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DIGITAL MATERIAL

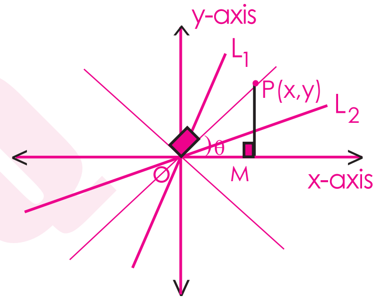
PAIR OF LINES - INDEX

4.PAIR OF LINES

ADDITIONAL QUESTIONS WITH SOLUTIONS

1. If the equation $ax^2+2hxy+by^2=0$ represents a pair of lines then the equation of the pair of angular bisectors is $h(x^2-y^2)-(a-b)xy=0$

Proof: Method I:



Let the two lines of $ax^2+2hxy+by^2=0$ be $y=m_1x$, $y=m_2x$

then we know that $m_1m_2=\frac{a}{b}, m_1+m_2=\frac{-2h}{b}$

In the diagram, let L_1 & L_2 represent the lines

$y=m_1x$, $y=m_2x$ with inclinations θ_1,θ_2 where $m_1=\tan\theta_1$ and $m_2=\tan\theta_2$

$$\therefore \tan\theta_1.\tan\theta_2=m_1m_2=\frac{a}{b}, \tan\theta_1+\tan\theta_2=m_1+m_2=-\frac{2h}{b} \quad ...(1)$$

Let P(x,y) be any point on the angular bisectors with inclination θ or $90^o+\theta$

Now, from ΔPOM, $\tan\theta=\frac{PM}{OM}=\frac{y}{x}$ ....(2)

$$\text{Also, } \theta=\frac{\theta_1+\theta_2}{2} \text{ or } \theta=90^o+\left(\frac{\theta_1+\theta_2}{2}\right) \Rightarrow 2\theta=\theta_1+\theta_2 \text{ or } 2\theta=180^o+(\theta_1+\theta_2)$$

$$\Rightarrow \tan 2\theta=\tan(\theta_1+\theta_2) \text{ or } \tan 2\theta=\tan\left(180^o+(\theta_1+\theta_2)\right)=\tan(\theta_1+\theta_2)$$

$$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta}=\frac{\tan\theta_1+\tan\theta_2}{1-\tan\theta_1\tan\theta_2} \Rightarrow \frac{2\left(\frac{y}{x}\right)}{1-\frac{y^2}{x^2}}=\frac{\frac{-2h}{b}}{1-\frac{a}{b}} \quad \text{[From (1) \& (2)]}$$

$$\Rightarrow \frac{\frac{2y}{x}}{\frac{x^2-y^2}{x^2}}=\frac{\frac{-2h}{b}}{\frac{b-a}{b}} \Rightarrow \frac{2xy}{x^2-y^2}=\frac{2h}{a-b} \Rightarrow \frac{x^2-y^2}{a-b}=\frac{xy}{h}$$

$$\Rightarrow h(x^2-y^2)=(a-b)(xy) \Rightarrow h(x^2-y^2)-(a-b)xy=0$$

Method II: Let the two lines of $ax^2+2hxy+by^2=0$ be $y=m_1x$ & $y=m_2x$ then we know that,

$$m_1+m_2=-\frac{2h}{b},\quad m_1m_2=\frac{a}{b}$$

The angular bisectors of $m_1x-y=0$ and $m_2x-y=0$ are

$$\frac{m_1x-y}{\sqrt{m_1^2+1}}=\pm\frac{m_2x-y}{\sqrt{m_2^2+1}}\Rightarrow\frac{(m_1x-y)^2}{m_1^2+1}=\frac{(m_2x-y)^2}{m_2^2+1}$$

$$\Rightarrow(m_2^2+1)(m_1x-y)^2=(m_1^2+1)(m_2x-y)^2$$

$$\Rightarrow(m_2^2+1)(m_1x-y)^2-(m_1^2+1)(m_2x-y)^2=0$$

$$\Rightarrow(m_2^2+1)(m_1^2x^2-2m_1xy+y^2)-(m_1^2+1)(m_2^2x^2-2m_2xy+y^2)=0$$

Now, take $x^2,-2xy,y^2$ common in the above equation

$$\Rightarrow x^2\left((m_2^2+1)m_1^2-(m_1^2+1)m_2^2\right)-2xy\left((m_2^2+1)m_1-(m_1^2+1)m_2\right)+y^2\left((m_2^2+1)-(m_1^2+1)\right)=0$$

$$\Rightarrow x^2\left(m_2^2m_1^2+m_1^2-m_1^2m_2^2-m_2^2\right)-2xy\left(m_2^2m_1+m_1-m_1^2m_2-m_2\right)+y^2\left(m_2^2+1-m_1^2-1\right)=0$$

$$\Rightarrow x^2(m_1^2-m_2^2)-2xy\left(m_1-m_2-m_1m_2(m_1-m_2)\right)+y^2(m_2^2-m_1^2)=0$$

$$\Rightarrow x^2\left((m_1-m_2)(m_1+m_2)\right)-2xy\left((m_1-m_2)(1-m_1m_2)\right)+y^2\left((m_2-m_1)(m_1+m_2)\right)=0$$

Now, take $-(m_1-m_2)$ common in the above equation

$$\Rightarrow-(m_1-m_2)\left((-x^2)(m_1+m_2)+2xy(1-m_1m_2)+y^2(m_1+m_2)\right)=0$$

$$\Rightarrow(-x^2)(m_1+m_2)+2xy(1-m_1m_2)+y^2(m_1+m_2)=0$$

$$\Rightarrow(-x^2)\left(-\frac{2h}{b}\right)+2xy\left(1-\frac{a}{b}\right)+y^2\left(-\frac{2h}{b}\right)=0$$

$$\Rightarrow x^2\frac{2h}{b}+2xy\left(\frac{b-a}{b}\right)-y^2\frac{2h}{b}=0\Rightarrow x^2h+xy(b-a)-y^2h=0$$

$$\Rightarrow h(x^2-y^2)-(a-b)xy=0$$

2. Show that the equation $x^2 - y^2 - x + 3y - 2 = 0$ represents a pair of perpendicular lines and find their equations.

Sol: (a) To show that the given equation represents a pair of perpendicular lines:

Comparing $x^2 - y^2 - x + 3y - 2 = 0$ with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

we get $a = 1$, $2h = 0 \Rightarrow h = 0$, $b = -1$, $2g = -1 \Rightarrow g = -1/2$, $2f = 3 \Rightarrow f = 3/2$, $c = -2$

(i) Consider $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= abc - af^2 - bg^2 \qquad (\because h = 0)$

$= (1)(-1)(-2) - (1)(3/2)^2 - (-1)(-1/2)^2$

$= 2 - 9/4 + 1/4 = 2 - 2 = 0$

(ii) $h^2 - ab = -ab = -1(-1) = 1 > 0$

(iii) $f^2 - bc = (3/2)^2 - (-1)(-2) = 9/4 - 2 = 1/2 > 0$

(iv) $g^2 - ac = (-1/2)^2 - (1)(-2) = 1/4 + 2 = 9/2 > 0$

Also, $a + b = 1 + (-1) = 1 - 1 = 0$

$\therefore$ The given equation represents the pair of perpendicular lines.

(b) To find the separate equation of the given pair of lines:

Let $x^2 - y^2 - x + 3y - 2 = (x - y)(x + y) - x + 3y - 2$

$\Rightarrow x^2 - y^2 - x + 3y - 2 = (x - y + n_1)(x + y + n_2)$

Comparing the co-efficients of x and y terms on both sides then we get

$n_1 + n_2 = -1$ ....(1)

$\Rightarrow n_1 - n_2 = 3$ ....(2)

Now (1) + (2) $\Rightarrow 2n_1 = 2 \Rightarrow n_1 = 1$

(1) – (2) $\Rightarrow 2n_2 = -4 = 0 \Rightarrow n_2 = -2$

The separate equations of the pair of lines are $x - y + 1 = 0$ and $x + y - 2 = 0$