# WELCOME

## STAR 'QR CODE'

## DIGITAL MATERIAL

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# 5. 3D - COORDINATES

## ADDITIONAL QUESTIONS WITH SOLUTIONS

**1. Show that the points O(0, 0, 0), A(2, –3, 3), B(–2, 3, –3) are collinear. Find the ratio in which each point divides the segment joining the other two**

**Sol:** Let O=(0, 0, 0), A=(2, –3, 3), B=(–2, 3, –3)

$$OA=\sqrt{(0-2)^2+(0+3)^2+(0-3)^2}=\sqrt{4+9+9}=\sqrt{22}$$

$$OB=\sqrt{(0+2)^2+(0-3)^2+(0+3)^2}=\sqrt{4+9+9}=\sqrt{22}$$

$$AB=\sqrt{(2+2)^2+(-3-3)^2+(3+3)^2}=\sqrt{16+36+36}=\sqrt{88}=\sqrt{4\times22}=2\sqrt{22}$$

Now $OA+OB=\sqrt{22}+\sqrt{22}=2\sqrt{22}=AB$ $\therefore$ O, A, B are collinear


'O' divides $\overline{AB}$ in the ratio = OA : OB= $\sqrt{22}:\sqrt{22}=1:1$(internally)

'A' divides $\overline{OB}$ in the ratio= OA : AB= $\sqrt{22}:2\sqrt{22}=1:2$ (externally)

'B' divides $\overline{OA}$ in the ratio= AB : BO = $2\sqrt{22}:\sqrt{22}=2:1$(externally)

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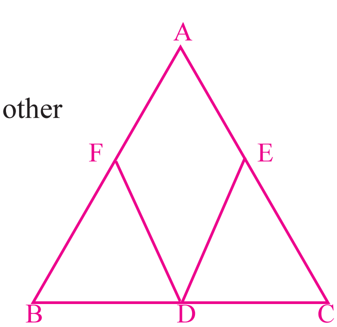
**2. If $D(x_1, y_1, z_1)$ $E(x_2, y_2, z_2)$ and $F(x_3, y_3, z_3)$ are the midpoints of the sides BC, CA and AB respectively of a triangle, find its vertices A, B and C.**

**Sol:** Given that $D(x_1, y_1, z_1)$ $E(x_2, y_2, z_2)$ and $F(x_3, y_3, z_3)$ are the midpoints of the sides BC, CA, AB respectively

Then AEDF form a Parallelogram.




We know that the diagonals of the parallelogram bisects each other

$\Rightarrow$ Mid point of AD = Mid point of EF

$$\Rightarrow \frac{A+D}{2}=\frac{E+F}{2}\Rightarrow A+D=E+F\Rightarrow A=E+F-D$$

$$\Rightarrow A=(x_2,y_2,z_2)+(x_3,y_3,z_3)-(x_1,y_1,z_1)$$

$$\Rightarrow A=(x_2+x_3-x_1, y_2+y_3-y_1, z_2+z_3-z_1)$$

Similarly $B=(x_3+x_1-x_2, y_3+y_1-y_2, z_3+z_1-z_2)$ and

$$C=(x_1+x_2-x_3, y_1+y_2-y_3, z_1+z_2-z_3)$$

**3. If H,G,S and I respectively denote orthocentre, centroid, circumcentre and in-centre of a triangle formed by the points (1,2,3), (2,3,1) and (3,1,2), then find H,G,S,I.**

**Sol:** Let A = (1,2,3), B=(2,3,1), C=(3,1,2)

$$AB = \sqrt{(2-1)^2 + (3-2)^2 + (1-3)^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$BC = \sqrt{(3-2)^2 + (1-3)^2 + (2-1)^2} = \sqrt{1+4+1} = \sqrt{6}$$

$$CA = \sqrt{(1-3)^2 + (2-1)^2 + (3-2)^2} = \sqrt{4+1+1} = \sqrt{6}$$

Since AB=BC=CA, ABC is an equilateral triangle.

In an equilateral triangle centroid, orthocentre, circumcentre and incentre coincide.

Now, centroid $G = \left(\frac{1+2+3}{3}, \frac{2+3+1}{3}, \frac{3+1+2}{3}\right) = (2,2,2)$

$\therefore$ H = (2,2,2), S=(2,2,2), I = (2,2,2).

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**4. Find the incentre of the triangle formed by the points (0,0,0), (3,0,0) and (0,4,0).**

**Sol :** Let $A(x_1,y_1,z_1)$=(0,0,0), $B(x_2,y_2,z_2)$= (3,0,0), $C(x_3,y_3,z_3)$=(0,4,0).

$$a = BC = \sqrt{9+16+0} = 5$$

$$b = CA = \sqrt{0+16+0} = 4$$

$$c = AB = \sqrt{9+0+0} = 3$$

$$\therefore \text{ Incentre } I = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}, \frac{az_1+bz_2+cz_3}{a+b+c}\right)$$

$$= \left(\frac{5(0)+4(3)+3(0)}{5+4+3}, \frac{5(0)+4(0)+3(4)}{5+4+3}, \frac{5(0)+4(0)+3(0)}{5+4+3}\right) = \left(\frac{12}{12}, \frac{12}{2}, \frac{0}{12}\right)$$

$$= (1,1,0)$$

**5. A(3, 2, 0), B(5, 3, 2), C(–9, 6, –3) are vertices of a triangle. $\overline{AD}$ is the bisector of $\angle BAC$ meet $\overline{BC}$ at D. Find the co-ordinates of D.**

**Sol:** The vertices of the triangle are A(3, 2, 0), B(5, 3, 2), C(–9, 6, –3)

We know that the bisector of $\angle BAC$ divides $\overline{BC}$ in the ratio AB : AC internally

Let $\overline{AD}$ is the bisector of $\angle BAC$ meet $\overline{BC}$ at D $\Rightarrow$ BD : DC = AB : AC ..........(1)

$$AB = \sqrt{(3-5)^2 + (2-3)^2 + (0-2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$

$$AC = \sqrt{(3+9)^2 + (2-6)^2 + (0+3)^2} = \sqrt{144+16+9} = \sqrt{169} = 13$$

From (1) BD : DC = 3 : 13

$$\therefore D = \left(\frac{3(-9)+13(5)}{3+13}, \frac{3(6)+13(3)}{3+13}, \frac{3(-3)+13(2)}{3+13}\right)$$

$$= \left(\frac{-27+65}{16}, \frac{18+39}{16}, \frac{-9+26}{16}\right) = \left(\frac{38}{16}, \frac{57}{16}, \frac{17}{16}\right)$$

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**6. If A(4, 8, 12) B(2, 4, 6) C(3, 5, 4) and D(5, 8, 5) are four points, show that the line $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect.**

**Sol:** Given points are A(4, 8, 12) B(2, 4, 6) C(3, 5, 4) and D(5, 8, 5)

Let E divides $\overline{AB}$ in the ratio k : 1–k ( Taking ratio in this way makes calculations simple)

$$\Rightarrow E = \left[\frac{k(2)+(1-k)(4)}{k+1-k}, \frac{k(4)+(1-k)(8)}{k+1-k}, \frac{k(6)+(1-k)(12)}{k+1-k}\right] = (4 - 2k, 8 - 4k, 12 - 6k) .........(1)$$

Let E divides $\overline{CD}$ in the ratio t : 1–t

$$\Rightarrow E = \left[\frac{t(5)+(1-t)(3)}{t+1-t}, \frac{t(8)+(1-t)(5)}{t+1-t}, \frac{t(5)+(1-t)(4)}{t+1-t}\right] = (3 + 2t, 5 + 3t, 4 + t) ............(2)$$

From (1) & (2); (4 – 2k, 8 – 4k, 12 – 6k) = (3 + 2t, 5 + 3t, 4 + t)

4 – 2k = 3 + 2t $\Rightarrow$ 2k + 2t – 1 = 0 ...........(3)

8 – 4k = 5 + 3t $\Rightarrow$ 4k + 3t – 3 = 0............(4)

12 – 6k = 4 + t $\Rightarrow$ 6k + t – 8 = 0 ............ (5)

Solving (3) & (4),

$$\frac{k}{(2)(-3)-(-1)(3)} = \frac{t}{(-1)(4)-(2)(-3)} = \frac{1}{(2)(3)-(2)(4)}$$

$$\Rightarrow \frac{k}{-6+3}=\frac{t}{-4+6}=\frac{1}{6-8}\Rightarrow \frac{k}{-3}=\frac{t}{2}=\frac{1}{-2}\Rightarrow k=\frac{3}{2}, t=-1$$

Substituting the values of k and t in equation (5), we have

$6\left(\frac{3}{2}\right)-1-8=9-1-8=0$ . So values of k and t satisfies the equation (5).

Hence the given lines $\overline{AB}$ and $\overline{CD}$ intersect each other.

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**7. Find the point of intersection of the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ where A = (7, –6, 1) , B = (17, –18, –3), C = (1, 4, –5) and D = (3, –4, 11)**

**Sol:** The given points are A(7, –6, 1) B(17, –18, –3) C(1, 4, –5) and D(3, –4, 11)

Let E divides $\overline{AB}$ in the ratio k : 1–k

$$\Rightarrow E=\left[\frac{k(17)+(1-k)(7)}{k+1-k},\frac{k(-18)+(1-k)(-6)}{k+1-k},\frac{k(-3)+(1-k)(1)}{k+1-k}\right]=(7+10k, -6-12k, 1-4k).........(1)$$

Let E divides $\overline{CD}$ in the ratio t : 1–t

$$\Rightarrow E=\left[\frac{t(3)+(1-t)(1)}{t+1-t},\frac{t(-4)+(1-t)(4)}{t+1-t},\frac{t(11)+(1-t)(-5)}{t+1-t}\right]=(1+2t, 4-8t, -5+16t)............(2)$$

From (1) & (2) $\Rightarrow$ (7+10k, –6–12k, 1–4k) = (1+2t, 4–8t, –5+16t)

7 + 10k = 1 + 2t $\Rightarrow$ 10k – 2t + 6 = 0 $\Rightarrow$ 5k – t + 3 = 0..........(3)

–6 – 12k = 4 – 8t $\Rightarrow$ 12k – 8t + 10 = 0 $\Rightarrow$ 6k – 4t + 5 = 0...........(4)

1 – 4k = –5 + 16t $\Rightarrow$ 4k + 16t – 6 = 0 $\Rightarrow$ 2k + 8t – 3 = 0 ........... (5)

Solving (3) & (4),

$$\frac{k}{(-1)(5)-(3)(-4)}=\frac{t}{(3)(6)-(5)(5)}=\frac{1}{(5)(-4)-(-1)(6)}$$

$$\Rightarrow \frac{k}{-5+12}=\frac{t}{18-25}=\frac{1}{-20+6}\Rightarrow \frac{k}{7}=\frac{t}{-7}=\frac{1}{-14}\Rightarrow k=\frac{-1}{2}, t=\frac{1}{2}$$

Substituting the values of k and t in equation (5) then $2\left(\frac{-1}{2}\right)+8\left(\frac{1}{2}\right)-3=-1+4-3=0$ .

k and t values satisfies the equation (5).

Hence the given lines $\overline{AB}$ and $\overline{CD}$ intersect each other at E.

From (2), $E=\left(1+2\left(\frac{1}{2}\right),4-8\left(\frac{1}{2}\right),-5+16\left(\frac{1}{2}\right)\right)=(2,0,3)$ .