WELCOME

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DIGITAL CONTENT MATERIAL

HYPERBOLA - INDEX

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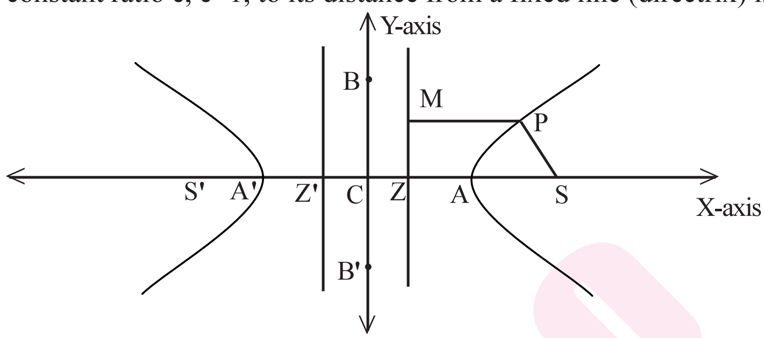
5. HYPERBOLA

Hyperbola: The locus of a point in a plane, which moves such that its distance from a fixed point (focus) bears a constant ratio e, e>1, to its distance from a fixed line (directrix) is called a hyperbola



1.The **eccentricity** of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\mathbf{e}=\frac{\sqrt{\mathbf{a}^2+\mathbf{b}^2}}{\mathbf{a}}$ $\because b^2=a^2(e^2-1)$, Also $\mathbf{a^2e^2=a^2+b^2}$

2.The **foci** of the hyperbola are **S=(ae, 0), S'(−ae, 0) i.e.,** $(\pm\sqrt{\mathbf{a}^2+\mathbf{b}^2},0)$

3.The **feet of the directrices** are $\mathbf{Z}=\left(\frac{\mathbf{a}}{\mathbf{e}},0\right)$, $\mathbf{Z'}=\left(-\frac{\mathbf{a}}{\mathbf{e}},0\right)$ & the equation of the directrices is $\mathbf{x}=\pm\frac{\mathbf{a}}{\mathbf{e}}$

4.1. **AA'** is called the **transverse axis**, **BB'** is called the **conjugate axis** of the hyperbola

4.2. The **length** of the **transverse axis** is **2a** and the **equation** of the **transverse axis** is **y=0**
The **length** of **conjugate axis** is **2b** and the **equation** of the **conjugate axis** is **x=0**

5.1. **A(a,0), A'(-a,0)** are called the vertices of the hyperbola on the **transverse axis**
B(0,b), B'(0, -b) are called the vertices on the **conjugate axis**.

5.2. The **equation** of the **tangents** at the **vertices** is **x=±a**

6.1.The line segments passing through the foci and perpendicular to the transverse axis are called the **latusrecta** of the hyperbola.

6.2. The **equation** of the **latus recta** is **x=±ae**

6.3. The ends of the **latus recta** are $\left(\mathbf{ae},\pm\frac{\mathbf{b}^2}{\mathbf{a}}\right)$ **and** $\left(-\mathbf{ae},\pm\frac{\mathbf{b}^2}{\mathbf{a}}\right)$

6.4. The length of the **latus rectum** is $\frac{\mathbf{2b}^2}{\mathbf{a}}$

7.1. If P is any point on the hyperbola then **SP** is known as the **focal distance** of the point **P** w.r.t the focus **S**.

7.2. The **focal distance** of the point $\mathbf{P(x_1,y_1)}$ on the hyperbola w.r.to the focus **S** is $\mathbf{SP=ex_1-a}$
and the **focal distance** of the point **P** on the hyperbola w.r.to the focus **S'** is $\mathbf{S'P=ex_1+a}$

8. The equation of the **auxiliary circle** of the hyperbola is $x^2+y^2=a^2$

9. The equation of the **director circle** of the hyperbola is $x^2+y^2=a^2-b^2$

II. The following results hold true w.r.to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$:

1. **Notation**: $S=\frac{x^2}{a^2}-\frac{y^2}{b^2}-1$; $S_1=\frac{x_1x}{a^2}-\frac{y_1y}{b^2}-1$; $S_{11}=\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1$; $S_{12}=\frac{x_1x_2}{a^2}-\frac{y_1y_2}{b^2}-1$
2. **Relative positions** of a point $P(x_1,y_1)$ and the hyperbola S=0
   (i) The point $P(x_1,y_1)$ lies on the hyperbola $S=0 \Leftrightarrow S_{11}=0$
   (ii) The point $P(x_1,y_1)$ lies inside the hyperbola $S=0 \Leftrightarrow S_{11}>0$
   (iii) The point $P(x_1,y_1)$ lies outside the hyperbola $S=0 \Leftrightarrow S_{11}<0$
3. The equation of the chord joining the points $A(x_1,y_1)$ & $B(x_2,y_2)$ on the hyperbola S=0 is $S_1+S_2=S_{12}$
4. The equation of the tangent at $P(x_1,y_1)$ on the hyperbola S=0 is $S_1=0$
5. The equation of the normal at $P(x_1,y_1)$ on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$
6. The condition for the line y=mx+c to be a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $c^2=a^2m^2-b^2$.
7. The equation of the tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ having slope m is $y=mx\pm\sqrt{a^2m^2-b^2}$
8. Two tangents can be drawn from an external point $P(x_1,y_1)$ to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ &

   if m_1,m_2 are the slopes of the two tangents then $m_1+m_2=\frac{2x_1y_1}{x_1^2-a^2}$, $m_1m_2=\frac{y_1^2+b^2}{x_1^2-a^2}$
9. The combined equation of the pair of tangents drawn from an external point $P(x_1,y_1)$ to the hyperbola S=0 is $S_1^2=S_{11}S$.
10. The equation of the **asymptotes** of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$ (or) $y=\pm\frac{b}{a}x$
11. The angle between the **asymptotes** of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $2\text{Tan}^{-1}\frac{b}{a}$ (or) $2\text{Sec}^{-1}e$

III. Parametric treatment:

12. The parametric point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $P(a\sec\theta, b\tan\theta)$ and is simply denoted by θ.
13. The equation of the tangent at $P(\theta)$ on the hyperbola S=0 is $\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$
14. The equation of the normal at $P(\theta)$ on the hyperbola S=0 is $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$

ADDITIONAL QUESTIONS WITH SOLUTIONS

1 **A circle cuts the rectangular hyperbola xy =1 in the points (x_r, y_r), r = 1,2,3,4. Prove that $x_1x_2x_3x_4 = y_1y_2y_3y_4 = 1$**

Sol: Let the circle be $x^2 + y^2 = a^2$. The parametric point on the hyperbola xy =1 is (t, 1/t).

The points of intersection of the circle $x^2 + y^2 = a^2$ and the hyperbola xy =1 are given by

$$t^2 + \frac{1}{t^2} = a^2 \Rightarrow t^4 + 1 = a^2t^2 \Rightarrow t^4 - a^2t^2 + 1 = 0 \Rightarrow t^4 + 0.t^3 - a^2t^2 + 0.t + 1 = 0$$

If $t_1t_2t_3$ and t_4 are the roots of the above biquadratic equation in t then $t_1t_2t_3t_4=1$

But xy = 1 and (t, 1/t) is a point on it.

Hence $t_1t_2t_3t_4=1 \Rightarrow (x_1x_2x_3x_4)(y_1y_2y_3y_4) = 1=(1)(1) \Rightarrow x_1x_2x_3x_4 = y_1y_2y_3y_4 = 1$

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2 **Show that the equation $\frac{x^2}{9-c} + \frac{y^2}{5-c} = 1$ represents**

(i) an ellipse if 'c' is a real constant less than 5.

(ii) a hyperbola if 'c' is any real constant between 5 and 9.

(iii) Show that each ellipse in (i) and each hyperbola (ii) has foci at the two points (±2, 0), independent of the value 'c'.

Sol: (i) The given equation is $\frac{x^2}{9-c} + \frac{y^2}{5-c} = 1$ .....(1)

(1) represents the equation of an ellipse if $9 - c > 0$ and $5 - c > 0$

$\Rightarrow c - 9 < 0$ and $c - 5 < 0 \Rightarrow c < 9$ and $c < 5 \Rightarrow c < 5$

(ii) (1) represents the equation of a hyperbola if $9-c>0$ and $5-c<0$

$\Rightarrow c - 9 < 0$ and $c - 5 > 0 \Rightarrow c < 9$ and $c > 5 \Rightarrow 5 < c < 9$

(iii) If the equation $\frac{x^2}{9-c} + \frac{y^2}{5-c} = 1$ represents an ellipse then $a^2 = 9 - c$, $b^2 = 5 - c$

Now, $a^2 - b^2 = (9-c) - (5-c) = 9-c-5+c = 4$

$\Rightarrow a^2e^2 = 4 \Rightarrow ae = 2$ [$\because b^2=a^2(1-e^2)=a^2-a^2e^2 \Rightarrow a^2-b^2=a^2e^2$]

$\therefore$ Foci = (±ae,0) = (±2,0).........(2)

If the equation $\frac{x^2}{9-c} + \frac{y^2}{5-c} = 1$ represents a hyperbola then $a^2 = 9-c$, $b^2 = c-5$

Now, $a^2 + b^2 = (9-c) + (c-5) = 9-c+c-5 = 4$

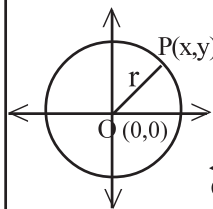
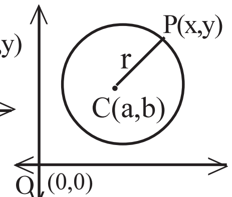
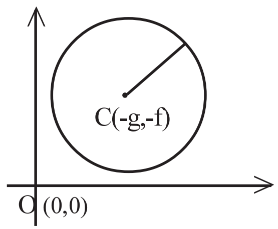
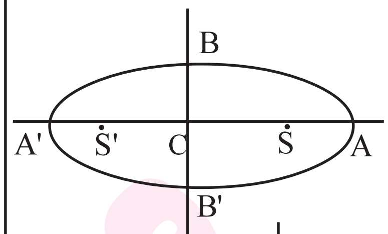
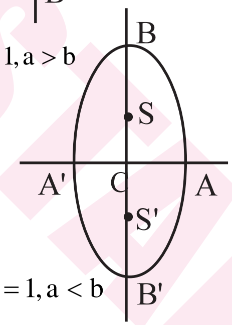
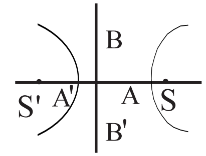
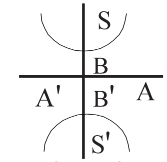
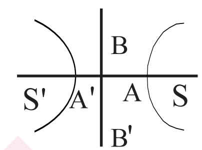
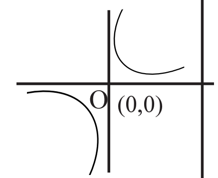
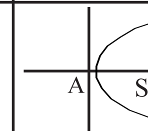
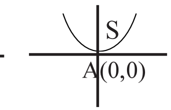
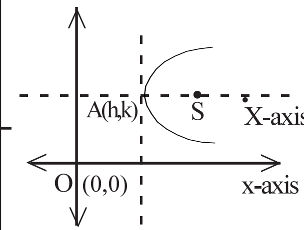
$\Rightarrow a^2e^2 = 4 \Rightarrow ae = 2$ [$\because b^2=a^2(e^2-1)=a^2e^2-a^2 \Rightarrow a^2+b^2=a^2e^2$]

$\therefore$ Foci = (±ae,0) = (±2,0)........(3)

From (2) and (3), we note that coordinates of foci are same and are independent of 'c'.

TABLE-II

THE CONIC TABLE

CONCEPT	CIRCLE	ELLIPSE	HYPERBOLA	PARABOLA
1. Figure & Equation of the Conic in cartesian form	 **1.** $x^2+y^2=r^2$  **2.** $(x-a)^2+(y-b)^2=r^2$  **3.** $x^2+y^2+2gx+2fy+c=0$	 **1.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$  **2.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a<b$	 **1.** $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$  **2.** $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$  **3.** $x^2-y^2=a^2$  **4.** $xy=c^2$	 **1.** $y^2=4ax$  **2.** $x^2=4ay$  **3.** $(y-k)^2=4a(x-h)$ (OR) $x=ly^2+my+n$
2. Parametric equations	1. $x=r\cos\theta$; $y=r\sin\theta$ 2. $x=a+r\cos\theta$; $y=b+r\sin\theta$	1. $x=a\cos\theta$; $y=b\sin\theta$, a>b	1. $x=a\sec\theta$; $y=b\tan\theta$ $x=a\cosh t$; $y=b\sinh t$ 2. $x=ct$, $y=c/t$	1. $x=at^2$; $y=2at$
3. Property of the conic	$CP=r$	$SP/PM=e$, $e<1$ $SP+S'P=2a$	$SP/PM=e$, $e>1$ $SP-S'P=2a$	$SP=PM$
4. Conditions for $ax^2+2hxy+by^2+2gx+2fy+c=0$ to represent the given conic	$a=b, h=0$, $g^2+f^2-ac\geq 0$	$h^2<ab$, $\Delta=abc+2fgh-af^2-bg^2-ch^2\neq 0$	1. $h^2>ab$, $\Delta\neq 0$ 3. $h^2>ab$, $\Delta\neq 0$, $a+b=0$ ☞ for asympotes $\Delta=0$	$h^2=ab$, $\Delta\neq 0$

CONCEPT	CIRCLE	ELLIPSE	HYPERBOLA	PARABOLA
5. Eccentricity	$e = 0$	1. $e = \frac{\sqrt{a^2-b^2}}{a} = \sqrt{1-\frac{b^2}{a^2}}, <1$ $\boxed{b^2 = a^2(1-e^2)}$, $\boxed{a^2e^2 = a^2-b^2}$ 2. $e = \frac{\sqrt{b^2-a^2}}{b} = \sqrt{1-\frac{a^2}{b^2}}, <1$ $\boxed{a^2 = b^2(1-e^2)}$, $\boxed{b^2e^2 = b^2-a^2}$	1. $e = \frac{\sqrt{a^2+b^2}}{a} = \sqrt{1+\frac{b^2}{a^2}}, >1$ $\boxed{b^2 = a^2(e^2-1)}$, $\boxed{a^2e^2 = a^2+b^2}$ 2. $e' = \frac{\sqrt{b^2+a^2}}{b} = \sqrt{1+\frac{a^2}{b^2}}, >1$ $\boxed{a^2 = b^2(e'^2-1)}$, $\boxed{b^2e'^2 = b^2+a^2}$	$e = 1$
6. Centre, Focus (foci), Vertex (Vertices)	1. Centre C (0,0) 2. Centre C(a,b) 3. Centre C(-g, -f)	1. Centre C(0,0) Vertices: A(a,0), A' (-a,0) B(0,b), B'(0,-b) Foci : S(ae,0),S'(-ae,0) i.e., $(\pm\sqrt{a^2-b^2}, 0)$ 2. Centre C(0,0) Vertices: B(0,b), B' (0,–b) A(a,0), A'(–a,0) Foci : S(0,be),S'(0,–be) i.e., $(0, \pm\sqrt{b^2-a^2})$	1. Centre C(0,0) Vertices: A(a,0), A' (-a,0) B(0,b), B'(0,-b) Foci : S(ae,0),S'(-ae,0) i.e., $(\pm\sqrt{a^2+b^2}, 0)$ 2. Centre C(0,0) Vertices: B(0,b), B' (0,-b) A(a,0), A'(-a,0) Foci : S(0,be'),S'(0,-be') i.e., $(0, \pm\sqrt{a^2+b^2})$	1. Vertex A(0,0) Focus S(a,0) 2. Vertex A(0,0) Focus S(0,a) 3. Vertex A(h,k) Focus S(a+h, k)
7. Radius, Latus rectum (L.R)	1. Radius = r 2. Radius = r 3. Radius = $\sqrt{g^2+f^2-c}$	1. Length of L.R = $2b^2/a$ ends of L.R = $(\pm ae, \pm b^2/a)$ Equation of L.R is $x = \pm ae$ 2. Length of L.R = $2a^2/b$ ends of L.R = $(\pm a^2/b, \pm be)$ Equation of L.R is $y = \pm be$	1. Length of L.R = $2b^2/a$ ends of L.R = $(\pm ae, \pm b^2/a)$ Equation of L.R is $x = \pm ae$ 2. Length of L.R = $2a^2/b$ ends of L.R = $(\pm a^2/b, \pm be')$ Equation of L.R is $y = \pm be'$	1. Length of L.R = 4a Ends of L.R = L(a,2a), L' (a,–2a) Equation of L.R is x = a 2. Length of L.R = 4a Ends of L.R=L(–2a, a), L'(2a, a) Equation of L.R is y = a 3. Length of L.R = 4a Ends of L.R=(h+a, ±2(h+a)) Equation of L.R is x = h+a

CONCEPT	CIRCLE	ELLIPSE	HYPERBOLA	PARABOLA
8. Directrix	Line at Infinity	1. Foot of the directrix $Z(\pm a/e, 0)$ Equation of the directrix is $x = \pm a/e$ i.e., $\sqrt{(a^2-b^2)}x = \pm a^2$ 2. Foot of the directrix $Z(0, \pm b/e)$ Equation of the directrix is $y = \pm b/e$ i.e., $\sqrt{(b^2-a^2)}y = \pm b^2$	1. Foot of the directrix $Z(\pm a/e, 0)$ Equation of the directrix is $x = \pm a/e$ i.e., $\sqrt{(a^2+b^2)}x = \pm a^2$ 2. Foot of the directrix $Z(0, \pm b/e')$ Equation of the directrix is $y = \pm b/e'$ i.e., $\sqrt{(a^2+b^2)}y = \pm b^2$	1. Foot of the directrix $(-a, 0)$ Equation is $x = -a$ 2. Foot of the directrix $(0, -a)$ Equation is $y = -a$ 3. Foot of the directrix $(h-a, k)$ Equation is $x-h+a = 0$
9. Tangent at the Vertex	$x = \pm a$	1. $x = \pm a$ 2. $y = \pm b$	1. $x = \pm a$ 2. $y = \pm b$	1. $x = 0$ (i.e., the y-axis) 2. $y = 0$ (i.e., the x-axis) 3. $x-h = 0$
10. Axis / Axes	1. $y=0$	1. Major axis is x-axis of length 2a &Minor axis is y-axis of length 2b 2. Major axis is y-axis of length 2b &Minor axis is x-axis of length 2a	1. Tranverse axis is x-axis of length 2a & Conjugate axis is y-axis of length 2b 2. Transverse axis is y-axis of length 2b & Conjugate axis is x-axis of length 2a	1. $y = 0$ (i.e., the x-axis) 2. $x = 0$ (i.e., the y-axis) 3. $y-k = 0$ (i.e., a line parallel to the x-axis)
11. Notation	$S \equiv x^2+y^2-r^2$ $S \equiv x^2+y^2+2gx+2fy+c$ $S_1 \equiv xx_1+yy_1+g(x+x_1)+f(y+y_1)+c$ $S_{11} \equiv x_1^2+y_1^2+2gx_1+2fy_1+c$ $S_{12} \equiv x_1x_2+y_1y_2+g(x_1+x_2)+f(y_1+y_2)+c$	$S = \frac{x^2}{a^2}+\frac{y^2}{b^2}-1$ $S_1 = \frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1$ $S_{11} = \frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$ $S_{12} = \frac{x_1x_2}{a^2}+\frac{y_1y_2}{b^2}-1$	$S = \frac{x^2}{a^2}-\frac{y^2}{b^2}-1$ $S_1 = \frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1$ $S_{11} = \frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1$ $S_{12} = \frac{x_1x_2}{a^2}-\frac{y_1y_2}{b^2}-1$	$S = y^2-4ax$ $S_1 = yy_1-2a(x+x_1)$ $S_{11} = y_1^2-4ax_1$ $S_{12} = y_1y_2-2a(x_1+x_2)$
12. Chord joining (x_1,y_1)&(x_2,y_2) on the conic & Parametric form	$S_1+S_2 = S_{12}$ 1. $x\cos\frac{\alpha+\beta}{2}+y\sin\frac{\alpha+\beta}{2} = r\cos\frac{\alpha-\beta}{2}$ 3. $(x+g)\cos\frac{\alpha+\beta}{2}+(y+f)\sin\frac{\alpha+\beta}{2} = r\cos\frac{\alpha-\beta}{2}$	$S_1+S_2 = S_{12}$ $\frac{x}{a}\cos\frac{\alpha+\beta}{2}+\frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$	$S_1+S_2 = S_{12}$ $\frac{x}{a}\cos\frac{\alpha-\beta}{2}-\frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha+\beta}{2}$	$S_1+S_2 = S_{12}$ $y(t_1+t_2) = 2x+2at_1t_2$
13. Chord with (x_1,y_1) as mid point	$S_1 = S_{11}$	$S_1 = S_{11}$	$S_1 = S_{11}$	$S_1 = S_{11}$

CONCEPT	CIRCLE	ELLIPSE	HYPERBOLA	PARABOLA
14. Tangent at (x_1,y_1) on S=0 and Slope of the Tangent	$S_1=0$ Slope $=\frac{-(x_1+g)}{(y_1+f)}$	$S_1=0$ Slope $=-\frac{b^2x_1}{a^2y_1}$	$S_1=0$ Slope $=\frac{b^2x_1}{a^2y_1}$	1. $S_1=0$; Slope $=2a/y_1$ 2. $S_1=xx_1-2a(y+y_1)=0$ Slope $=x_1/2a$
15. Tangential Condition for the line $lx+my+n=0$ or $y=mx+c$	$n^2=r^2(l^2+m^2)$ $c^2=r^2(1+m^2)$	$n^2=a^2l^2+b^2m^2$ $c^2=a^2m^2+b^2$	$n^2=a^2l^2-b^2m^2$ $c^2=a^2m^2-b^2$	1. $ln=am^2$ $c=a/m$ 2. $mn=al^2$ $c=-am^2$
16. Equation of the Tangent with slope 'm'	1. $y=mx\pm r\sqrt{1+m^2}$ 2. $y-b=m(x-a)\pm r\sqrt{1+m^2}$ 3. $y+f=m(x+g)\pm r\sqrt{1+m^2}$	$y=mx\pm\sqrt{a^2m^2+b^2}$	$y=mx\pm\sqrt{a^2m^2-b^2}$	1. $y=mx+(a/m)$ 2. $y=mx-am^2$
17. Point of contact of $lx+my+n=0$ or $y=mx+c$	$\left(\frac{-lr^2}{n},\frac{-mr^2}{n}\right)$ $\left(\frac{-mr^2}{c},\frac{r^2}{c}\right)$	$\left(\frac{-la^2}{n},\frac{-mb^2}{n}\right)$ $\left(\frac{-ma^2}{c},\frac{b^2}{c}\right)$	$\left(\frac{-la^2}{n},\frac{mb^2}{n}\right)$ $\left(\frac{-ma^2}{c},\frac{-b^2}{c}\right)$	$\left(\frac{n}{l},\frac{-2am}{l}\right),\left(\frac{-2al}{m},\frac{n}{m}\right)$ $\left(\frac{c}{m},2c\right)$ or $\left(\frac{a}{m^2},\frac{2a}{m}\right)$
18. Equation of the Tangent in parametric form	1. $x\cos\theta+y\sin\theta=r$ 2. $(x+g)\cos\theta+(y+f)\sin\theta=\sqrt{g^2+f^2-c}$	$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$	$\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$	$yt=x+at^2$ (The slope is 1/t)
19. Normal at (x_1,y_1) on S=0	1. $x/x_1=y/y_1$ 3. $(y-y_1)/(y_1+f)=(x-x_1)/(x_1+g)$	$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$	$\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$	$y-y_1=\frac{-y_1}{2a}(x-x_1)$
20. Equation of the normal in parametric form	$x\cos\theta-y\sin\theta=0$	$\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$	$\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$	$y+xt=2at+at^3$ $t_1t_2t_3=y_1/a$; $t_1+t_2+t_3=0$ $t_1t_2+t_2t_3+t_3t_1=(2a-x_1)/a$
21. Equation of the normal with slope 'm'	1. $y=mx$ 2. $y-b=m(x-a)$ 3. $y+f=m(x+g)$	$y=mx\pm m\sqrt{\frac{a^2-b^2}{a^2+b^2m}}$	$y=mx\pm m\sqrt{\frac{a^2+b^2}{a^2-b^2m}}$	$y=mx-2am-am^3$ Foot of normal: $(am^2,-2am)$
22. Normal condition for $lx+my+n=0$	3. $lg+mf-n=0$	$\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$	$\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{(a^2+b^2)^2}{n^2}$	$al^3+2alm^2+m^2n=0$

CONCEPT	CIRCLE	ELLIPSE	HYPERBOLA	PARABOLA
23. Polar /Chord of contact of (x_1,y_1) w.r.to S=0	$S_1 \equiv xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$ $S_1 \equiv (x_1+g)+(y_1+f)y+x_1g+y_1f+c=0$	$S_1 = \frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1=0$	$S_1 = \frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1=0$	$S_1 = yy_1-2a(x+x_1)=0$
24. Pole of the line $lx+my+n=0$ w.r.t the conic S=0	1. $\left(\frac{-lr^2}{n},\frac{-mr^2}{n}\right)$ 3. $\left(-g+\frac{lr^2}{N},-f+\frac{mr^2}{N}\right)$, $N = lg+mf-n$	$\left(\frac{-la^2}{n},\frac{-mb^2}{n}\right)$	$\left(\frac{-la^2}{n},\frac{mb^2}{n}\right)$	1. $\left(\frac{n}{l},\frac{-2am}{l}\right)$ 2. $\left(\frac{-2al}{m},\frac{n}{m}\right)$
25. Pair of tangents from (x_1,y_1) to S=0 Sum & Product of slopes of Tangents	$S_1^2 = S_{11}S$ $m_1+m_2=\frac{2x_1y_1}{x_1^2-r^2}$ $m_1m_2=\frac{y_1^2-r^2}{x_1^2-r^2}$	$S_1^2 = S_{11}S$ $m_1+m_2=\frac{2x_1y_1}{x_1^2-a^2}$ $m_1m_2=\frac{y_1^2-b^2}{x_1^2-a^2}$	$S_1^2 = S_{11}S$ $m_1+m_2=\frac{2x_1y_1}{x_1^2-a^2}$ $m_1m_2=\frac{y_1^2+b^2}{x_1^2-a^2}$	$S_1^2 = S_{11}S$ $m_1+m_2=\frac{y_1}{x_1}$ $m_1m_2=\frac{a}{x_1}$
26. Angle θ between the tangents	$\theta=\mathrm{Tan}^{-1}\frac{2r\sqrt{S_{11}}}{S_{11}-r^2}$ or $\mathrm{Tan}\frac{\theta}{2}=\frac{r}{\sqrt{S_{11}}}$	$\tan\theta=\frac{2ab\sqrt{S_{11}}}{x_1^2+y_1^2-a^2-b^2}$	$\tan\theta=\frac{2ab\sqrt{S_{11}}}{x_1^2-y_1^2-a^2+b^2}$	$\tan\theta=\frac{\sqrt{S_{11}}}{x_1+a}$
27. Condition for (x_1,y_1)&(x_2,y_2) to be conjugate	$S_{12}=0$	$S_{12}=0$	$S_{12}=0$	$S_{12}=0$
28. Condition for $l_1x+m_1y+n_1=0$, $l_2x+m_2y+n_2=0$ to be conjugate	$n_1n_2=r^2(l_1l_2+m_1m_2)$	$n_1n_2=a^2l_1l_2+b^2m_1m_2$	$n_1n_2=a^2l_1l_2-b^2m_1m_2$	1. $2am_1m_2=l_1n_2+l_2n_1$ 2. $2al_1l_2=m_1n_2+m_2n_1$
29. Locus of point of intersection of $\perp^r$ tangents (Director circle)	$x^2+y^2=2r^2$	$x^2+y^2=a^2+b^2$	$x^2+y^2=a^2-b^2$	$x=-a$ (the Directrix)
30. Locus of the feet of perpendiculars from foci (Auxillary Circle)	$x^2+y^2=r^2$	$x^2+y^2=a^2$	$x^2+y^2=a^2$	$x=0$ (Tangent at the vertex)