5. PERMUTATIONS & COMBINATIONS

Sections	No. of periods (18)	Weightage in IPE [2x2+1x4+1x7 =15]
1. Permutations	12	2 or 4 or 7 marks
2. Combinations	6	2 or 4 or 7 marks

What we do in 'Permutations and Combinations' is just ‘counting without actually counting’. This topic is related to various number of applications in day-to-day life. Applications of this topic are seen in sets, functions, relations, Binomial theorem, Probability etc.,

'Fundamental principle of counting' is the most underlying basic concept of this topic. 'Permutation' is an 'arrangement' (order is given importance) of things and 'combination' is merely a 'selection' (order is given no importance).

The word 'arrangement' is synonymous with 'permutation'.

The terms 'Group, selection, set, team, choosing, committee, class' etc., refer to 'combination'.

The number of permutations of n things taken r at a time is denoted by ${}^nP_r$. ${}^nC_r$ is used to denote the number of combinations of n things taken r at a time.

Consider a simple case of <u>taking </u>(???) two digits out of digits 1,2,3.

The total number of 2 digited numbers is 6 viz., 12, 23, 31, 21, 32, 13

Total number of 2 digited sets is 3 viz., {1,2}, {2,3}, {3,2}

Total number of ordered pairs is 9 viz., (1,1), (2,2), (3,3), (1,2), (2,3), (3,1), (2,1),(3,2), (1,3) Total number of 2 element multi sets is 6 viz., {1,1}, {2,2}, {3,3}, {1,2}, {2,3}, {3,1} Like this, we encounter with so many varieties of practical questions. To solve such type of problems, we have to understand and analyse the questions properly and proceed accordingly.

SYNOPSIS POINTS

1. **Fundamental Principle of counting** : If an operation A can be performed in m ways and another operation B, which is independent of A can be performed in n ways, then both the operations A and B can be performed in m·n ways

2. **Fundamental principle of Addition**: If a work A can be done in m ways, another work B which is independent of A, can be done in n ways, then the number of ways that, atleast one of A,B (i.e., A or B) can be done is (m+n).

3. The number of permutations of n dissimilar things taken r at a time is

$$^{n}P_r = \frac{n!}{(n-r)!} = n(n-1)(n-2).....(n-r+1)$$ =product of r consecutive integers starting from n

= number of one one function that can be defined from A to B, where O(A)=n, O(B)=r

4.1. The number of permutations of 'n' dissimilar things taken r at a time, when repetition of things is allowed any number of times is n^r.

4.2. The number of permutations of n dissimilar things taken r at a time with **at least** one repetition is $n^r - {}^{n}p_r$.

4.3. The number of functions that can defined from a set containing m elements into a set containing n elements is n^m.

5. The number of linear permutations of 'n' things, in which there are p alike things of one kind, q alike things of the second kind, r alike things of the third kind and the rest are different is $\frac{n!}{p!q!r!}$

6. The number of circular permutations of 'n' dissimilar things taken all at a time is (n–1)!.

7. The number of circular permutations of n things, when orientation of things is not considered, is $\frac{(n-1)!}{2}$ (half of the actual number of circular permutations).

8. The number of combinations of 'n' dissimilar things taken 'r' at a time is $^{n}C_r = \frac{n!}{(n-r)!r!} = \frac{^{n}P_r}{r!}$

9.1. $^{n}C_r = {}^{n}C_{n-r}$

9.2. $^{n}C_r + {}^{n}C_{r-1} = {}^{(n+1)}C_r$

9.3. If $^{n}C_r = {}^{n}C_s$ then r=s or r+s=n

10. The number of diagonals in a regular polygon of n sides is $\frac{n(n-3)}{2}$

11. If m.n are distinct positive integers, then the number of ways of dividing 'm+n' things into two groups containing 'm' things and 'n' things respectively is $\frac{(m+n)!}{m!n!}$

12. The number of selections of (p+q) things taken (i) any number (0 to p+q) at a time when p things are alike of one kind and q things are alike of other kind= (p+1)(q+1) (ii) one or more at a time (i.e.,atleast one at a time)=(p+1)(q+1)–1.

13. The total number of different selections of n different things taken
(i) any number (0 to n) at a time = 2^n
(ii) atleast one at a time (i.e., one or more at a time)=$2^n - 1$

ADDITIONAL QUESTIONS WITH SOLUTIONS

1 **Find the number of all 4 letter words that can be formed using the letters of the word EQUATION . How many of these words begin with E? How many end with N? How many begin with E and end with N?**

Sol: **(i)** The given word EQUATION contains 8 letters.

So, the number of 4 letter words formed from it = $^8P_4 = 8\times7\times6\times5 = 1680$

(ii) 4 letter words begining with E:

E			

Fill the first place with E.

Then the remaining 3 places can be filled with the remaining 7 letters in $^7P_3 = 7\times6\times5 = 210$ ways.

(iii) 4 letter words ending with N:

			N

Fill the last place with N. Then the remaining 3 places can be filled with the remaining 7 letters in $^7P_3 = 7\times6\times5 = 210$ ways.

(iv) 4 letter words beginning with E and ending with N:

E			N

Fill the first place with E and last place with N.

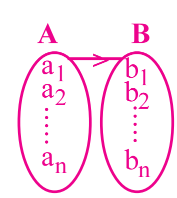
Then the remaining 2 places can be filled with remaining 6 letters in $^6P_2 = 6\times5 = 30$ ways.

---

2 **Find the number of functions from a set A with m elements to a set B with n elements.**

Sol: To get a function, all the m elements of A must be mapped the n elements of B.

Let A = $\{a_1, a_2, \ldots\ldots a_m\}$ and B = $\{b_1, b_2\ldots\ldots, b_n\}$.



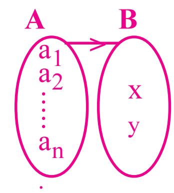
First element a_1 can be mapped with any element of B in n ways.

Second element a_2 can be mapped with any element of B in n ways.

Similarly, m^{th} element a_m can be mapped with any element of B in n ways.

$\therefore$ Total number of functions = $n \times n \times \ldots\ldots \times n$ (m times) = n^m.

3 **Find the number of surjections from a set A with n elements to a set B with 2 elements when $n > 1$.**



Sol: We remember that in a surjection (on-to function) all the elements of codomain B must have preimages from the set of elements of domain A.

Let $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{x , y\}$.

$\therefore$ Total number of functions from A to B $= n^m = 2^n$

Among these 2^n functions there are two functions which are not actually surjections.

One is when all the elements of A correspond with x and the other is when all the elements of A correspond with y.

Excepting these two, all the remaining are surjections.

Hence, the number of surjections from A to B is $2^n - 2$

---

4 **Find the number of numbers less than 2000 that can be formed using the digits 1, 2, 3,4 if repetition is allowed.**

Sol: All the single digited numbers, two digited numbers, three digited numbers and the four digited numbers using the digits 1, 2, 3, 4 are less than 2000

(i) The number of single digited numbers formed using the given digits = 4

(ii) The number of two digited numbers formed using the given 4 digits when repetition is allowed $= 4 \times 4 = 16$

(iii) The number of three digited numbers formed using the given digits $= 4 \times 4 \times 4 = 64$

(iv) All the 4 digited numbers starting with 1 using 1,2,3,4 are all less than 2000.

When 1 is placed in the first place the remaining 3 places can be filled in $4^3 = 64$ ways.

$\therefore$ The total number of numbers less than 2000 $= 4 + 16 + 64 + 64 = 148$

5 **Find the number of 4-digits numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 that are divisible by (i) 2 (ii) 3 when the repetition is allowed**

Sol: **i) Divisible by 2:**

			2,4,6

A number is divisible by 2, if the units place is occupied by an even digit.

The last place (units place) can be filled with any one of the evens 2,4,6 in 3 ways.

Since repetition is allowed, the number of ways of filling the remaining 3 places with each of the 6 digits is 6^3.

$\therefore$ The required number of numbers = $3 \times 6^3 = 3 \times 216 = 648$.

ii) Numbers divisible by 3:

The number of ways of filling the first 3 places with the given 6 digits is $6^3 = 216$.

Now, after filling up the first 3 places with three digits, we can fill up the units place in 6 ways.In this pattern, we get 6 consecutive positive integers. Now we use the result that out of any six consecutive integers only two numbers are divisible by 3. In this way, the units place can be filled in 2 ways.

$\therefore$ The required number of numbers $216 \times 2 = 432$.

---

6 **Find the number of 4 - digit numbers which can be formed using the digits 0, 2, 5, 7, 8 that are divisible by (i) 2 (ii) 4 when repetition is allowed.**

Sol: **i) Divisible by 2:**

A number is divisible by 2, if the units place is occupied by an even digit.

The last place (units place) can be filled with any one of the evens 0,2,8 in 3 ways.

Third place (tens place) can be filled with any one of the 5 digits in 5 ways.

Second place (hundreds place) can be filled with any one of the 5 digits in 5 ways.

First place (thousands place) can be filled with any one of the non-zero digits 2,5,7,8 in 4 ways.

$\therefore$ The required number of numbers = $3 \times 5 \times 5 \times 4 = 300$

ii) Divisible by 4:

A number is divisible by 4, if the last two places is divisible by 4.

From among the given digits the possible number of numbers divisible by 4 are 00, 08, 20, 28, 52, 72, 80, 88 . That is 8 ways.

The second place can be filled in 5 ways.

The first place except 0 can be filled in 4 ways

$\therefore$ The required number of numbers = $8 \times 4 \times 5 = 160$

7 **Find the number of 4- digit numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 which are divisible by 6 when repetition of the digits is allowed**

Sol: The first place can be filled 'without 0 digit' using 1,2,3,4,5 in 5 ways.

The middle two places can be filled with 6 digits in 6^2 ways ($\because$ repetition is allowed)

We can fill the last place (units place) with any one of 0,1,2,3,4,5,6.

This can be done in 6 ways. But among such 6 consecutive numbers only one number will be divisible by 6. So the units place can be filled in 1 way.

$\therefore$ The required number of 4 digit numbers = $5 \times 6^2 \times 1 = 5 \times 36 = 180$

---

8 **In how many ways can the letters of the word CHEESE be arranged so that no two E's come together?**

Sol: The given word contains 6 letters with 1 C, 1 H, 3 E's and 1 S.

Since no two E's come together, first arrange the remaining 3 letters. This is done in 3! ways.

Then we have 4 gaps between them.

The 3E's can be arranged in these 4 gaps in $\frac{{}^4P_3}{3!} = 4$ ways

$\therefore$ The required number of arrangements = $3! \times 4 = 24$

---

9 **Find the number of ways of arranging the letters of the word MISSING so that the two S's are together and the two I's are together.**

Sol: The word MISSING contains 7 letters in which there are 2I's are alike, 2S's are alike and rest are different.

Treating the 2S's as one unit and 2I's as one unit, we have altogether 5 units.

These can be arranged in 5! ways.

The 2S's can be arranged among themselves in 1 way. Also, the 2I's can be arranged in 1 way.

$\therefore$ The required number of arrangements = $5! \times 1 \times 1 = 120$

---

10 **Find the number of ways arranging the letters of the word SPECIFIC . In how many of them (i) the two C's come together? (ii) the two I's do not come together?**

Sol: **(i)** The given word has 8 letters in which there are 2I's and 2C's(alike letters).

$\therefore$ Total number of arrangements = $\frac{n!}{p!q!} = \frac{8!}{2!2!} = \frac{8\times7\times6\times5\times4\times3\times2\times1}{2\times1\times2\times1} = 10,080$

(ii)Two C's come together?

Treat the 2C's as one unit. The 2C's among themselves can be arranged in 1 way.

We have 6+1 = 7 letter units in which two letters (I's) are alike.

Hence, the required number of arrangements $=\frac{n!}{p!}=\frac{7!}{2!}=\frac{7\times6\times5\times4\times3\times2\times1}{2\times1}=2520$

(iii) Two I's do not come together?

Keeping the 2I's aside, the remaining 6 letters with 2C's alike can be arranged in $\frac{6!}{2!}=360$ ways.

In the set of 6 letters we find 7 gaps: –S–P–E–C–F–C–

The two I's can be arranged in these 7 gaps in $\frac{^7P_2}{2!}=\frac{7\times6}{2\times1}=21$ ways

Hence, the required number of arrangements $=\frac{6!}{2!}\times\frac{^7P_2}{2!}=360\times21=7560$

---

11 **Find the number of ways of arranging the letters of the word ASSOCIATIONS. In how many of them i) all the three S's come together ii) The two A's do not come together.**

Sol: (i) The given word ASSOCIATIONS has 12 letters in which there are 2 A's are alike, 3 S's are alike, 2 O's are alike 2 I's are alike and rest are different.

$\therefore$ Total number of arrangements $=\frac{(12)!}{2!3!2!2!}$

(ii) All the three S's come together:

Treat the 3 S's as one unit. Then we have 9 + 1 = 10 units in which 2A's are alike, 2O's are alike, 2 I's are alike and rest are different.They can be arranged in $\frac{(10)!}{2!2!2!}$ ways.

The 3S's among themselves can be arranged in 1 way.

$\therefore$ The required number of arrangements $=\frac{(10)!}{2!2!2!}$

(iii) The two A's do not come together.

Since 2 A's do not come together, first arrange the remaining 10 letters in which there are 3 S's are alike, 2 O's are alike 2 I's are alike and rest are different .

They can be arranged in $\frac{(10)!}{3!2!2!}$ ways. Then we have 11 gaps between them.

The 2 A's can be arranged in these 11 gaps in $\frac{^{11}P_2}{2!}$ ways.

$\therefore$ The required number of arrangements $=\frac{(10)!}{3!2!2!}\times\frac{^{11}P_2}{2!}$

12 **Find the number of ways of arranging the letters of the word SINGING so that,**
(i) they begin and end with I (ii) the two G's come together
(iii) relative positions of vowels and consonants are not disturbed.

Sol: **(i) Words beginning and ending with I:**

I						I

First we fill the first and last place with 2 I's in 1 way.

The number of ways of filling the remaining 5 places with the 5 letters S,N,G,N,G= $\frac{5!}{2!2!}$ =30

Hence, the required number of permutations is 30.

(ii) Words with two G's come together:
Treat the two G's as one unit.These 2 G's among themselves can be arranged in 1way.
The number of arrangements with the remaining six letters in which 2 I's and 2N's are

alike = $\frac{6!}{2!2!}$ =180 ways.

Hence the number of required permutations is 180.

(iii) Words with relative positions of vowels and consonants are not disturbed.

C	V	C	C	V	C	C

In the word SINGING the 2 vowels(2I's) are alike.

These 2 I's among themselves can be arranged in 1 way.

The number of arrangements with the remaining 5 consonants in which 2 Ns and 2G's

are alike = $\frac{5!}{2!2!} = 30$

$\therefore$ Required number of arrangements = 1 ×30=30

---

13 **Out of 3 different books on Economics, 4 different books on political science and 5 different books on Geography, how many collections can be made, if each collection consists of (i) exactly one book of each subject (ii) atleast one book of each subject.**

Sol: **(i) Selecting exactly one book of each subject :**

One Economics Book out of 3 can be selected in $^3C_1 = 3$ ways

One Political science Book out of 4 can be selected in $^4C_1 = 4$ ways

One Geography Book out of 5 can be selected in $^5C_1 = 5$ ways

From the counting principle the required number of ways = 3×4×5=60

(ii) Selecting atleast one book of each subject :

Number of selections of atleast one book of each subject

$(2^{n_1}-1)(2^{n_2}-1)(2^{n_3}-1) = (2^3-1)(2^4-1)(2^5-1) = 7\times15\times31 = 3255$

14 **To pass an examination a student has to pass in each of the three papers. In how many ways can a student fail in the examination?**

Sol: For each of the 3 papers there are 2 choices P or F.

Total number of choices for 3 papers = $2^3 = 8$ choices. Among there is only one way to pass.

$\therefore$ Required number of ways of failing $= 2^3 - 1 = 8 - 1 = 7$.

---

15 **A teacher wants to take 10 students to a park. He can take exactly 3 students at a time and will not take the same group of 3 students more than once. Find the num ber of times (i) each student can go to the park (ii) the teacher can go to the park.**

Sol: (i) To find the number of times a specific student can go to the park, we have to select 2 more students from the remaining 9 students. This can be done in ${}^9C_2$ ways.

Hence the no.of times that each student can go to the park = ${}^9C_2 = \dfrac{9\times8}{2\times1} = 36$

(ii) The teacher can go to the park ${}^{10}C_3$ times because he can select any 3 students out of 10 in ${}^{10}C_3$ ways

$\therefore$ The number of times the teacher can go to the park = ${}^{10}C_3 = \dfrac{10\times9\times8}{3\times2\times1} = 120$

---

16 **There are 8 railway stations along a railway line. In how many ways can a train be stopped at 3 of these stations such that no two of them are consecutive?**

Sol: We have 3 halting stations and 5 Non-Halting stations (NH)

— NH_1 — NH_2 — NH_3 — NH_4 — NH_5 —

To satisfy the given restriction the train has to stop at any 3 of the 6 gaps.

And, these can be selected in ${}^6C_3 = \dfrac{6\times5\times4}{3\times2\times1} = 20$ ways

---

17 **A double decker minibus has 8 seats in the lower deck and 10 seats in the upper deck. Find the number of ways of arranging 18 persons in the bus if 3 children want to go to the upper deck and 4 old people can not go to the upper deck.**

Sol: Allowing 3 children to the upper deck and 4 old people to the lower deck, we are left with 11 people and 11 seats (7 seats in the upper deck and 4 in the lower deck).

We can select 7 people for the upper deck out of 11 people in ${}^{11}C_7$ ways.

The remaining 4 persons go to the lower deck.

Now we can arrange 10 persons (3 children and 7 others) in the upper deck and 8 persons (4 old people and 4 others) in the lower deck in (10)! and (8)! ways respectively.

Hence the required number of arrangements = ${}^{11}C_7 \times 10! \times 8!$

18 **There are m points in a plane out of which p points are collinear and no three of the points are collinear unless all the three are from these p points. Find the number of different i) straight lines passing through pairs of distinct points.**
ii) triangles formed by joining these points (by line segments).

Sol: **(i)** From the given m points, by drawing straight lines passing through 2 distinct points at a time, we are supposed to get ${}^{m}C_2$ number of lines. But since p out of these m points are collinear, by forming lines passing through these p points 2 at a time we get only one line instead of getting ${}^{p}C_2$ dummy lines.

Therefore, the number of different lines passing through pairs of distinct points is ${}^{m}C_2 - {}^{p}C_2 + 1$.

(ii) From the given m points, by joining 3 points at a time, we are supposed to get ${}^{m}C_3$ number of triangles.

Since p out of these m points are collinear by joining these 'p' points 3 at a time we do not get any triangle . Here we are supposed to get ${}^{p}C_3$ number of dummy triangles.

Hence the number of triangles formed by joining the given m points $= {}^{m}C_3 - {}^{p}C_3$.

---

19 **Find the number of ways of giving away 4 similar coins to 5 boys if each boy can be given any number (less than or equal to 4) of coins.**

Sol: The 4 similar coins can be divided into various groups as follows.

i) One group containing 4 coins

ii) Two groups containing 1, 3 coins respectively

iii) Two groups containing 3, 1 coins respectively

iv) Two groups containing 2, 2 coins respectively

v) Three groups containing 1, 1, 2 coins respectively

vi) Three groups containing 1, 2, 1, coins respectively

vii) Three groups containing 2, 1, 1 coins respectively

viii) Four groups containing 1, 1, 1, 1 coins respectively

The required number of ways of giving 4 coins to the above groups

$$= {}^{5}C_1 + \left(2 \times {}^{5}C_2\right) + {}^{5}C_2 + \left({}^{5}C_3 \times \frac{3!}{2!}\right) + {}^{5}C_4 = 5 + 20 + 10 + 30 + 5 = 70 \text{ ways.}$$

20 **Find the number of zeros in 100!.** EAM Q

Sol: $100! = 2^{\alpha}3^{\beta}5^{\gamma}7^{\delta}....$ Here $\alpha = \left[\frac{100}{2}\right]+\left[\frac{100}{2^2}\right]+\left[\frac{100}{2^3}\right]+\left[\frac{100}{2^4}\right]+... = 50 + 25 + 12 + 6 + 3 + 1 = 97$

$$\gamma = \left[\frac{100}{5}\right]+\left[\frac{100}{5^2}\right] = 20 + 4 = 24$$

Thus 2 occurs 97 times and 5 occurs 24 times in 100!.

To get a 10 we require a 2 and a 5 .

Out of ninety seven 2's we take twenty four 2's to join with twenty four 5's.

Hence, the number of zeros in 100! =Number of 10's in 100!= 24. [lesser of 24, 97]

Now, the number of zeros in 100! is 24. [Since $10 = 2 \times 5$]

---

21 **A class contains 4 boys and g girls. Every Sunday, five students with atleast 3boys go for a picnic. A different group is being sent every week. During the picnic, the class teacher gives each girl in the group a doll. If the total number of dolls distributed is 85, find g.**

Sol: Number of boys = 4, Number of girls = g

Each group should contain atleast 3 boys. So it can be done in 2 ways as shown in the table.

	boys (4)	girls (g)
Group(G_1)	3	2
Group(G_2)	4	1

The number of girls in group of type $G_1 = [{}^4C_3 \times {}^gC_2] \times 2$ [$\because$ Group G_1 contains 2 girls each]

The number of girls in group of type $G_2 = [{}^4C_4 \times {}^gC_1] \times 1$ [$\because$ Group G_2 contains 1 girl]

But each girl in the group is given one doll.

Given number of dolls distributed = 85 $\Rightarrow [{}^4C_3 \times {}^gC_2] \times 2 + [{}^4C_4 \times {}^gC_1] \times 1 = 85$

$$\Rightarrow 4 \times \frac{g(g-1)}{2} \times 2 + 1 \times g \times 1 = 85 \Rightarrow 4g^2 - 4g + g - 85 = 0$$

$$\Rightarrow 4g^2 - 3g - 85 = 0 \Rightarrow 4g^2 - 20g + 17g - 85 = 0 \Rightarrow 4g(g-5) + 17(g-5) = 0$$

$$\Rightarrow (g-5)(4g+17) = 0 \Rightarrow g-5=0 \text{ (or) } 4g+17=0 \Rightarrow g=5 \quad \left[\because g \neq \frac{-17}{4}\right]$$

Hence number of girls (g) = 5

THE ILLUSTRATIVE TABLE OF PERMUTATIONS AND COMBINATIONS

1.1. The number of combinations of 2 elements out of {1,2,3} : $[{}^{n}C_{r}] = {}^{3}C_{2} = 3$; {{1,2}, {2,3}, {3,1}}

1.2. The number of selections of 3 elements out of {a,b,c} : $[{}^{n}C_{n}] = {}^{3}C_{3} = 1$; {a,b,c}

2.1. The number of permutations of 2 elements out of {1,2,3} : $[{}^{n}P_{r}] = {}^{3}P_{2} = 6$; {(1,2), (2,1), (2,3), (3,2), (1,3), (3,1)}

2.2. The number of permutations of 3 elements out of {a,b,c} : $[{}^{n}P_{n} = n!] = {}^{3}P_{3} = 3! = 6$; {(a,b,c),(a,c,b),(b,a,c),(b,c,a),(c,a,b),(c,b,a)}

2.3. The number of one one functions from A={1,2} to B={3,4,5}: $[{}^{n}P_{r}] = {}^{3}P_{2} = 6$;

f_1={(1, 3),(2, 4)}; f_2={(1, 3),(2, 5)}; f_3={(1, 4),(2, 3)}; f_4={(1, 4),(2, 5)}; f_5={(1, 5),(2, 4)}; f_6={(1, 5),(2, 3)}

3.1. The number of permutations of 2 elements out of {1,2,3} with repetitions: $[n^r] = 3^2 = 9$; {(1 2), (2 1), (2 3), (3 2), (1 3), (3 1),(1 1),(2 2),(3 3)}

3.2. The number of sequences of 3 elements out of {a,b} with repetitions: $[n^r] = 2^3 = 8$; {(a a a),(a a b),(a b a),(b a a),(b b a),(b a b),(a b b),(bb b)}

3.3. The number of functions from A={1,2} to B={3,4,5}: $[n^r] = 3^2 = 9$;

f_1={(13),(24)}; f_2={(13),(25)}; f_3={(14),(23)}; f_4={(14),(25)}; f_5={(15),(24)}; f_6={(15),(23)}; f_7={(13),(23)}; f_8={(14),(24)}; f_9={(15),(25)}

3.4. The number of permutations of 2 elements out of {1,2,3}with atleast one repetition: $n^r - {}^{n}P_{r} = 3^2 - {}^{3}P_{2} = 3$: {(11), (22), (33)}

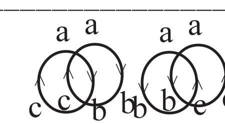
3.5. The number of onto functions from A={1,2,3} to B={4,5}: $[2^n - 2] = 2^3 - 2 = 6$:

f_1={(14),(24),(35)}; f_2={(15),(25),(34)}; f_3={(14),(25),(34)}; f_4={(15),(24),(35)}; f_5={(14),(25),(35)}; f_6={(15)(24)(34)}

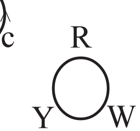
4.1. The number of combinations of 2 elements from {1,2,3} with repetitions: $[{}^{(n+r-1)}C_{r}] = {}^{(3+2-1)}C_{2} = {}^{4}C_{2} = 6$; {12}, {23}, {31}, {11}, {22},{33}

4.2. The number of terms in the expansion of $(a+b+c)^2$: $[{}^{(n+r-1)}C_{r}] = {}^{(3+2-1)}C_{2} = {}^{4}C_{2} = 6$; $a^2+b^2+c^2+2ab+2bc+2ca$

5.1. The number of circular permutations of arranging 3 persons around a table: [(n–1)!]=(3–1)!=2!=2;



5.2. The number of circular permutations of arranging 1 Red, 1 Yellow, 1 White flower $\left[\dfrac{(n-1)!}{2}\right] = \dfrac{(3-1)!}{2} = \dfrac{2!}{2} = 1$;



6.1. The number of permutations of A,S,S taken all at a time: $\left[\frac{n!}{r!}\right] = \frac{3!}{2!} = 3$; {(AAS)(SAA),(ASA)}

6.2. The number of permutations of A,S,S taken 2 at a time: $\left[\frac{{}^nP_r}{r!}\right] = \frac{{}^3P_2}{2!} = 3$; {(SS)(SA),(AS)}

7.1. The number of two digited numbers of 0,1,2 taken 2 at a time: $\square, \square$ = 2x2=4; (12), (21), (20),(10)

7.2. The number of permutations of 0,1,2 taken 2 at a time with repetitions: $\square, \square$ = 2x3=6; (12), (21), (20),(10),(11),(22)

8.1. The number of selections of a,b,c taken any number at a time: $[{}^nC_0+{}^nC_1+...+{}^nC_n=2^n]=2^3=8$; { },{a},{b},{c},{ab},{bc},{ca},{abc}

8.2. The number of selections of a,b,c taken atleast one: $[{}^nC_1+{}^nC_2+......+{}^nC_n=2^n-1]=2^3-1=7$; {a},{b},{c},{ab},{bc},{ca}, {abc}

9.1. The number of selections of 2 Maths books and 1 Physics book taken any number at a time [(p+1)(q+1)]=(2+1)(1+1)=6;
{MMP},{MM},{MP}{M}, {P},{ }

9.2. The number of divisors of 18 $[n = p_1^{\alpha_1}.p_2^{\alpha_2}....p_k^{\alpha_k} \Rightarrow (\alpha_1+1)(\alpha_2+1).......(\alpha_k+1)] \Rightarrow 18 = 2x3x3 = 2^1.3^2 \Rightarrow (1+1)(2+1) = 6$;
1, 2, 3, 2x3, 3x3, 2x3x3 $\Rightarrow$ 1, 2,3,6,9,18 $\Rightarrow$ 6 divisors.

10.1. The number of ways of dividing A,B,C,D,E into 2 groups G_1, G_2 of 2,3 respectively = ${}^{(m+n)}C_m$.

$\left[\frac{(m+n)!}{m!n!}\right] = \frac{5!}{2!\,3!} = 10$; $\left|\frac{G_1}{G_2}\right| : \left\{\frac{AB}{CDE}\right\},\left\{\frac{AC}{BDE}\right\},\left\{\frac{AD}{BCE}\right\},\left\{\frac{AE}{BCD}\right\},\left\{\frac{BC}{ADE}\right\},\left\{\frac{BD}{ACE}\right\},\left\{\frac{BE}{ACD}\right\},\left\{\frac{CD}{ABE}\right\},\left\{\frac{CE}{ABD}\right\},\left\{\frac{DE}{ABC}\right\}$

10.2. The number of ways of dividing A,B,C,D into 2 equal groups : $\left[\frac{(kn)!}{k!(n!)^k} = \frac{2n!}{2!(n!)^2}\right] = \frac{4!}{2(2!)^2} = 3$; $\left|\frac{G_1}{G_2}\right| : \left\{\frac{AB}{CD}\right\},\left\{\frac{AC}{BD}\right\},\left\{\frac{AD}{BC}\right\}$

10.3. The number of ways of dividing A,B,C,D into 2 persons P,Q equally $\left[\frac{(kn)!}{(n!)^k} = \frac{2n!}{(n!)^2}\right] = \frac{4!}{(2!)^2} = 6$; $\left|\frac{P}{Q}\right| : \left\{\frac{AB}{CD}\right\},\left\{\frac{CD}{AB}\right\},\left\{\frac{BC}{AD}\right\},\left\{\frac{AD}{BC}\right\},\left\{\frac{AC}{BD}\right\},\left\{\frac{BD}{AC}\right\}$