WELCOME

STAR 'QR CODE'

DIGITAL MATERIAL

STRAIGHT LINES-INDEX

LEVEL-I

STRAIGHT LINES-I

1. Find the slope of the line passing through the points$(-p, q),(q, -p),(pq \neq 0)$

Sol: Slope of the line passing through $A(x_1,y_1)=(-p, q)$ and $B(x_2,y_2)=(q, -p)$ is

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{q-(-p)}{-p-q}=\frac{q+p}{-(p+q)}=-1$$

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2. If the linear equations ax+by+c=0 (a,b,c≠0) and lx+my+n=0 represent the same line and $r=\frac{l}{a}=\frac{n}{c}$, write the value of r in terms m and b.

Sol: If ax+by+c=0 and lx+my+n=0 represent the same line

then the corresponding coefficients are proportional $\Rightarrow \frac{l}{a}=\frac{m}{b}=\frac{n}{c}=r \qquad \therefore r=\frac{m}{b}$

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3. If the product of the intercepts made by the straight line xtanα+ysecα = 1, $\left(0\le\alpha<\frac{\pi}{2}\right)$, on the co-ordinates axes is equal to sinα, find α.

Sol: Given line is x tanα+ysecα = 1 $\Rightarrow \frac{x}{\cot\alpha}+\frac{y}{\cos\alpha}=1$

Product of intercepts is sinα ⇒(cotα)(cosα) = sinα $\Rightarrow \left(\frac{\cos\alpha}{\sin\alpha}\right)(\cos\alpha)=\sin\alpha$

$\Rightarrow \cos^2\alpha=\sin^2\alpha \Rightarrow \tan^2\alpha=1 \Rightarrow \tan\alpha=\pm1 \Rightarrow \alpha=45^0, \left(0\le\alpha<\frac{\pi}{2}\right)$

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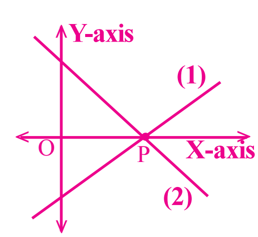
4. The line $\frac{x}{a}-\frac{y}{b}=1$ meets the X-axis at P. Find the equation of the line perpendicular to this line at P.



Sol : Equation of the given line is $\frac{x}{a}-\frac{y}{b}=1$ ....(1)

Equation of X-axis is y=0 $\Rightarrow \frac{x}{a}-0=1 \Rightarrow x=a \Rightarrow P=(a,0)$

From (1), Equation of the line perpendicular to the given line is $\frac{x}{b}+\frac{y}{a}=k$ ....(2)

If this line pass through P(a,0) then $\frac{a}{b}+0=k \Rightarrow k=\frac{a}{b}$

$\therefore$ From (2), the equation of the required line is $\frac{x}{b}+\frac{y}{a}=\frac{a}{b}$

5. If the sum of the reciprocals of the intercepts made by a variable straight line on the axes of coordinates is a constant, then prove that the line always passes through a fixed point.

Sol: Equation of the line in the intercept form is $\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{1}{a}(x)+\frac{1}{b}(y)=1$.........(1)

But sum of the reciprocals is a constant k

$\Rightarrow \frac{1}{a}+\frac{1}{b}=k \Rightarrow \frac{1}{ak}+\frac{1}{bk}=1 \Rightarrow \frac{1}{a}\left(\frac{1}{k}\right)+\frac{1}{b}\left(\frac{1}{k}\right)=1$ ....(2)

Comparing (1) and (2) we get $x=\frac{1}{k}$ and $y=\frac{1}{k}$

$\therefore$ (1) always pass through the fixed point $\left(\frac{1}{k},\frac{1}{k}\right)$

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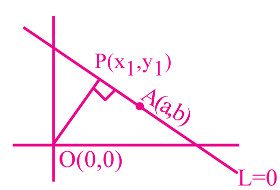
6. Find the locus of the foot of the perpendicular from the origin to a variable straight line which always passes through a fixed point (a,b).

Sol: Given the variable straight line L=0 passes through the fixed point A(a,b).

Let $P(x_1,y_1)$ be a point on the locus.

Then $P(x_1,y_1)$ is the foot of the perpendicular from the origin to the line.



Here AP $\perp$ OP$\Rightarrow$Slope of AP $\times$Slope of OP= -1

$$\Rightarrow \left(\frac{y_1-b}{x_1-a}\right)\left(\frac{y_1-0}{x_1-0}\right)=-1 \Rightarrow \left(\frac{y_1-b}{x_1-a}\right)\left(\frac{y_1}{x_1}\right)=-1 \Rightarrow \frac{y_1^2-by_1}{x_1^2-ax_1}=-1 \Rightarrow y_1^2-by_1=-(x_1^2-ax_1)$$

$\Rightarrow x_1^2+y_1^2-ax_1-by_1=0$

$\therefore$ Equation of locus of $P(x_1,y_1)$ is $x^2+y^2-ax-by=0$

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7. Let PS be the median of the triangle with vertices P(2,2) Q(6,–1) and R(7,3). Find the equation of the straight line passing through (1,–1) and parallel to the median PS.

Sol: Given that P(2,2) Q(6,–1), R(7,3) are the vertices of ΔPQR and PS is a median.

$\Rightarrow$ S is the mid point of QR$\Rightarrow$S$=\left(\frac{6+7}{2},\frac{-1+3}{2}\right)=\left(\frac{13}{2},1\right)$

Slope of PS $=\frac{1-2}{\frac{13}{2}-2}=\frac{-1}{\frac{9}{2}}=-\frac{2}{9}$

$\therefore$ Equation of the parallel line passing through (1,–1) with slope –2/9 is

$$y+1=-\frac{2}{9}(x-1) \Rightarrow 9y+9=-2x+2=0 \Rightarrow 2x+9y+7=0$$

STRAIGHT LINES-II

8. Find the points on the line 4x–3y–10=0 which are at a distance of 5 units from the point (1,–2).

Sol: Here, $P(x_1,y_1)=(1,-2)$, $|r|=5$,

Slope of the line 4x–3y–10=0 is $m=-\left(\frac{4}{-3}\right)=\frac{4}{3}$

$\Rightarrow m=\tan\theta=\frac{4}{3}\Rightarrow \cos\theta=\frac{3}{5},\ \sin\theta=\frac{4}{5}$

∴ The required points are given by $(x_1\pm r\cos\theta,\ y_1\pm r\sin\theta)$

$=\left(1+5\left(\frac{3}{5}\right),\ -2+5\left(\frac{4}{5}\right)\right)=(1+3,-2+4)=(4,2)$

and $\left(1-5\left(\frac{3}{5}\right),\ -2-5\left(\frac{4}{5}\right)\right)=(1-3,-2-4)=(-2,-6)$

∴ The required points are (4,2) and (–2,–6)

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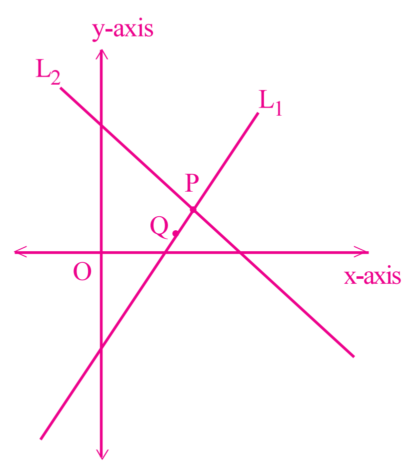
9. A straight line is parallel to the line $y=\sqrt{3}x$ passes through Q(2,3) and cuts the line 2x+4y–27=0 at P then find the length of PQ.

Sol: Slope of the given line $y=\sqrt{3}x$ is $\sqrt{3}$



$\Rightarrow$ Slope of the line (L_1) PQ is $m=\sqrt{3}$

$\Rightarrow \tan\theta=\sqrt{3}=\tan 60^\circ\Rightarrow \theta=60^\circ$

Let $Q=(x_1, y_1)=(2, 3)$ and PQ = r

The parametric point on the line L_1 which is at a distance r from Q (2,3) is

$P=(x_1+r\cos\theta, y_1+r\sin\theta)=(2+r\cos 60^\circ, 3+r\sin 60^\circ)$

$=\left(2+r\left(\frac{1}{2}\right),3+r\left(\frac{\sqrt{3}}{2}\right)\right)=\left(2+\frac{r}{2},3+\frac{r\sqrt{3}}{2}\right)$

But the point P lies on the line(L_2) 2x + 4y – 27 = 0

$\Rightarrow 2\left(2+\frac{r}{2}\right)+4\left(3+\frac{r\sqrt{3}}{2}\right)-27=0\Rightarrow 2\left(\frac{4+r}{2}\right)+4\left(\frac{6+r\sqrt{3}}{2}\right)-27=0$

$\Rightarrow (4+r)+2(6+r\sqrt{3})-27=0\Rightarrow r+2r\sqrt{3}+4+12-27=0$

$\Rightarrow r(1+2\sqrt{3})-11=0\Rightarrow r(1+2\sqrt{3})=11$

10. The intercepts of a straight line on the axes of co-ordinates are a and b. If p is the length of the perpendicular drawn from the origin to this line. Write the value of p in terms of a and b.

Sol: The equation of the line is $\frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow \frac{bx+ay}{ab}=1 \Rightarrow bx+ay=ab \Rightarrow bx+ay-ab=0$

The perpendicular distance from O(0,0)to the above line is $p=\frac{|-ab|}{\sqrt{a^2+b^2}}$

$\Rightarrow p^2=\frac{a^2b^2}{a^2+b^2} \Rightarrow p=\frac{ab}{\sqrt{a^2+b^2}}$

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11. Line L has intercepts a and b on both axes of co-ordinates. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q on the transformed axes then prove that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

Sol: Equation of the line with intercepts a,b is $\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{a}+\frac{y}{b}-1=0......(1)$

MAINS Q

If X,Y denote the coordinates of any point P(x,y) after rotation.

The equation of the line with intercepts p,q is $\frac{X}{p}+\frac{Y}{q}=1 \Rightarrow \frac{X}{p}+\frac{Y}{q}-1=0......(2)$

But distance is invariant under rotation of axes.

$\therefore$ Perpendicular distance from the origin O(0,0) to (1) & (2) is same.

$\therefore \frac{|-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=\frac{|-1|}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}} \Rightarrow \frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}=\frac{1}{\frac{1}{p^2}+\frac{1}{q^2}} \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

STRAIGHT LINES-III

12. Find the equation of the straight line passing through the point of intersection of the lines x+y+1 = 0 and 2x–y+5 = 0 and containing the point (5,–2).

MAINS Q EAM Q

Semi Sol: The equation of any line passing through the point of intersection of the lines x+y+1=0, 2x–y+5=0 is (x+y+1)+ λ(2x–y+5)=0 . ...(1) ,λ∈R

Substituing (5,–2) in (1) we get λ = – 4/17.

$$\Rightarrow (x+y+1)-\frac{4}{17}(2x-y+5)=0 \Rightarrow 17(x+y+1)-4(2x-y+5)=0$$

$$\Rightarrow 17x+17y+17-8x+4y-20=0 \Rightarrow 9x+21y-3=0 \Rightarrow 3x+7y-1=0$$

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13. Find the equation of the line passing through the point of intersection of 2x–5y+1=0, x–3y–4 = 0 and making equal intercepts on the axes.

EAM Q

Semi Sol: Solving 2x–5y+1=0, x–3y–4=0 we get Point of intersection= (–23, –9)

Let the intercepts made by the required line be a, a

Then the equation of the line is $\frac{x}{a}+\frac{y}{a}=1 \Rightarrow x+y=a$

But this line passes through (–23, –9) ⇒ –23 –9=a ⇒ a= –32

∴ Equation of the required line is x+y= –32 ⇒ x + y + 32 = 0

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14. Find the perpendicular distance between the point of intersection of 3x+2y+4=0, 2x+5y–1=0 and the line 7x+24y=15.

EAM Q

Semi Sol: Solving 3x+2y+4=0, 2x+5y–1=0 we get Point of intersection= (–2, 1)

The perpendicular distance from (–2, 1) to the line 7x + 24y – 15 = 0 is

$$=\left|\frac{7(-2)+24(1)-15}{\sqrt{7^2+24^2}}\right|=\left|\frac{-14+24-15}{\sqrt{49+576}}\right|=\frac{5}{25}=\frac{1}{5}$$

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15. Determine whether or not the four straight lines with equations x + 2y – 3 = 0, 3x + 4y – 7 = 0, 2x + 3y – 4 = 0 and 4x + 5y – 6 = 0 are concurrent.

Sol: First we solve the first two equations x + 2y – 3 = 0 ....(1)

3x + 4y – 7 = 0 ....(2)

$$\frac{x}{2(-7)-(-3)(4)}=\frac{y}{-3(3)-1(-7)}=\frac{1}{1(4)-2(3)}$$

$$\Rightarrow \frac{x}{-14+12}=\frac{y}{-9+7}=\frac{1}{4-6} \Rightarrow \frac{x}{-2}=\frac{y}{-2}=\frac{1}{-2} \Rightarrow x=1,\ y=1$$

$\therefore$ Point of intersection P=(1,1)

Substituting x=1, y=1 in $2x + 3y - 4 = 0$...........(3) we have $2(1)+3(1)-4 = 5-4 \neq 0$

$\therefore$ Lines (1),(2),(3) lines are not concurrent.

Substituting x=1, y=1 in $4x + 5y - 6 = 0$...........(4) we have $4(1)+5(1)-6 = 9-6 \neq 0$

$\therefore$ Lines (1),(2),(4) lines are not concurrent.

Hence the given four lines are not concurrent.

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16. Show that the straight lines (a–b)x+(b–c)y=c–a, (b–c)x+(c–a)y= a–b, (c–a)x+(a–b)y=b–c are concurrent.

Sol: Let $L_1 \equiv (a-b)x+(b-c)y+(a-c)=0$...(1)

$L_2 \equiv (b-c)x+(c-a)y+(b-a)=0$...(2),

$L_3 \equiv (c-a)x+(a-b)y+(c-b)=0$......(3)

Adding (1),(2) & (3) we have $L_1+L_2+L_3 = 1.L_1+1.L_2+1.L_3$

$\Rightarrow[(a-b)+(b-c)+(c-a)]\ x+[(b-c)+(c-a)+(a-b)]y+[(a-c)+(b-a)+(c-b)] = (0)x+(0)y+0=0$

$\therefore$ The given 3 lines are concurrent.

Applied Result: $L_1 \equiv a_1x+b_1y+c_1=0, L_2 \equiv a_2x+b_2y+c_2=0, L_3 \equiv a_3x+b_3y+c_3=0$ are three lines such that for non-zero roots $\lambda_1,\lambda_2,\lambda_3$, if $\lambda_1L_1+\lambda_2L_2+\lambda_3L_3=0$ then the lines $L_1=0, L_2=0, L_3=0$ are concurrent.

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17. Find the area of the triangle formed by the straight lines 2x – y – 5 = 0, x – 5y + 11= 0 and x + y – 1 = 0.

Semi Sol: Given lines are $2x - y - 5 = 0$ .... (1), $x - 5y + 11 = 0$ .... (2), $x + y - 1 = 0$ ....(3)

Solving (1) & (2),we get A = (4, 3)

Solving (1) & (3),we get B = (2, –1)

Solving (2) & (3),we get C = (–1, 2)

The area of the triangle formed with the vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ is

$$\Delta = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|4(-1-2) + 2(2-3) - 1(3+1)| = \frac{1}{2}|4(-3) + 2(-1) - 1(4)| = \frac{1}{2}|-18| = 9 \text{ sq.units}$$

18. Show that the straight lines x + y = 0, 3x + y – 4 = 0 and x + 3y – 4 = 0 form an isosceles triangle.

Sol: Given lines are $x + y = 0$........ (1), $3x + y - 4 = 0$....... (2), $x + 3y - 4 = 0$ ......(3)

If 'A' is the angle between (1),(2) then

$$\cos A = \frac{|a_1a_2+b_1b_2|}{\sqrt{(a_1^2+b_1^2)(a_2^2+b_2^2)}} = \frac{|1(3)+1(1)|}{\sqrt{(1^2+1^2)(3^2+1^2)}} = \frac{4}{\sqrt{(2)(10)}} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} \Rightarrow A = \text{Cos}^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

If 'B' is the angle between (2),(3) then

$$\cos B = \frac{|a_1a_2+b_1b_2|}{\sqrt{(a_1^2+b_1^2)(a_2^2+b_2^2)}} = \frac{|3(1)+1(3)|}{\sqrt{(3^2+1^2)(1^2+3^2)}} = \frac{6}{\sqrt{10}\sqrt{10}} = \frac{6}{10} = \frac{3}{5} \Rightarrow B = \text{Cos}^{-1}\left(\frac{3}{5}\right)$$

If 'C' is the angle between (1),(3) then

$$\cos C = \frac{|a_1a_2+b_1b_2|}{\sqrt{(a_1^2+b_1^2)(a_2^2+b_2^2)}} = \frac{|1(1)+1(3)|}{\sqrt{(1^2+1^2)1^2+3^2}} = \frac{4}{\sqrt{2}\sqrt{10}} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} \Rightarrow C = \text{Cos}^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

Here A=C. So the given lines form an Isosceles triangle.

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19. If the four straight lines ax+by+p=0, ax+by+q=0, cx+dy+r=0 and cx+dy+s=0 form a parallelogram, show that the area of the parallelogram so formed is $\left|\frac{(p-q)(r-s)}{bc-ad}\right|$

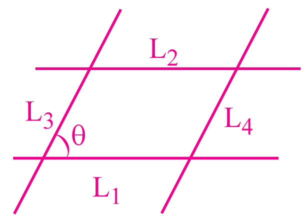
Sol: Let L_1=ax+by+p=0, L_2=ax+by+q=0, L_3=cx+dy+r=0, L_4=cx+dy+s=0

Clearly $L_1 \parallel L_2$ and $L_3 \parallel L_4$.

If θ is the angle between L_1 and L_3 then area of the parallelogram is $\Delta = \frac{d_1d_2}{\sin\theta}$

Now d_1= distance between L_1 and L_2 = $\frac{|p-q|}{\sqrt{a^2+b^2}}$

d_2= distance between L_3 and L_4 = $\frac{|r-s|}{\sqrt{c^2+d^2}}$



From L_1 & L_3, $\cos\theta = \frac{|ac+bd|}{\sqrt{(a^2+b^2)(c^2+d^2)}}$

$$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{\frac{(a^2+b^2)(c^2+d^2)-(ac+bd)^2}{(a^2+b^2)(c^2+d^2)}} = \frac{|bc-ad|}{\sqrt{(a^2+b^2)(c^2+d^2)}}$$

∴ Area of the parallelogram $\Delta = \frac{d_1d_2}{\sin\theta} = \left|\frac{(p-q)(r-s)}{bc-ad}\right|$

STRAIGHT LINES-IV

LAQs on (Circumcentre, Orthocentre, Incentre)

20. Find the circumcenter of the triangle whose vertices are (1,3), (0,–2) and (–3, 1)

Sol: Let S(x,y) be the circumcentre of the triangle ABC where A=(1,3), B=(0, –2), C=(–3,1)

We know that SA=SB=SC i.e., SA=SB and SB=SC $\Rightarrow SA^2=SB^2$ and $SB^2=SC^2$

Now, $SA^2=SB^2 \Rightarrow (x-1)^2+(y-3)^2=(x-0)^2+(y+2)^2$

$\Rightarrow (x^2-2x+1)+(y^2-6y+9) = x^2+(y^2+4y+4)$

$\Rightarrow -2x-10y+6=0 \Rightarrow -2(x+5y-3)=0 \Rightarrow x+5y-3=0$ ....(1)

Also $SB^2 = SC^2 \Rightarrow (x-0)^2+(y+2)^2=(x+3)^2+(y-1)^2$

$\Rightarrow x^2+(y^2+4y+4)=(x^2+6x+9)+(y^2-2y+1) \Rightarrow 6x-6y+6=0 \Rightarrow 6(x-y+1)=0 \Rightarrow x-y+1=0$ ....(2)

Solving (1) and (2), we get S; $x+5y-3=0$

$x-y+1=0$

$6y-4=0 \Rightarrow 6y=4 \Rightarrow y=2/3$

$(2) \Rightarrow x-\frac{2}{3}+1=0 \Rightarrow x=\frac{2}{3}-1=\frac{2-3}{3}=-\frac{1}{3}$

$\therefore$ the circumcentre of the given triangle is S (x,y) = (1/3, 2/3)

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21. Find the circumcentre of the triangle whose vertices are (1,0),(–1,2),(3,2)

Sol: We take circumcentre as S(x,y) and vertices A=(1, 0), B=(–1, 2), C=(3, 2), then

SA=SB=SC ($\because$ S is Equidistant from A,B,C)

Take, SA= SB $\Rightarrow \sqrt{(x-1)^2+(y-0)^2} = \sqrt{(x+1)^2+(y-2)^2}$

Squaring on both sides we get $(x-1)^2+(y-0)^2=(x+1)^2+(y-2)^2$

$\Rightarrow (\cancel{x^2}+1-2x)+(\cancel{y^2}) = (\cancel{x^2}+1+2x)+(\cancel{y^2}+4-4y)$

$\Rightarrow -2x+1=2x-4y+5 \Rightarrow 4x-4y+4=0 \Rightarrow x-y+1=0$ ....(1)

Again take SB=SC

$\Rightarrow \sqrt{(x+1)^2+(y-2)^2} = \sqrt{(x-3)^2+(y-2)^2}$

Squaring on both sides we get $\Rightarrow (x+1)^2+(y-2)^2 = (x-3)^2+(y-2)^2$

$\Rightarrow (\cancel{x^2}+1+2x)=(\cancel{x^2}+9-6x) \Rightarrow 8x-8=0 \Rightarrow x=1$ ....(2)

Solving (1) and (2), we get S;

From (1), $1-y+1=0 \Rightarrow y=2$ $\therefore$ Circumcentre S(x,y) = (1, 2)

22. Find the orthocentre of the triangle whose sides are x+y+10=0, x–y–2=0, 2x+y–7=0

Sol: Let x+y+10=0....(1), x–y–2=0 ....(2) , 2x+y–7=0 .....(3)

represent the sides of ΔABC.

Let O be the orthocentre of ΔABC

First we find the equation of the altitude through A

Solving (1) and (2), we get A; x+y+10=0

x–y–2=0

$$\Rightarrow \frac{x}{-2+10} = \frac{y}{10+2} = \frac{1}{-1-1} \Rightarrow \frac{x}{8} = \frac{y}{12} = \frac{-1}{2}$$

$\Rightarrow x = -4, y = -6 \quad \therefore A = (-4, -6)$

The slope of the opposite side BC, 2x+y–7=0 is –2

⇒ the slope of its perpendicular is 1/2

The equation of the altitude passing through A(–4,–6) and with slope 1/2 is

y+6=(1/2)(x+4) ⇒ 2y+12= x+4=0 ⇒ x–2y–8=0 .... (4)

Now, we find the equation of the altitude through B

Solving (1) & (3) we get B; x+y+10=0

2x+y–7=0

$$\Rightarrow \frac{x}{-7-10} = \frac{y}{20+7} = \frac{1}{1-2} \Rightarrow \frac{x}{-17} = \frac{y}{27} = -1$$

$\Rightarrow x = 17, y = -27 \qquad \therefore B = (17, -27)$

The slope of the opposite side AC, x–y–2=0 is 1

⇒ the slope of its perpendicular is –1

The equation of the altitude through B(17,–27) and with slope –1 is y+27=–1(x–17)

⇒ x+y+10=0 .....(5)

Solving (4) & (5) we get orthocentre O;

(4)–(5) ⇒ –3y –18 =0 ⇒3y = –18⇒ y = –6

From (4), x =2y+8= 2(–6)+8 = –12+8 = –4.

∴ Orthocentre = (–4, –6)

23. Find the circumcentre of the triangle whose sides are x+y+2=0,5x–y–2=0,x–2y+5=0

Sol: Let x+y+2=0 .... (1), 5x–y–2=0 .... (2), x–2y+5=0....(3) represent the sides AB,BC,CA of ΔABC respectively

Solving (1) & (3),we get A; $x+y+2=0$

$x-2y+5=0$

$(1)-(3) \Rightarrow 3y-3=0 \Rightarrow y=1$

$\therefore (1) \Rightarrow x+1+2=0 \Rightarrow x=-3 \quad \therefore A=(-3,1)$

Solving (1) & (2),we get B; $x+y+2=0$

$5x-y-2=0$

$(1)+(2) \Rightarrow 6x = 0 \Rightarrow x=0$

$\therefore (1) \Rightarrow 0+y+2=0 \Rightarrow y=-2 \quad \therefore B=(0,-2)$

Solving (2) & (3),we get C; $x-2y+5=0$

$(2)\times 2 \Rightarrow 10x-2y-4=0$

$\Rightarrow -9x + 9=0 \Rightarrow 9x=9 \Rightarrow x=1$

$\therefore (3) \Rightarrow 1-2y+5=0 \Rightarrow 2y=6 \Rightarrow y=3 \quad \therefore C=(1,3)$

Let S(x,y) be the circumcentre of ΔABC with vertices A(–3,1), B(0,–2), C(1,3)

$$\Rightarrow SA=SB \Rightarrow SA^2=SB^2 \Rightarrow (x+3)^2+(y-1)^2=(x-0)^2+(y+2)^2$$

$$\Rightarrow (x^2+6x+9)+(y^2-2y+1)=x^2+(y^2+4y+4)$$

$$\Rightarrow 6x-6y+6=0 \Rightarrow 6(x-y+1)=0 \Rightarrow x-y+1=0 \text{ ....(4)}$$

Also $SB=SC \Rightarrow SB^2=SC^2 \Rightarrow (x-0)^2+(y+2)^2=(x-1)^2+(y-3)^2$

$$\Rightarrow x^2+(y^2+4y+4)=(x^2-2x+1)+(y^2-6y+9)$$

$$\Rightarrow 2x+10y-6=0 \Rightarrow 2(x+5y-3)=0 \Rightarrow x+5y-3=0 \text{ ....(5)}$$

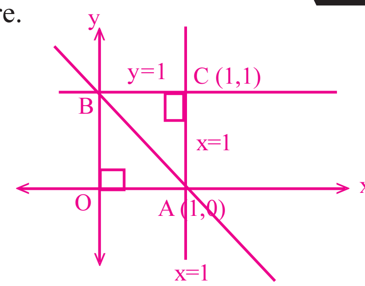
Solving (4) & (5) we get the circumcentre S; $(4)-(5) \Rightarrow -6y+4=0 \Rightarrow 6y=4 \Rightarrow y=2/3$

$(4) \Rightarrow x-\frac{2}{3}+1=0 \Rightarrow x=\frac{2}{3}-1=\frac{2-3}{3}=\frac{-1}{3}$ $\therefore$ the circumcentre ΔABC is $S=\left(\frac{-1}{3},\frac{2}{3}\right)$

24. Find the (i) circumcentre (ii) incentre of the triangle formed by the lines x=1, y=1 and x+y=1

EAM Q

Sol: Given lines and vertices of ΔABC are shown in the figure.



Equation of AB is x+y=1

Equation of AC is x=1

Equation of BC is y=1

Solving these equations we get

$A(x_1,y_1)=(1,0), B(x_2,y_2)=(0,1), C(x_3,y_3)=(1,1)$

(i) **Circumcentre:** It is clear that ΔABC is a right angled triangle.

$\therefore$ Circumcentre S = Mid point of the hypotenuse AB

$$S=\left(\frac{1+0}{2},\frac{0+1}{2}\right)=\left(\frac{1}{2},\frac{1}{2}\right)$$

(ii) Also $a=BC=\sqrt{(0-1)^2+(1-1)^2}=\sqrt{1+0}=1$

$$b=CA=\sqrt{(1-1)^2+(1-0)^2}=\sqrt{0+1}=1$$

$$c=AB=\sqrt{(1-0)^2+(0-1)^2}=\sqrt{1+1}=\sqrt{2}$$

$\therefore$ Incenter of ΔABC is $I=\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_3}{a+b+c}\right)$

Now $\frac{ax_1+bx_2+cx_3}{a+b+c}=\frac{1(1)+1(0)+\sqrt{2}(1)}{1+1+\sqrt{2}}=\frac{1+\sqrt{2}}{2+\sqrt{2}}=\frac{1+\sqrt{2}}{\sqrt{2}(\sqrt{2}+1)}=\frac{1}{\sqrt{2}}$

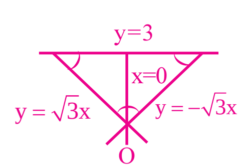
and $\frac{ay_1+by_2+cy_3}{a+b+c}=\frac{1(0)+1(1)+\sqrt{2}(1)}{1+1+\sqrt{2}}=\frac{1+\sqrt{2}}{2+\sqrt{2}}=\frac{1+\sqrt{2}}{\sqrt{2}(\sqrt{2}+1)}=\frac{1}{\sqrt{2}}$

$\therefore$ Incenter $I=\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

---

25. Find the incenter of the triangle formed by the straight lines $y=\sqrt{3}x, y=-\sqrt{3}x$, y=3

Sol: The straight lines $y=\sqrt{3}x$ and $y=-\sqrt{3}x$ make angles 60° and 120° with x-axis.



Since y=3 is a horizontal line, the triangle formed by the given three lines is Equilateral.

So its Incenter(I) is same as the Centroid (G).

It is at a distance of 2 units from the origin vertex. [$\because$ G divides the median in the ratio 2:1]

Incentre lies on the angular bisector y-axis.

$\therefore$ Incentre of the triangle is I=(0,2)

26. Find the incenter of the triangle formed by the straight lines. x + 1 = 0, 3x – 4y = 5 and 5x + 12y = 27

Sol: Given lines are x + 1 = 0 .... (1), 3x – 4y –5 = 0 .... (2), 5x + 12y –27 = 0....(3)

Solving (1) & (2),we get A; $x + 1 = 0 \Rightarrow x = -1$

Put x= –1 in $3x - 4y - 5 = 0 \Rightarrow 3(-1) - 4y - 5 = 0 \Rightarrow 4y = -8 \Rightarrow y = -2$

$\therefore A=(-1,-2)$

Solving (1) & (3),we get B; $x + 1 = 0 \Rightarrow x = -1$

Put x= –1 in $5x + 12y - 27 = 0 \Rightarrow 5(-1) + 12y - 27 = 0 \Rightarrow 12y = 32 \Rightarrow \Rightarrow y = \frac{32}{12} = \frac{8}{3}$

$$\therefore B = \left(-1, \frac{8}{3}\right)$$

Solving (2) & (3),we get C;

$3\times(2) \Rightarrow 9x - 12y - 15 = 0$..........(4)

(3) $\Rightarrow 5x + 12y - 27 = 0$

Adding (3) & (4) we have $14x - 42 = 0 \Rightarrow 14x = 42 \Rightarrow x = 3$

Put x=3 in $3x - 4y - 5 = 0 \Rightarrow 3(3) - 4y - 5 = 0 \Rightarrow 4y = 4 \Rightarrow y = 1$

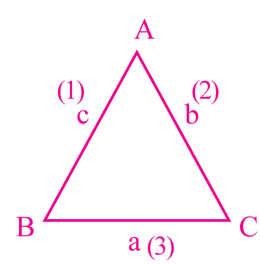
$\therefore C = (3,1)$

Vertices of ΔABC are $A(x_1, y_1) = (-1,-2)$, $B(x_2, y_2) = (-1, 8/3)$ and $C(x_3, y_3) = (3,1)$

$$\text{Now, } a = BC = \sqrt{(3+1)^2 + \left(1 - \frac{8}{3}\right)^2} = \sqrt{4^2 + \left(\frac{-5}{3}\right)^2} = \sqrt{16 + \frac{25}{9}} = \sqrt{\frac{144+25}{9}} = \sqrt{\frac{169}{9}} = \frac{13}{3}$$

$$b = AC = \sqrt{(3+1)^2 + (1+2)^2} = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$c = AB = \sqrt{(-1+1)^2 + \left(\frac{8}{3} + 2\right)^2} = \sqrt{\left(\frac{14}{3}\right)^2} = \frac{14}{3}$$



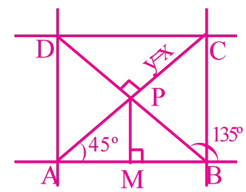
$$\therefore \text{ Incenter of } \Delta ABC \text{ is } I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

$$= \left(\frac{\frac{13}{3}(-1) + \frac{14}{3}(3) + 5(-1)}{\frac{13}{3} + \frac{14}{3} + 5}, \frac{\frac{13}{3}(-2) + \frac{14}{3}(1) + 5\left(\frac{8}{3}\right)}{\frac{13}{3} + \frac{14}{3} + 5}\right) = \left(\frac{14}{42}, \frac{28}{42}\right) = \left(\frac{1}{3}, \frac{2}{3}\right)$$

PQ **Find the incenter of the triangle formed by the straight lines. x + y – 7 = 0, x – y + 1 = 0 and x – 3y + 5 = 0** $[\text{Ans}: (3, 1+\sqrt{5})]$

LEVEL -II PROBLEMS

27. Each side of a square is of length 4 units. The centre of the square is (3,7) and one of its diagonals is parallel to y=x. Find the co-ordinates of its vertices.



Sol: Let ABCD be the square of side length 4 units.

Point of intersection of the diagonals is the centre P(3,7)

Let M be the foot of the perpendicular from P on to AB.

Then M is the mid point of AB. $\therefore$ AM=MB=PM=2

Since a diagonal is parallel to y = x, its sides are parallel to the co-ordinate axes.

Now P=(3,7) $\Rightarrow$M= (3,7–2)=(3,5)

Now M= (3,5)$\Rightarrow$ A=(3–2,5)=(1,5) and M= (3,5) $\Rightarrow$ B= (3+2, 5)= (5,5)

Again A= (1, 5)$\Rightarrow$ D(1, 5+4)= (1,9) and B= (5, 5)$\Rightarrow$ C(5, 5+4)= (5,9)

(or) Side of square a =4 $\Rightarrow$ diagonal $d=\sqrt{2}a=\sqrt{2}(4)$

A, C are at adistance $\frac{4\sqrt{2}}{2}=2\sqrt{2}$ from the center P(3,7) .

Also y=x$\Rightarrow$ m=1= tan 45° $\Rightarrow$ θ= 45°. Now A,C are given by the parametric equations

$$(x,y)=(x_1\pm r\cos\theta, y_1\pm r\sin\theta)=\left(3\pm 2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right), 7\pm 2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\right)\Rightarrow A=(1,5),\ C=(5,9)$$

Also by taking θ=135° we get B= (5,5), D=(1,9)

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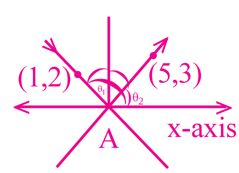
28. A ray of light passing through the point (1, 2) reflects on the X– axis at a point A and the reflected ray passes through the point (5, 3). Find the coordinates of A.



Sol: If the inclination of the line is θ_1 then inclination of its refelcted ray is $\theta_2=(180°-\theta_1)$
Hence, if the slope of a ray is m then slope of its reflected ray is –m
The equation of the ray passing through (1, 2) having the slope m is

$y-2=m(x-1)\Rightarrow m=\frac{y-2}{x-1}$ ....(1)

The equation of the reflected ray passing through (5, 3)

having the slope –m is $y-3=-m(x-5)\Rightarrow m=\frac{y-3}{5-x}$ ....(2)

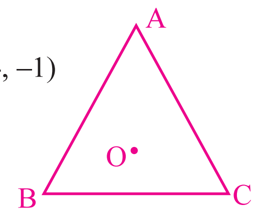
Solving (1) & (2) we get the point of reflection.

From (1) & (2), $\frac{y-2}{x-1}=\frac{y-3}{5-x}$

But A lies on the X axis. So put y=0$\Rightarrow \frac{-2}{x-1}=\frac{-3}{5-x}\Rightarrow$2(5–x)=3(x–1)

$\Rightarrow 10-2x=3x-3\Rightarrow 5x=13\Rightarrow x=\frac{13}{5}$ $\qquad \therefore A=\left(\frac{13}{5},0\right)$

29. Show that the origin is with in the triangle whose angular points are (2, 1) (3, –2) and (–4, –1).



Semi Sol: Let the vertices of the triangle are A = (2, 1), B = (3, –2) , C = (–4, –1)

Equation of the line passing through the points A(2,1), B(3, –2) is

$$y-1=\frac{-2-1}{3-2}(x-2)\Rightarrow y-1=-3(x-2)\Rightarrow 3x+y-7=0$$

Similarly, equation of the line through the points B(3, –2), C(–4, –1) is $x+7y+11=0$

Equation of the line passing through the points A(2, 1), C(–4, –1) is $x-3y+1=0$

(i) Equation of the side AB is $L = 3x + y - 7 = 0$...........(1)

Put C (–4, –1) in (1) $\Rightarrow L_{11} = 3(-4) - 1 - 7 = -20 < 0$

Put O (0, 0) in (1) $\Rightarrow L_{22} = 3(0) + 0 - 7 = -7 < 0$

Here L_{11}, L_{22} have same signs. So the point C and Origin O lie on the same side of AB.

(ii) Equation of the side BC is $L = x + 7y + 11 = 0$...........(2)

Here L_{11}, L_{22} have same signs. So the point A and Origin O lie on the same side of BC.

(iii) Equation of the side AC is $L = x - 3y + 1 = 0$...........(3)

Here L_{11}, L_{22} have same signs. So the point B and Origin O lie on the same side of AC.

From the above 3 cases we see that the origin lies on the same side of the sides $\overline{AB}, \overline{BC}, \overline{CA}$

$\therefore$ The origin lies with in the triangle ABC.

(or) If O is any point inside the triangle then verify that the sum of the areas formed by ΔAOB, ΔBOC, ΔAOC = Area of ΔABC.

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30. A straight line L with negative slope passes through the point (8, 2) and cuts positive co-ordinate axes at the points P and Q. Find the minimum value of OP+OQ as L varies, when O is the origin.

MAINS Q

Sol: Equation of the line passing through A(8, 2) with negative slope '–m' is $y - 2 = -m(x - 8)$

$\Rightarrow y - 2 = -mx + 8m \Rightarrow mx + y = 2 + 8m$

$$\Rightarrow \frac{mx}{2+8m}+\frac{y}{2+8m}=1\Rightarrow \frac{x}{\left(\frac{2+8m}{m}\right)}+\frac{y}{(2+8m)}=1$$

Here the X - intercept $OP=\frac{2+8m}{m}$ and the Y- intercept $OQ = 2 + 8m$

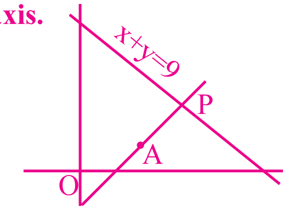
$$\therefore OP+OQ=\left(\frac{2+8m}{m}\right)+(2+8m)=\frac{2}{m}+\frac{8m}{m}+2+8m=\frac{2}{m}+8+2+8m=\left(\frac{2}{m}+8m\right)+10$$

Now $\left(\frac{2}{m}+8m\right)+10\geq 2\sqrt{\left(\frac{2}{m}\right)(8m)}+10$ $\quad(\because A.M\geq G.M\Rightarrow \frac{a+b}{2}\geq\sqrt{ab}\Rightarrow a+b\geq 2\sqrt{ab})$

$$=2(4)+10=18$$

$\therefore OP + OQ \geq 18$. Hence the minimum value of OP + OQ is 18

31. A straight line L is drawn through the point A(2, 1) such that its point of intersection with the straight line x + y = 9 is at a distance of $3\sqrt{2}$ from A. Find the angle which the line L makes with the positive direction of the X-axis.



Sol: Let P be the parametric point on the line AP.

The coordinate of $A=(x_1,y_1)=(2,1)$ and $AP=r=3\sqrt{2}$

Then $P=(x_1+r\cos\theta,\ y_1+r\sin\theta) = (2+3\sqrt{2}\cos\theta,\ 1+3\sqrt{2}\sin\theta)$

But P also lies on the line x+y=9.

$\Rightarrow (2+3\sqrt{2}\cos\theta) + (1+3\sqrt{2}\sin\theta) = 9 \Rightarrow 3\sqrt{2}\cos\theta + 3\sqrt{2}\sin\theta = 9-3$

$\Rightarrow 3\sqrt{2}(\cos\theta+\sin\theta) = 6 \Rightarrow \cos\theta+\sin\theta = \dfrac{6}{3\sqrt{2}} = \dfrac{2}{\sqrt{2}}$

$\Rightarrow \cos\theta+\sin\theta = \dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}} \Rightarrow \cos\theta = \dfrac{1}{\sqrt{2}}$ and $\sin\theta = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = 45^{\circ}$

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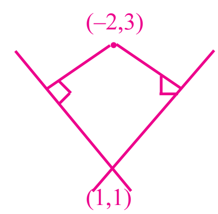
32. Find the equation of the straight lines passing through (1, 1) and which are at a distance of 3 units from (–2, 3).



Sol: Let slope of the required line be m

Equation of any line passing through A(1, 1) with slope m is $y-1 = m(x-1)$..........(1)

$\Rightarrow mx - y + (1-m) = 0$

But distance from B(–2, 3) to the above line is 3 units.

$\Rightarrow \dfrac{|m(-2)-3+(1-m)|}{\sqrt{m^2+1}} = 3 \Rightarrow \dfrac{|-3m-2|}{\sqrt{m^2+1}} = 3 \Rightarrow 3\sqrt{m^2+1} = |3m+2|$

$\Rightarrow 9(m^2+1) = (3m+2)^2 \Rightarrow \cancel{9m^2}+9 = \cancel{9m^2}+12m+4 \Rightarrow 12m = 5 \Rightarrow m = \dfrac{5}{12}\ (\text{or})\ m = \infty$

$\therefore$ The slope of the required line is 5/12 (or) ∞.

Equation of the line passing through (1, 1) with slope $\dfrac{5}{12}$ is $y-1 = \dfrac{5}{12}(x-1)$

$\Rightarrow 12(y-1) = 5(x-1) \Rightarrow 12y-12 = 5x-5 \Rightarrow 5x-12y+7=0$

Also the equation of the vertical line passing through the point (1, 1) with slope ∞ is x=1

---

33. Find the equation of the straight lines passing through the point of intersection of the lines 3x + 2y + 4 = 0, 2x + 5y = 1 and whose distance from (2, –1) is 2.

Sol: The equation of any line passing through the point of intersection of

$L_1 \equiv 3x+2y+4=0,\ L_2 \equiv 2x+5y-1=0$ is $L_1+\lambda L_2 = 0,\ \lambda\in R$

$\Rightarrow (3x+2y+4)+\lambda(2x+5y-1) = 0$ ....(1)

$\Rightarrow (3+2\lambda)x+(2+5\lambda)y+(4-\lambda) = 0$

The perpendicular distance from (2, –1) to the above line is 2

$\therefore \dfrac{|(3+2\lambda)2+(2+5\lambda)(-1)+(4-\lambda)|}{\sqrt{(3+2\lambda)^2+(2+5\lambda)^2}} = 2 \Rightarrow \dfrac{|-2\lambda+8|}{\sqrt{(3+2\lambda)^2+(2+5\lambda)^2}} = 2$

$\Rightarrow \cancel{2}|(-\lambda+4)| = \cancel{2}\sqrt{(3+2\lambda)^2+(2+5\lambda)^2}$

$\Rightarrow (-\lambda+4)^2 = (3+2\lambda)^2 + (2+5\lambda)^2$

$\Rightarrow \lambda^2 + 16 - 8\lambda = (9 + 4\lambda^2 + 12\lambda) + (4 + 25\lambda^2 + 20\lambda) \Rightarrow 28\lambda^2 + 40\lambda - 3 = 0$

$\Rightarrow 28\lambda^2 - 2\lambda + 42\lambda - 3 = 0 \Rightarrow 2\lambda(14\lambda - 1) + 3(14\lambda - 1) = 0 \Rightarrow (2\lambda + 3)(14\lambda - 1) = 0$

$\Rightarrow 2\lambda + 3 = 0 \text{ (or) } 14\lambda - 1 = 0 \Rightarrow \lambda = \frac{-3}{2} \text{ (or) } \lambda = \frac{1}{14}$

(i) Put $\lambda = -\frac{3}{2}$ in (1) then $(3x + 2y + 4) - \frac{3}{2}(2x + 5y - 1) = 0$

$\Rightarrow 2(3x + 2y + 4) - 3(2x + 5y - 1) = 0 \Rightarrow -11y + 11 = 0 \Rightarrow y - 1 = 0$

(ii) Put $\Rightarrow \lambda = \frac{1}{14}$ in (1) then $(3x + 2y + 4) + \frac{1}{14}(2x + 5y - 1) = 0$

$\Rightarrow 14(3x + 2y + 4) + (2x + 5y - 1) = 0 \Rightarrow 44x + 33y + 55 = 0 \Rightarrow 4x + 3y + 5 = 0$

---

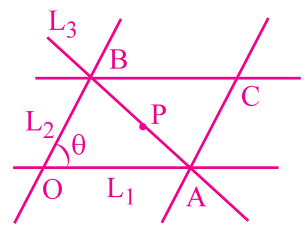
34. Two adjacent sides of a parallelogram are given by 4x+5y=0, 7x+2y=0 and one diagonal is 11x+7y=9. Find the equations of the remaining sides and the other diagonal.

Semi Sol: Given sides are $L_1=4x+5y=0$........(1), $L_2=7x+2y=0$........(2), diagonal $L_3=11x+7y=9$........(3)

Solving (1) & (2) we get point of intersection O(0,0)

Solving (1) & (3) we get A= $\left(\frac{5}{3}, \frac{-4}{3}\right)$

Solving (2) & (3) we get B= $\left(\frac{-2}{3}, \frac{7}{3}\right)$



Points of intersection are O(0,0), A(5/3, – 4/3), B(–2/3, 7/3)

Mid point of AB is P= (1/2, 1/2) ⇒ Slope of OP is 1⇒ Equation of OC is x = y.

Mid point of OC= P. Hence we get C=(1,1)

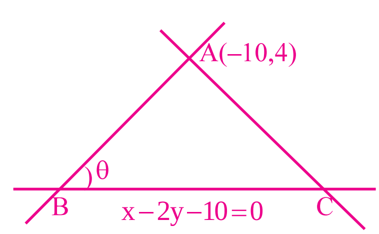
∴ Equation of side AC is 7x+2y–9=0; Equation of side BC is 4x+5y–9=0

---

35. Find the equation of the straight lines passing through the point (–10, 4) and making an angle θ with the line x – 2y = 10 such that tan θ = 2.

Sol: Let slope of the required line be m

The slope of the given line x – 2y – 10 = 0 is $-\frac{a}{b} = \frac{-1}{-2} = \frac{1}{2}$



The angle between the lines is θ ⇒ $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow 2 = \left|\frac{2m-1}{2+m}\right| \Rightarrow 4(2+m)^2 = (2m-1)^2 \Rightarrow 4(4 + m^2 + 4m) = 4m^2 + 1 - 4m$

$\Rightarrow 16 + 4m^2 + 16m = 4m^2 + 1 - 4m \Rightarrow 16m + 4m = 1 - 16 \Rightarrow 20m = -15 \Rightarrow m = -3/4$ & $m = \infty$

Equation of the other line passing through A(–10, 4) with slope –3/4 is

$y - 4 = -\frac{3}{4}(x + 10) \Rightarrow 4(y-4) = -3(x+10) \Rightarrow 4y - 16 = -3x - 30 \Rightarrow 3x + 4y + 14 = 0$

Equation of the vertical line passing through the point A(–10, 4) with slope ∞ is x=–10[∵x=k]

36. The hypotenuse of a right angled isosceles triangle has its ends at the points (1,3) and (–4,1). Find the equations of the legs of the triangle.

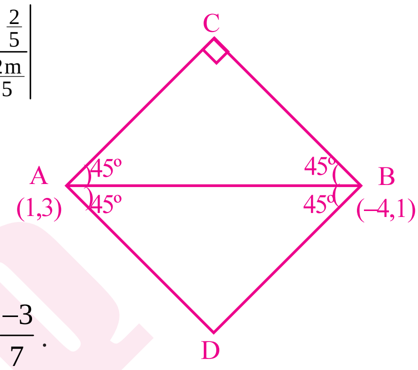
Sol: Let A=(1,3) and B=(–4,1) be the ends of hypotenuse AB of ΔABC & ΔABD.

We require the equations of $\overleftrightarrow{AC}$ & $\overleftrightarrow{BC}$; $\overleftrightarrow{AD}$ & $\overleftrightarrow{BD}$

Let m be the slope of the side $\overleftrightarrow{AC}$

Slope of $\overleftrightarrow{AB}$ is $\dfrac{1-3}{-4-1}=\dfrac{2}{5}$

Angle between the lines is θ=45°. Then $\tan 45^\circ=\left|\dfrac{m-\frac{2}{5}}{1+\frac{2m}{5}}\right|$



$\Rightarrow 1=\dfrac{|5m-2|}{|5+2m|}\Rightarrow\dfrac{5m-2}{5+2m}=\pm1\Rightarrow 5m-2=\pm(5+2m)$

$\Rightarrow 3m=7$ (or) $7m+3=0 \Rightarrow m=\dfrac{7}{3}$ or $\dfrac{-3}{7}$

Case (i): For ΔACB

If the slope of $\overleftrightarrow{AC}$ is $\dfrac{7}{3}$, then the slope of $\overleftrightarrow{BC}$ will be $\dfrac{-3}{7}$.

Equation of AC through A(1,3) with slope $\dfrac{7}{3}$ is $y-3=\dfrac{7}{3}(x-1)$

$\Rightarrow 3(y-3)=7(x-1)\Rightarrow 3y-9=7x-7\Rightarrow 7x-3y+2=0$

Equation of BC through B(–4,1) with slope –3/7 is $y-1=\dfrac{-3}{7}(x+4)$

$\Rightarrow 7(y-1)=-3(x+4)\Rightarrow 7y-7=-3x-12\Rightarrow 3x+7y+5=0$

Case (ii): For ΔABD

If the slope of AD is $\dfrac{-3}{7}$ then slope of BD is $\dfrac{7}{3}$.

Equation of AD through A(1,3) with slope –3/7 is $y-3=\dfrac{-3}{7}(x-1)$

$\Rightarrow 7(y-3)=-3(x-1)\Rightarrow 7y-21=-3x+3\Rightarrow 3x+7y-24=0$

Equation of BD through B(–4,1) with slope 7/3 is $y-1=\dfrac{7}{3}(x+4)$

$\Rightarrow 3(y-1)=7(x+4)\Rightarrow 3y-3=7x+28\Rightarrow 7x-3y+31=0$

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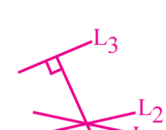
37. A person standing at the junction (crossing) of two straight paths represented by the equations 2x – 3y + 4 = 0 and 3x + 4y – 5 = 0 wants to reach the path whose equation is 6x – 7y + 8 = 0 in the least time. Find the equation of the path the he should follow.

Sol: The required path equation passes through the point of intersection of given two lines & perpendicular to the other line.

Given lines are L_1=2x – 3y + 4 = 0 .. (1), L_2=3x + 4y – 5 = 0 .. (2), L_3=6x – 7y + 8 = 0 .(3)

Solving (1) & (2),we get P; 2x – 3y + 4 = 0 ;

3x + 4y – 5 = 0



$\Rightarrow\dfrac{x}{(-3)(-5)-(4)(4)}=\dfrac{y}{(4)(3)-(2)(-5)}=\dfrac{1}{(2)(4)-(-3)(3)}$

$\Rightarrow \frac{x}{15-16} = \frac{y}{12+10} = \frac{1}{8+9} \Rightarrow \frac{x}{-1} = \frac{y}{22} = \frac{1}{17} \Rightarrow x = \frac{-1}{17}, y = \frac{22}{17}$ $\quad\therefore P = \left(\frac{-1}{17}, \frac{22}{17}\right)$

$\therefore$ The junction of two straight paths (1) & (2) is $P = \left(\frac{-1}{17}, \frac{22}{17}\right)$

Slope of the 3rd line $6x - 7y + 8 = 0$ is $6/7 \Rightarrow$ Slope of its perpendiculars is $-7/6$

$\therefore$ Equation of the line passing through $P = \left(\frac{-1}{17}, \frac{22}{17}\right)$ with slope $-7/6$ is

$$y - \frac{22}{17} = \frac{-7}{6}\left(x + \frac{1}{17}\right) \Rightarrow \frac{17y-22}{17} = \frac{-7}{6}\left(\frac{17x+1}{17}\right) \Rightarrow 6(17y-22) = -7(17x+1)$$

$\Rightarrow 102y - 132 = -119x - 7 \Rightarrow 119x + 102y - 125 = 0$.

This is the required equation of the path.

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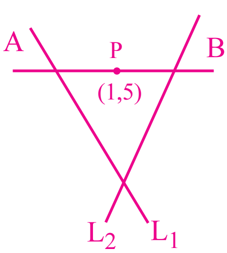
38. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain its equation.

Sol: Given lines are $L_1 \equiv 5x - y + 4 = 0$ ....(1), $L_2 \equiv 3x + 4y - 4 = 0$ ....(2)

Let $A = (x_1, y_1)$ and $B = (x_2, y_2)$ and $P(1, 5)$ be the midpoint of the portion $\overline{AB}$

Mid point of AB = P.



$\therefore \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (1, 5)$

$\frac{x_1 + x_2}{2} = 1 \Rightarrow x_1 + x_2 = 2 \Rightarrow x_2 = 2 - x_1$ ....(3)

$\frac{y_1 + y_2}{2} = 5 \Rightarrow y_1 + y_2 = 10 \Rightarrow y_2 = 10 - y_1$ ....(4)

But $A(x_1, y_1)$ lies on the line $5x - y + 4 = 0 \Rightarrow 5x_1 - y_1 + 4 = 0$ ....(5)

Also $B(x_2, y_2)$ lies on the line $3x + 4y - 4 = 0 \Rightarrow 3x_2 + 4y_2 - 4 = 0$

$\Rightarrow 3(2 - x_1) + 4(10 - y_1) - 4 = 0$ [$\because$ From (3) & (4)]

$\Rightarrow 6 - 3x_1 + 40 - 4y_1 - 4 = 0 \Rightarrow -3x_1 - 4y_1 + 42 = 0 \Rightarrow 3x_1 + 4y_1 - 42 = 0$ ....(6)

Solving (5) & (6), we get A; $5x_1 - y_1 + 4 = 0$

$3x_1 + 4y_1 - 42 = 0$

$$\Rightarrow \frac{x_1}{(-1)(-42) - (4)(4)} = \frac{y_1}{(4)(3) - (5)(-42)} = \frac{1}{(5)(4) - (-1)(3)}$$

$$\Rightarrow \frac{x_1}{42 - 16} = \frac{y_1}{12 + 210} = \frac{1}{20 + 3} \Rightarrow \frac{x_1}{26} = \frac{y_1}{222} = \frac{1}{23} \Rightarrow x_1 = \frac{26}{23}, \quad y_1 = \frac{222}{23} \qquad \therefore A\left(\frac{26}{23}, \frac{222}{23}\right)$$

Slope of the line through P (1,5) and $A\left(\frac{26}{23}, \frac{222}{23}\right)$ is $m = \frac{\frac{222}{23} - 5}{\frac{26}{23} - 1} = \frac{\frac{222 - 115}{23}}{\frac{26 - 23}{23}} = \frac{107}{3}$

$\therefore$ Equation of the line passing through P(1,5) with slope $\frac{107}{3}$ is $y-y_1=m(x-x_1)$

$\Rightarrow y-5=\frac{107}{3}(x-1) \Rightarrow 3(y-5) = 107(x-1) \Rightarrow 3y-15 = 107x - 107$

$\Rightarrow$ 107x – 3y – 92 = 0

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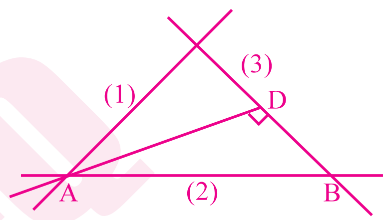
39. Prove that the feet of the perpendiculars from the origin on the lines x+y=4, x+5y=26 and 15x–27y=424 are collinear.

Sol: Given lines x+y–4=0....(1), x+5y–26=0....(2), 15x–27y–424=0....(3)

Let $P(x_1,y_1)$ be the foot of the perpendicular from (0,0) on x+y–4=0



$\Rightarrow \frac{x_1-0}{1}=\frac{y_1-0}{1}=\frac{-(0+0-4)}{1+1}=\frac{4}{2}=2$

$\Rightarrow x_1-0=2,\ y_1-0=2 \Rightarrow x_1=2,\ y_1=2$

$\therefore$ P=(2,2)

Let $Q(x_2,y_2)$ be the foot of the perpendicular from (0,0) on x+5y–26=0

$\Rightarrow \frac{x_2-0}{1}=\frac{y_2-0}{5}=\frac{-(0+0-26)}{1+25}=\frac{26}{26}=1$

$\Rightarrow x_2=1,\ y_2=5 \Rightarrow Q=(1,5)$

Let $R(x_3,y_3)$ be the foot of the perpendicular from (0,0) on 15x–27y–424=0

$\Rightarrow \frac{x_3-0}{15}=\frac{y_3-0}{-27}=\frac{-(0+0-424)}{225+729}=\frac{212}{477} \Rightarrow x_3=\frac{15\times 212}{477}=\frac{1060}{159};\ y_3=\frac{-27\times 212}{477}=-12$

$\therefore R=\left(\frac{1060}{159},-12\right)$

Now we find the equation of the line passing through P,Q

Slope of the line passing through P (2,2) and Q(1,5) is $m=\frac{5-2}{1-2}=\frac{3}{-1}=-3$

$\therefore$ Equation of the line passing through P(2,2) with slope –3 is y–2=–3(x–2)

$\Rightarrow$ y–2= –3x+6 $\Rightarrow$ 3x+y–8= 0

Put $R\left(\frac{1060}{159},-12\right)$ in the above equation we have $3\left(\frac{1060}{159}\right)-12-8=20-20=0$

$\therefore$ The three feet of the perpendiculars from the origin and given lines are collinear.

40. A triangle is formed by the lines ax + by + c = 0, *l*x + my + n = 0 and px + qy + r = 0. Given that the triangle is not right angled, show that the straight line

$$\frac{ax+by+c}{ap+bq}=\frac{lx+my+n}{lp+mq}$$ **passes through the orthocentre of the triangle.**

Sol: The sides of given triangle are

ax + by + c = 0...............(1)

*l*x + my + n = 0...............(2)

px + qy + r = 0 ....(3)

Equation of any line passing through the point of intersection of (1) and (2) is

$\Rightarrow$ (ax + by + c) + λ (*l*x + my + n) = 0...............(4)

$\Rightarrow$ (a + λ*l*) x + (b + λm) y + (c + nλ) = 0

If this line perpendicular to (3), then product of slopes of (3) and (4) is –1

$$\Rightarrow\left(\frac{-p}{q}\right)\left(\frac{-(a+\lambda l)}{(b+\lambda m)}\right)=-1$$

$\Rightarrow$ p(a+ λ*l*) + q(b + λm) = 0

$$\Rightarrow \lambda(lp+mq)+(ap+bq)=0$$

$$\Rightarrow \lambda=-\left(\frac{ap+bq}{lp+mq}\right)$$

From (4), the equation of AD is $(ax+by+c)-\left(\frac{ap+bq}{lp+mq}\right)(lx+my+n)=0$

$$\Rightarrow\frac{ax+by+c}{ap+bq}=\frac{lx+my+n}{lp+mq}$$

This is the equation of the altitude through A.

So it passes through the Orthocentre of the triangle.