\frac{1}{2}\log\left|x^2+3x+12\right|-\frac{1}{2}\int\frac{dx}{\left(x-\frac{3}{2}\right)^2+\left(\frac{\sqrt{39}}{4}\right)^2}$

$=\frac{1}{2}\log\left|x^2+3x+12\right|-\frac{1}{2}\frac{1}{2\sqrt{39}}Tan^{-1}\left(\frac{x+3/2}{\sqrt{39}/2}\right)+c=\frac{1}{2}\log\left|x^2+3x+12\right|-\frac{1}{\sqrt{39}}Tan^{-1}\left(\frac{2x+3}{\sqrt{39}}\right)+c$

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13 **Evaluate** $\int\frac{1}{(1+\sqrt{x})\sqrt{(x-x^2)}}dx$

Sol: Put $x = t^2 \Rightarrow dx = 2t\,dt$. Also $x-x^2=t^2-t^4=t^2(1-t^2)$

$\therefore I=\int\frac{1}{(1+\sqrt{x})\sqrt{(x-x^2)}}dx=\int\frac{1}{(1+t)}\frac{(2t)dt}{\sqrt{t^2(1-t^2)}}=2\int\frac{1}{(1+t)}\frac{dt}{\sqrt{(1-t^2)}}$ ...(1)

Put $1+t=\frac{1}{z}\Rightarrow dt=-\frac{1}{z^2}dz$. $\quad$ Also $t=\frac{1}{z}-1=\frac{1-z}{z}$

$\Rightarrow 1-t^2=1-\left(\frac{1-z}{z}\right)^2=\frac{z^2-(1-2z+z^2)}{z^2}=\frac{2z-1}{z^2}$

$\therefore I=2\int\frac{1}{\left(\frac{1}{z}\right)\sqrt{\frac{2z-1}{z^2}}}\left(\frac{-1}{z^2}\right)dz=-2\int\frac{1}{\sqrt{2z-1}}dz=(-2).\frac{1}{2}.(2\sqrt{2z-1})+c=-2(\sqrt{2z-1})+c$

$=-2\left[\sqrt{2\left(\frac{1}{1+t}\right)-1}\right]+c=-2\left[\sqrt{\frac{2-1-t}{1+t}}\right]+c=-2\left[\sqrt{\frac{1-t}{1+t}}\right]+c=-2\left[\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\right]+c$

$=-2\left[\sqrt{\frac{\left(1-\sqrt{x}\right)^2}{(1+\sqrt{x})(1-\sqrt{x})}}\right]+c=\frac{-2(1-\sqrt{x})}{\sqrt{1-x}}+c=\frac{2(\sqrt{x}-1)}{\sqrt{1-x}}+c$

The INTEGRATION TABLE

☞ The Integral of f(x) is F(x)

No.	Type	Result / Rule
1.	T-1	$\int f(x)dx = F(x)$
2.	T-2.1	$\int f(ax+b)dx = \frac{1}{a}F(ax+b)$
3.	T-2.2	$\int \frac{f'(x)}{f(x)}dx = \log\|f(x)\|$
4.	T-2.3.1	$\int (f(x))^n f'(x)dx = \frac{(f(x))^{n+1}}{n+1}$
5.	T-2.3.2	$\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)}$
6.	T-2.4.1	$\int f(g(x))g'(x)dx = F(g(x))$
7.	T-2.4.2	$\int f(x^n)x^{n-1}dx = \frac{1}{n}F(x^n)$
8.	T-3	$\int \frac{1}{\sqrt{a^2-x^2}}dx = \text{Sin}^{-1}\frac{x}{a}$
		$\int \frac{1}{\sqrt{a^2+x^2}}dx = \text{Sinh}^{-1}\frac{x}{a}$
		$\int \frac{1}{\sqrt{x^2-a^2}}dx = \text{Cosh}^{-1}\frac{x}{a}$
		$\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\text{Sin}^{-1}\frac{x}{a}$
		$\int \sqrt{a^2+x^2}dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\text{Sinh}^{-1}\frac{x}{a}$
		$\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\text{Cosh}^{-1}\frac{x}{a}$
		$\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\left(\text{Sec}^{-1}\frac{x}{a}\right)$
		$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\left(\text{Tan}^{-1}\frac{x}{a}\right)$
		$\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\|$
		$\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\|$
9.	T-4.1	$\int \frac{px+q}{(x+a)(x+b)}dx = \int\left(\frac{A}{x+a}+\frac{B}{x+b}\right)dx$
		$= A\log(x+a) + B\log(x+b)$
10.	T-4.2	$\int \frac{dx}{x(1+x^n)} = \frac{1}{n}\log\left(\frac{x^n}{1+x^n}\right)$
		$\int \frac{dx}{x(1-x^n)} = \frac{1}{n}\log\left(\frac{x^n}{1-x^n}\right)$
		$\int \frac{dx}{x(x^n-1)} = \frac{1}{n}\log\left(\frac{x^n-1}{x^n}\right)$
11.	T-5.1	**Integration by parts**
		$\int uvdx = u\int vdx - \int\left[\frac{du}{dx}(\int vdx)\right]dx$
		$\int udv = uv - \int (du)v$
12.	T-5.2	$\int e^x(f(x)+f'(x))dx = e^x f(x)$
13.	T-6	$I_n = \int \sin^n xdx = \frac{-\sin^{n-1}x\cos x}{n} + \left(\frac{n-1}{n}\right)I_{n-2}$
		$I_n = \int \tan^n xdx = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$
		$I_{m,n} = \int \sin^m x\cos^n x\, dx = \frac{\sin^{m+1}x\cos^{n-1}x}{m+n} + \frac{n-1}{m+n}I_{m,n-2}$
14.	T-7.1	$\int \frac{dx}{a+b\sin x}$, $\int \frac{dx}{a+b\cos x}$
		put $\tan\frac{x}{2} = t \Rightarrow dx = \frac{2dt}{1+t^2}$
		$\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$
15.	T-7.2	$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$
		multiply the Nr & Dr by $\sec^2 x$ and put tanx=t
16.	T-7.3	$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}dx$
		express the Nr. as follows: acosx+bsinx= $A(c\cos x + d\sin x) + B\frac{d}{dx}(c\cos x + d\sin x)$
17.	T-8.1	$\int \frac{(px+q)dx}{ax^2+bx+c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$
		$\int (px+q)\sqrt{ax^2+bx+c}\,dx$
		express px+q as follows: $px+q = A + B\frac{d}{dx}(ax^2+bx+c)$
18.	T-8.2.1	$\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$
		put $px+q = \frac{1}{t}$
19.	T-8.2.2	$\int \frac{dx}{(px^2+q)\sqrt{ax^2+b}}$
		put $x = \frac{1}{t}$ and in the resulting integrand put $\sqrt{\text{quadratic}} = y$
20.	T-8.3	$\int \frac{ax+b}{\sqrt{px+q}}dx$, $\int \frac{dx}{(ax+b)\sqrt{px+q}}$
		$\int (ax+b)\sqrt{px+q}\,dx$
		put $\sqrt{px+q} = t$ i.e, $px+q = t^2$
21.	T-8.4	$\int \frac{(x^2+1)dx}{x^4+kx^2+1}$, $\int \frac{(x^2-1)dx}{x^4+kx^2+1}$
		divide the Nr.& Dr. by x^2 then
		Put $x - \frac{1}{x} = t$ if the Nr. is $1 + \frac{1}{x^2}$
		Here $x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$
		Put $x + \frac{1}{x} = t$ if the Nr. is $1 - \frac{1}{x^2}$
		Here $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$
22.	T-8.5	$\int \frac{ax+b}{\sqrt[n]{px+q}}dx$
		put $\sqrt[n]{px+q} = t$
		i.e, $px+q = t^n$

THE ILLUSTRATIVE INTEGRATION TABLE

S.No	Type	Result / Rule	Illustrations
1	Type-1	$\int f(x)dx = F(x)$	**1.** $\int \frac{1}{x}dx = \log x + c$ **2.** $\int \frac{1}{1+x^2}dx = \mathrm{Tan}^{-1}x + c$ **3.** $\int 5^x dx = 5^x \log_5 e + c$ **4.** $\int e^{\frac{1}{2}\log x}dx = \int \sqrt{x}dx = \frac{2}{3}x^{\frac{3}{2}} + c$
2	Type-2.1	$\int f(ax+b)dx = \frac{1}{a}F(ax+b)$	**1.** $\int \cos(2x+3)dx = \frac{1}{2}\sin(2x+3) + c$ **2.** $\int \frac{1}{\sqrt{5x+6}}dx = \frac{2}{5}\sqrt{5x+6} + c$ **3.** $\int \csc 3x \cot 3x dx = -\frac{1}{3}\csc 3x + c$
3	Type-2.2	$\int \frac{f'(x)}{f(x)}dx = \log\lvert f(x)\rvert$	**1.** $\int \frac{2x+3}{x^2+3x+4}dx = \log(x^2+3x+4) + c$ **2.** $\int \frac{dx}{x\log x} = \log(\log x)$ **3.** $\int \frac{\cos x - \sin x}{\sin x + \cos x}dx = \log(\sin x + \cos x) + c$
4.	Type-2.3.1	$\int f(x)^n f'(x)dx = \frac{(f(x))^{n+1}}{n+1}$	**1.** $\int \sin^5 x \cos x dx = \frac{\sin^6 x}{6} + c$ **2.** $\int \frac{\log x}{x}dx = \frac{(\log x)^2}{2} + c$ **3.** $\int \frac{\mathrm{Tan}^{-1}x}{1+x^2}dx = \frac{(\mathrm{Tan}^{-1}x)^2}{2} + c$
5.	Type-2.3.2	$\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)}$	**1.** $\int \frac{\cos x}{\sqrt{\sin x}}dx = 2\sqrt{\sin x} + c$ **2.** $\int \frac{x+1}{\sqrt{x^2+2x+3}}dx = \sqrt{x^2+2x+3} + c$ **3.** $\int \frac{\sec^2 x dx}{\sqrt{a+b\tan x}} = \frac{2}{b}\sqrt{a+b\tan x} + c$
6.	Type-2.4.1	$\int f(g(x))g'(x)dx = F(g(x))$	**1.** $\int \frac{e^{\mathrm{Sin}^{-1}x}}{\sqrt{1-x^2}}dx = e^{\mathrm{Sin}^{-1}x} + c$ **2.** $\int \frac{1}{x}\cot(\log x)dx = \log\sin(\log x)$ **3.** $\int \frac{\cos(\mathrm{Tan}^{-1}x)}{1+x^2}dx = \sin(\mathrm{Tan}^{-1}x) + c$
7.	Type-2.4.2	$\int f(x^n)x^{n-1}dx = \frac{1}{n}F(x^n)$	**1.** $\int x\cos x^2 dx = \frac{1}{2}\sin x^2 + c$ **2.** $\int x^2 \sec^2 x^3 dx = \frac{1}{3}\tan x^3 + c$ **3.** $\int x^3 \sin x^4 dx = \frac{-1}{4}\cos x^4 + c$
8.	Type-3	$\int \frac{1}{\sqrt{a^2-x^2}}dx = \mathrm{Sin}^{-1}\frac{x}{a}$	**1.** $\int \frac{dx}{\sqrt{4-x^2}} = \mathrm{Sin}^{-1}\frac{x}{2} + c$ **2.** $\int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{dx}{\sqrt{3^2-(2x)^2}} = \frac{1}{2}\mathrm{Sin}^{-1}\frac{2x}{3} + c$ **3.** $\int \frac{dx}{\sqrt{2ax-x^2}} = \int \frac{dx}{\sqrt{a^2-(x-a)^2}} = \mathrm{Sin}^{-1}\frac{x-a}{a} + c$ **4.** $\int \frac{dx}{\sqrt{15-4x-4x^2}} = \frac{1}{2}\int \frac{dx}{\sqrt{2^2-\left(x+\frac{1}{2}\right)^2}} = \frac{1}{2}\mathrm{Sin}^{-1}\left(\frac{x+(1/2)}{2}\right) + c$
		$\int \frac{1}{\sqrt{a^2+x^2}}dx = \mathrm{Sinh}^{-1}\frac{x}{a}$	**1.** $\int \frac{dx}{\sqrt{3+x^2}} = \mathrm{Sinh}^{-1}\frac{x}{\sqrt{3}} + c$ **2.** $\int \frac{dx}{\sqrt{2x^2+3x+4}} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(x+\frac{3}{4}\right)^2+\frac{23}{16}}} = \frac{1}{\sqrt{2}}\mathrm{Sinh}^{-1}\left(\frac{x+\frac{3}{4}}{\frac{\sqrt{23}}{4}}\right) + c$
		$\int \frac{1}{\sqrt{x^2-a^2}}dx = \mathrm{Cosh}^{-1}\frac{x}{a}$	**1.** $\int \frac{dx}{\sqrt{x^2-1}} = \mathrm{Cosh}^{-1}x + c$ **2.** $\int \frac{dx}{\sqrt{x^2-16}} = \mathrm{Cosh}^{-1}\frac{x}{4} + c$

S.No	Type	Result / Rule	Illustrations
8.	Type-3	$\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\mathrm{Sin}^{-1}\frac{x}{a}$	**1.** $\int\sqrt{25-x^2}dx=\frac{x}{2}\sqrt{25-x^2}+\frac{25}{2}\mathrm{Sin}^{-1}\frac{x}{5}+c$ **2.** $\int\sqrt{5-4x-x^2}dx=\int\sqrt{3^2-(x+2)^2}dx=\frac{x+2}{2}\sqrt{5-4x-x^2}+\frac{9}{2}\mathrm{Sin}^{-1}\frac{x+2}{3}+c$
		$\int\sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\mathrm{Sinh}^{-1}\frac{x}{a}$	**1.** $\int\sqrt{1+x^2}dx=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\mathrm{Sinh}^{-1}x+c$ **2.** $\int\sqrt{3+x^2}dx=\frac{x}{2}\sqrt{3+x^2}+\frac{3}{2}\mathrm{Sinh}^{-1}\frac{x}{\sqrt{3}}+c$
		$\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\mathrm{Cosh}^{-1}\frac{x}{a}$	**1.** $\int\sqrt{x^2-16}dx=\frac{x}{2}\sqrt{x^2-16}-8\mathrm{Cosh}^{-1}\frac{x}{4}+c$ **2.** $\int\sqrt{x^2-5}dx=\frac{x}{2}\sqrt{x^2-5}-\frac{5}{2}\mathrm{Cosh}^{-1}\frac{x}{\sqrt{5}}+c$ **3.** $\int\sqrt{4x^2+4x-2}dx=\int\sqrt{(2x+1)^2-3}dx=\frac{1}{2}\left(\frac{2x+1}{2}\sqrt{4x^2+4x-2}-\frac{3}{2}\mathrm{Cosh}^{-1}\frac{2x+1}{\sqrt{3}}\right)+c$
		$\int\frac{dx}{a^2+x^2}=\frac{1}{a}\left(\mathrm{Tan}^{-1}\frac{x}{a}\right)$	**1.** $\int\frac{dx}{4+x^2}=\frac{1}{2}\mathrm{Tan}^{-1}\frac{x}{2}+c$ **2.** $\int\frac{dx}{2+x^2}=\frac{1}{\sqrt{2}}\mathrm{Tan}^{-1}\frac{x}{\sqrt{2}}+c$ **3.** $\int\frac{dx}{9+(2x+1)^2}=\frac{1}{2}\cdot\frac{1}{3}\mathrm{Tan}^{-1}\frac{2x+1}{3}+c$ **4.** $\int\frac{dx}{1+9x^2}=\frac{1}{3}\mathrm{Tan}^{-1}3x+c$ **5.** $\int\frac{dx}{x^2+2x+3}=\int\frac{dx}{(x+1)^2+(\sqrt{2})^2}=\frac{1}{\sqrt{2}}\mathrm{Tan}^{-1}\frac{x+1}{\sqrt{2}}+c$
		$\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\|$	**1.** $\int\frac{dx}{49-x^2}=\frac{1}{2x7}\log\left\|\frac{7+x}{7-x}\right\|$ **2.** $\int\frac{dx}{25-9x^2}=\frac{1}{2x5}\frac{1}{3}\log\left\|\frac{5+3x}{5-3x}\right\|$ **3.** $\int\frac{dx}{a^2-b^2x^2}=\frac{1}{b}\frac{1}{2a}\log\left\|\frac{a+bx}{a-bx}\right\|+c$
		$\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\|$	**1.** $\int\frac{dx}{2x^2-3}=\int\frac{dx}{(\sqrt{2}x)^2-(\sqrt{3})^2}=\frac{1}{\sqrt{2}}\cdot\frac{1}{2\sqrt{3}}.\log\left(\frac{\sqrt{2}x-\sqrt{3}}{\sqrt{2}x+\sqrt{3}}\right)$ **2.** $\int\frac{x^2dx}{x^2-9}=\int\left(1+\frac{9}{x^2-9}\right)dx=x+9.\frac{1}{2.3}\log\left(\frac{x-3}{x+3}\right)+c$
9.	Type-4.1	$\int\frac{(px+q)dx}{(x+a)(x+b)}=\int\left(\frac{A}{x+a}+\frac{B}{x+b}\right)dx$ $=A\log(x+a)+B\log(x+b)$	**1.** $\int\frac{(x-1)}{(x-2)(x-3)}dx=\int\left(\frac{2}{x-3}-\frac{1}{x-2}\right)dx=2\log(x-3)-\log(x-2)+c$
10.	Type-4.2	$\int\frac{dx}{x(1+x^n)}=\frac{1}{n}\log\left(\frac{x^n}{1+x^n}\right)$	**1.** $I=\int\frac{dx}{x(1+x^5)}=\int\frac{x^4}{x^5(1+x^5)}dx$, Put $x^5=t\Rightarrow5x^4dx=dt\Rightarrow I=\frac{1}{5}\int\frac{dt}{t(1+t)}=\frac{1}{5}\int\left(\frac{1}{t}-\frac{1}{1+t}\right)dt=\frac{1}{5}\log\left(\frac{t}{1+t}\right)=\frac{1}{5}\log\left(\frac{x^5}{1+x^5}\right)+c$

S.No	Type	Result / Rule	Illustrations
11.	Type-5.1	**Integration by parts:** $\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\left(\int v\,dx\right)\right]dx$ $\int u\,dv = uv - \int (du)\,v$	**1.** $\int x\sec^2 x\,dx = x\int \sec^2 x - \int \frac{d}{dx}(x)\int \sec^2 x\,dx = x\tan x - \int 1.\tan x = x\tan x - \log\lvert\sec x\rvert + c$ **2.** $\int \mathrm{Tan}^{-1}x\,dx = \int \mathrm{Tan}^{-1}x(1)dx = \mathrm{Tan}^{-1}x(x) - \int \frac{1}{1+x^2}x\,dx = x\mathrm{Tan}^{-1}x - \frac{1}{2}\log(1+x^2) + c$
12.	Type-5.2	$\int e^x(f(x)+f'(x))dx = e^x f(x)$	**1.** $\int e^x(x^2+2x)dx = e^x x^2 + c$ **2.** $\int e^x\left(\frac{x\log x+1}{x}\right)dx = \int e^x\left(\log x + \frac{1}{x}\right)dx = e^x\log x + c$
13.	Type-6	$I_n = \int \sin^n x\,dx = \frac{-\sin^{n-1}x\cos x}{n} + \left(\frac{n-1}{n}\right)I_{n-2}$	**1.** $\int \sin^4 x\,dx = I_4 = -\frac{\sin^3 x\cos x}{4} + \frac{3}{4}I_2 = -\frac{\sin^3 x\cos x}{4} + \frac{3}{4}\left(-\frac{\sin x\cos x}{2} + \frac{1}{2}I_0\right)$; here $I_0 = x$
14.	Type-7.1	$\int\frac{dx}{a+b\sin x}$, $\int\frac{dx}{a+b\cos x}$ put $\tan\frac{x}{2} = t \Rightarrow dx = \frac{2dt}{1+t^2}$ $\sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}$	**1.** $\int\frac{dx}{3+2\cos x}$; put $\tan\frac{x}{2} = t \Rightarrow dx = \frac{2dt}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2} \Rightarrow I = \int\frac{2dt}{(1+t^2)\left(3+2\left(\frac{1-t^2}{1+t^2}\right)\right)}$ $\Rightarrow I = 2\int\frac{dt}{5+t^2} = \frac{2}{\sqrt{5}}\mathrm{Tan}^{-1}\frac{t}{\sqrt{5}} = \frac{2}{\sqrt{5}}\mathrm{Tan}^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{5}}\right) + c$
15.	Type-7.2	$\int\frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$ multiply the Nr & Dr by $\sec^2 x$ and put tanx=t	**1.** $\int\frac{dx}{4\cos^2 x + 9\sin^2 x} = \int\frac{\sec^2 x\,dx}{4+9\tan^2 x}$; Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$; $I = \int\frac{dt}{2^2+(3t)^2} = \frac{1}{3}\cdot\frac{1}{2}\mathrm{Tan}^{-1}\left(\frac{3\tan x}{2}\right) + c$
16.	Type-7.3	$\int\frac{a\cos x + b\sin x}{c\cos x + d\sin x}dx$ express the Nr. as follows: acosx+bsinx= $A(c\cos x + d\sin x) +$ $B\frac{d}{dx}(c\cos x + d\sin x)$	**1.** $\int\frac{2\cos x + 3\sin x}{4\cos x + 5\sin x}dx$; put $2\cos x + 3\sin x = A(4\cos x + 5\sin x) + B\frac{d}{dx}(4\cos x + 5\sin x)$ $= A(4\cos x + 5\sin x) + B(-4\sin x + 5\cos x) \Rightarrow A = \frac{23}{41}, B = \frac{-2}{41}$ $\therefore I = \int\frac{2\cos x + 3\sin x}{4\cos x + 5\sin x}dx = \int\frac{\frac{23}{41}(4\cos x + 5\sin x) - \frac{2}{41}\frac{d}{dx}(4\cos x + 5\sin x)}{4\cos x + 5\sin x}dx$ $= \int\left(\frac{23}{41} - \frac{2}{41}\frac{\frac{d}{dx}(4\cos x + 5\sin x)}{4\cos x + 5\sin x}\right)dx = \frac{23}{41}x - \frac{2}{41}\log(4\cos x + 5\sin x) + c$

S.No	Type	Result / Rule	Illustrations
17	Type-8.1	$\int \frac{(px+q)dx}{ax^2+bx+c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$, $\int (px+q)\sqrt{ax^2+bx+c}\ dx$ express px+q as follows: $px+q = A + B\frac{d}{dx}(ax^2+bx+c)$	**1.** $\int \frac{x+2}{x^2+2x+3}dx$; put $x+2 = A + B(2x+2) \Rightarrow A = 1, B = 1/2 \Rightarrow I = \int \frac{1+\frac{1}{2}(2x+2)}{x^2+2x+3}dx$ $\Rightarrow I = \int \frac{dx}{(x+1)^2+(\sqrt{2})^2} + \frac{1}{2}\int \frac{2x+2}{x^2+2x+3}dx = \frac{1}{\sqrt{2}}Tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + \frac{1}{2}\log(x^2+2x+3)+c$
18	Type-8.2.1	$\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$ Put $px+q = \frac{1}{t}$	**1.** $\int \frac{dx}{(x-1)\sqrt{x^2+1}}$; $x-1 = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2}dt$ & $x = \frac{1+t}{t} \Rightarrow I = -\int \frac{dt}{\sqrt{t^2+t+\frac{1}{2}}} = -\frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{\left(t+\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}}$ $= -\frac{1}{\sqrt{2}}Sinh^{-1}(2t+1) = -\frac{1}{\sqrt{2}}Sinh^{-1}\left(\frac{2}{x-1}+1\right) = -\frac{1}{\sqrt{2}}Sinh^{-1}\left(\frac{x+1}{x-1}\right)^2$
19	Type-8.2.2	$\int \frac{dx}{(px^2+q)\sqrt{ax^2+b}}$ put $x = \frac{1}{t}$ & in the resulting integrand put $\sqrt{quadratic} = y$	**1.** $\int \frac{dx}{(x^2+1)\sqrt{2-x^2}}$; put $x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2}dt \Rightarrow I = -\int \frac{tdt}{(1+t^2)\sqrt{2t^2+1}}$ put $\sqrt{2t^2+1} = y \Rightarrow tdt = \frac{1}{2}ydy$ $\Rightarrow I = -\int \frac{dy}{y^2+1} = -Tan^{-1}y = -Tan^{-1}\sqrt{2t^2+1} = -Tan^{-1}\frac{\sqrt{2+x^2}}{x}+c$
20	Type-8.3	$\int \frac{ax+b}{\sqrt{px+q}}dx$, $\int \frac{dx}{(ax+b)\sqrt{px+q}}$ $\int (ax+b)\sqrt{px+q}\ dx$ put $\sqrt{px+q} = t$ i.e, $px+q = t^2$	**1.** $\int \frac{x+2}{\sqrt{2x+3}}dx$; put $\sqrt{2x+3} = t \Rightarrow 2x+3 = t^2 \Rightarrow x = \frac{t^2-3}{2} \Rightarrow I = \frac{1}{2}\int (t^2+1)dt = \frac{1}{2}\left(\frac{t^3}{3}+t\right)$ $= \frac{t}{2}\left(\frac{t^2+3}{3}\right) = \frac{\sqrt{2x+3}}{2}\left(\frac{2x+3+3}{3}\right) = \frac{\sqrt{2x+6}(x+3)}{3}+c$
21.	Type-8.4	$\int \frac{(x^2+1)dx}{x^4+kx^2+1}$, $\int \frac{(x^2-1)dx}{x^4+kx^2+1}$ divide the Nr&Dr by x^2 then Put $x-\frac{1}{x} = t$ if the Nr.is $1+\frac{1}{x^2}$ Here $x^2+\frac{1}{x^2} = \left(x-\frac{1}{x}\right)^2+2$ Put $x+\frac{1}{x} = t$ if the Nr.is $1-\frac{1}{x^2}$ Here $x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2-2$	**1.** $\int \frac{x^2+1}{x^4-x^2+1}dx = \int \frac{1+\frac{1}{x^2}}{x^2-1+\frac{1}{x^2}}dx$ put $x-\frac{1}{x} = t \Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt$; $x^2+\frac{1}{x^2}\left(x-\frac{1}{x}\right)^2+2 = t^2+2$ $\Rightarrow I = \int \frac{dt}{t^2+2-1} = \int \frac{dt}{t^2+1} = Tan^{-1}t = Tan^{-1}\left(x-\frac{1}{x}\right)+c$