

3. QUADRATIC EXPRESSIONS

Sections	No. of periods (10)	Weightage in IPE [1x2 + 1x4=6]
1. Quadratic Equations	4	2 Marks
2. Quadratic Expressions	4	4 Marks
3. Quadratic Inequations	2	2 Marks

- For $a, b, c \in \mathbb{C}$, $a \neq 0$ the equation $ax^2 + bx + c = 0$ is called quadratic equation in x
- For $a, b, c \in \mathbb{R}$, $a \neq 0$ the expression $ax^2 + bx + c$ is called real quadratic expression.
- For $a, b, c \in \mathbb{R}$, $a \neq 0$, $ax^2 + bx + c > 0$, $ax^2 + bx + c \leq 0$ etc., are called quadratic inequations.

- In Physics, it is learnt that, the path of a projectile in space is a parabola and its equation takes the quadratic form $f(x) = ax^2 + bx + c$.

- If a point P divides the line segment AB internally in the ratio such that

$$\frac{AB}{AP} = \frac{AP}{PB} \text{ then } P \text{ is said to be the point of 'golden section' and the ratio is } \frac{1+\sqrt{5}}{2}.$$

Such a division is considered pleasing to the eye. The rectangle with sides having this ratio is called golden rectangle. That ratio is obtained from the roots of a quadratic equation viz., $x^2 - x - 1 = 0$.

Translation : The equation $ax^2 + bx = c$ should be multiplied by $4a$ and b^2 should be added to both sides and then square root should be extracted to get the roots.

- In section 1, roots of quadratic equation and various properties regarding these, are discussed. Condition for existence of a common root for two quadratic equations is derived.
- In section 2, some important theorems on quadratic expressions are stated and proved. Tracing the changes in the signs, determining the maxima or minima (extreme values) of quadratic expressions, determining the range (limit) of a given rational expression etc., are illustrated.

In Section-3, Quadratic inequations are discussed.

SYNOPSIS POINTS

1. The roots of $ax^2+bx+c=0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and are generally denoted by α, β

2.1. The **sum** of the roots of $ax^2+bx+c=0$ is $\alpha + \beta = \frac{-b}{a} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

2.2. The **product** of the roots of $ax^2+bx+c=0$ is $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

3. The quadratic equation with roots α, β is $(x-\alpha)(x-\beta)=0$ i.e., $x^2-x(\alpha+\beta)+\alpha\beta=0$

4. The discriminant of $ax^2+bx+c=0$ is $\Delta=b^2-4ac$

5. Two quadratic equations $a_1x^2+b_1x+c_1=0$ and $a_2x^2+b_2x+c_2=0$ have a common root

if $(a_1b_2-a_2b_1)(b_1c_2-b_2c_1)=(c_1a_2-c_2a_1)^2$. If so, the common root is $\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

6. If $f(x)=ax^2+bx+c=0$ then the equation whose roots are

(i) reciprocal to the roots of $f(x)=0$ is $f\left(\frac{1}{x}\right)=0$ (ii) multiplied by k is $f\left(\frac{x}{k}\right)=0$

(iii) changed in signs to that of $f(x)=0$ is $f(-x)=0$ (iv) greater by k is $f(x-k)=0$

7.1. If $a>0$ then ax^2+bx+c has the minimum value $\frac{4ac-b^2}{4a}$ at $x = -\frac{b}{2a}$

7.2. If $a<0$ then ax^2+bx+c has the maximum value $\frac{4ac-b^2}{4a}$ at $x = -\frac{b}{2a}$

7.3. The vertex of the parabola $y=ax^2+bx+c$ is $\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$

8. Tracing the changes in the sign of quadratic expression ax^2+bx+c

Sign of Δ	Sign of the expression	Values of x
$\Delta=b^2-4ac<0$	a and ax^2+bx+c have the same sign	$\forall x \in \mathbb{R}$
$\Delta=b^2-4ac=0$	a and ax^2+bx+c have the same sign	$\forall x \in \mathbb{R} - \{-b/2a\}$
$\Delta=b^2-4ac>0$ α, β are roots of $ax^2+bx+c=0, \alpha<\beta$	a and ax^2+bx+c have the same sign	$x<\alpha$ or $x>\beta$
	a and ax^2+bx+c have opposite signs	$\alpha<x<\beta$

9. For $a<b$, $(x-a)(x-b)<0 \Leftrightarrow x \in (a,b)$, $(x-a)(x-b) \leq 0 \Leftrightarrow x \in [a,b]$

$(x-a)(x-b)>0 \Leftrightarrow x \in (-\infty, a) \cup (b, \infty)$, $(x-a)(x-b) \geq 0 \Leftrightarrow x \in (-\infty, a] \cup [b, \infty)$

$a^2 \leq x^2 \leq b^2 \Leftrightarrow x \in [-b, -a] \cup [a, b]$

ADDITIONAL QUESTIONS WITH SOLUTIONS

- 1** If x_1, x_2 are the roots of the quadratic equation $ax^2+bx+c=0$ and $c \neq 0$, find the value of $(ax_1+b)^{-2} + (ax_2+b)^{-2}$ in terms of a, b, c .

Sol: Given that x_1, x_2 are the roots of the equation $ax^2+bx+c=0$

$$\text{Sum of the roots } x_1 + x_2 = \frac{-b}{a}, \text{ Product of the roots } x_1 x_2 = \frac{c}{a}$$

$$x_1 \text{ is a root of } ax^2+bx+c=0 \Rightarrow ax_1^2 + bx_1 + c = 0 \Rightarrow x_1(ax_1 + b) = -c \Rightarrow ax_1 + b = -\frac{c}{x_1}$$

$$\therefore (ax_1 + b)^{-2} = \left(-\frac{c}{x_1}\right)^{-2} = \left(-\frac{x_1}{c}\right)^2 = \frac{x_1^2}{c^2}$$

$$\text{Similarly, we get } (ax_2 + b)^{-2} = \frac{x_2^2}{c^2}$$

$$\therefore (ax_1 + b)^{-2} + (ax_2 + b)^{-2} = \frac{x_1^2}{c^2} + \frac{x_2^2}{c^2} = \frac{x_1^2 + x_2^2}{c^2} = \frac{(x_1 + x_2)^2 - 2x_1 x_2}{c^2} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{c^2} = \frac{b^2 - 2ac}{a^2 c^2}$$

- 2** If α and β are the roots of $x^2+px+q=0$, form a quadratic equation whose roots are $(\alpha-\beta)^2$ and $(\alpha+\beta)^2$.

Sol: Given that α, β are the roots of the equation $x^2+px+q=0 \Rightarrow \alpha + \beta = -p$ and $\alpha\beta = q$

$$\text{Now sum of roots } (\alpha - \beta)^2 + (\alpha + \beta)^2 = 2(\alpha^2 + \beta^2) = 2[(\alpha + \beta)^2 - 2\alpha\beta] = 2[p^2 - 2q]$$

$$\text{Product of roots } (\alpha - \beta)^2 (\alpha + \beta)^2 = [(\alpha + \beta)^2 - 4\alpha\beta](\alpha + \beta)^2 = (p^2 - 4q)(p^2)$$

\therefore The required equation = $x^2 - (\text{Sum of the roots})x + (\text{Product of the roots}) = 0$

$$\Rightarrow x^2 - 2(p^2 - 2q)x + p^2(p^2 - 4q) = 0$$

- 3** If α, β are the roots of the quadratic equation of $ax^2 + bx + c = 0$ form a quadratic equation whose roots are $\alpha^2 + \beta^2$ and $\alpha^{-2} + \beta^{-2}$.

Sol: Given that α, β are the roots of $ax^2+bx+c=0$ then $\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$$

$$\alpha^{-2} + \beta^{-2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2} = \frac{\frac{b^2 - 2ac}{a^2}}{\frac{c^2}{a^2}} = \frac{b^2 - 2ac}{c^2}$$

The required equation = $x^2 - (\text{Sum of the roots})x + (\text{Product of the roots}) = 0$

$$\Rightarrow x^2 - [(\alpha^2 + \beta^2) + (\alpha^{-2} + \beta^{-2})]x + (\alpha^2 + \beta^2)(\alpha^{-2} + \beta^{-2}) = 0$$

$$\Rightarrow x^2 - \left[\left(\frac{b^2 - 2ac}{a^2} \right) + \left(\frac{b^2 - 2ac}{c^2} \right) \right]x + \left(\frac{b^2 - 2ac}{a^2} \right) \left(\frac{b^2 - 2ac}{c^2} \right) = 0$$

$$\Rightarrow x^2 - (b^2 - 2ac) \left[\frac{c^2 + a^2}{c^2 a^2} \right]x + \frac{(b^2 - 2ac)^2}{a^2 c^2} = 0$$

$$\Rightarrow a^2 c^2 x^2 - (b^2 - 2ac)(c^2 + a^2)x + (b^2 - 2ac)^2 = 0$$

4

Let α and β be the roots of the quadratic equation $ax^2 + bx + c = 0$. If $c \neq 0$, then form the quadratic equation whose roots are $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$.

EAM Q

Sol: Given that α, β are the roots of $ax^2 + bx + c = 0$ then $\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

$$\text{Sum of the roots } \frac{1-\alpha}{\alpha} + \frac{1-\beta}{\beta} = \frac{\beta(1-\alpha) + \alpha(1-\beta)}{\alpha\beta} = \frac{\alpha + \beta - 2\alpha\beta}{\alpha\beta} = \frac{\frac{-b}{a} - \frac{2c}{a}}{\frac{c}{a}} = \frac{-b-2c}{c}$$

$$\text{Product of the roots } \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{1-\beta}{\beta} \right) = \frac{1 - (\alpha + \beta) + \alpha\beta}{\alpha\beta} = \frac{1 + \frac{b}{a} + \frac{c}{a}}{\frac{c}{a}} = \frac{a+b+c}{c}$$

The required equation = $x^2 - (\text{Sum of the roots})x + (\text{Product of the roots}) = 0$

$$= x^2 - \left(\frac{-b-2c}{c} \right)x + \left(\frac{a+b+c}{c} \right) = 0 \Rightarrow cx^2 + (b+2c)x + (a+b+c) = 0$$

EQUATIONS REDUCIBLE TO QUADRATIC EQUATION

5 Solve $\sqrt{\frac{x}{x-3}} + \sqrt{\frac{x-3}{x}} = \frac{5}{2}$, when $x \neq 3$ and $x \neq 0$.

Sol: Put $\sqrt{\frac{x}{x-3}} = t \Rightarrow t + \frac{1}{t} = \frac{5}{2} \Rightarrow \frac{t^2 + 1}{t} = \frac{5}{2} \Rightarrow 2t^2 + 2 = 5t$

$$\Rightarrow 2t^2 - 5t + 2 = 0 \Rightarrow 2t^2 - 4t - t + 2 = 0$$

$$\Rightarrow 2t(t-2) - 1(t-2) = 0 \Rightarrow (2t-1)(t-2) = 0$$

$$\Rightarrow 2t-1=0 \text{ or } t-2=0 \Rightarrow t = \frac{1}{2} \text{ or } 2$$

If $t=2$ then $\sqrt{\frac{x}{x-3}} = 2 \Rightarrow \frac{x}{x-3} = 4 \Rightarrow x = 4x - 12 \Rightarrow 3x - 12 = 0 \Rightarrow x = 4$

If $t = \frac{1}{2}$ then $\sqrt{\frac{x}{x-3}} = \frac{1}{2} \Rightarrow \frac{x}{x-3} = \frac{1}{4} \Rightarrow 4x = x - 3 \Rightarrow 3x = -3 \Rightarrow x = -1$

\therefore The solution set is $\{4, -1\}$.

6 Solve $\sqrt{\frac{3x}{x+1}} + \sqrt{\frac{x+1}{3x}} = 2$, when $x \neq -1$ and $x \neq 0$.

EAM Q

Sol: Put $\sqrt{\frac{3x}{x+1}} = t \Rightarrow t + \frac{1}{t} = 2 \Rightarrow \frac{t^2 + 1}{t} = 2$

$$\Rightarrow t^2 + 1 = 2t \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0 \Rightarrow t-1=0 \Rightarrow t=1$$

If $t=1$ then $\sqrt{\frac{3x}{x+1}} = 1 \Rightarrow \frac{3x}{x+1} = 1 \Rightarrow 3x = x+1 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

\therefore The solution set is $\{1/2\}$.

7 Solve $2\left(x + \frac{1}{x}\right)^2 - 7\left(x + \frac{1}{x}\right) + 5 = 0$ when $x \neq 0$

Sol: Put $x + \frac{1}{x} = t$ then we get $2t^2 - 7t + 5 = 0 \Rightarrow (2t-5)(t-1) = 0 \Rightarrow t = \frac{5}{2}, 1$

Case (i): $t = \frac{5}{2} \Rightarrow x + \frac{1}{x} = \frac{5}{2} \Rightarrow \frac{x^2 + 1}{x} = \frac{5}{2} \Rightarrow 2x^2 + 2 = 5x \Rightarrow 2x^2 - 5x + 2 = 0$

$$\Rightarrow (x-2)(2x-1) = 0 \Rightarrow x = 2 \text{ or } 1/2$$

Case (ii): $t=1 \Rightarrow x + \frac{1}{x} = 1 \Rightarrow \frac{x^2 + 1}{x} = 1 \Rightarrow x^2 + 1 = x \Rightarrow x^2 - x + 1 = 0$

$$\Rightarrow x = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{3}i}{2} \quad \therefore \text{The roots are } 2, \frac{1}{2}, \frac{1 \pm \sqrt{3}i}{2}$$

8 Solve $\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) + 6 = 0$, when $x \neq 0$

EAM Q

Sol: Put $x + \frac{1}{x} = t \Rightarrow \left(x + \frac{1}{x}\right)^2 = t^2 \Rightarrow x^2 + \frac{1}{x^2} + 2x \cdot \frac{1}{x} = t^2 \Rightarrow x^2 + \frac{1}{x^2} = t^2 - 2$

Then the given equation becomes $(t^2 - 2) - 5t + 6 = 0 \Rightarrow t^2 - 5t + 4 = 0$
 $\Rightarrow t^2 - 4t - t + 4 = 0 \Rightarrow t(t - 4) - 1(t - 4) = 0 \Rightarrow (t - 1)(t - 4) = 0 \Rightarrow t = 1$ or 4

Case (i): $t = 1 \Rightarrow x + \frac{1}{x} = 1 \Rightarrow \frac{x^2 + 1}{x} = 1 \Rightarrow x^2 + 1 = x$
 $\Rightarrow x^2 - x + 1 = 0 \Rightarrow x = \frac{1 + \sqrt{1 - 4}}{2} = \frac{1 \pm \sqrt{3}i}{2}$

Case (ii): $t = 4 \Rightarrow x + \frac{1}{x} = 4 \Rightarrow \frac{x^2 + 1}{x} = 4 \Rightarrow x^2 + 1 = 4x \Rightarrow x^2 - 4x + 1 = 0$

$\Rightarrow x = \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$ \therefore The roots are $2 \pm \sqrt{3}, \frac{1 \pm \sqrt{3}i}{2}$

9 Solve $3^{1+x} + 3^{1-x} = 10$

Sol: The given equation can be written as $3 \cdot 3^x + \frac{3}{3^x} = 10$

Put $3^x = t$, then we get $3t + \frac{3}{t} = 10 \Rightarrow 3t^2 + 3 = 10t$

$\Rightarrow 3t^2 - 10t + 3 = 0 \Rightarrow 3t^2 - 9t - t + 3 = 0 \Rightarrow 3t(t - 3) - 1(t - 3) = 0$

$\Rightarrow (t - 3)(3t - 1) = 0 \Rightarrow t - 3 = 0$ or $3t - 1 = 0 \Rightarrow t = 3$ or $t = \frac{1}{3}$

If $t = 3$ then $3^x = 3^1 \Rightarrow x = 1$

If $t = \frac{1}{3}$ then $3^x = 3^{-1} \Rightarrow x = -1$ \therefore The roots are $1, -1$.

10 Solve $4^{x-1} - 3 \cdot 2^{x-1} + 2 = 0$

Sol: The given equation can be written as $(2^{x-1})^2 - 3 \cdot 2^{x-1} + 2 = 0$

Put $2^{x-1} = t$, then we get $t^2 - 3t + 2 = 0$

$\Rightarrow t^2 - 2t - t + 2 = 0 \Rightarrow t(t - 2) - 1(t - 2) = 0$

$\Rightarrow (t - 1)(t - 2) = 0 \Rightarrow t = 1$ or 2

If $t = 1$ then $2^{x-1} = 1 = 2^0 \Rightarrow x - 1 = 0 \Rightarrow x = 1$

If $t = 2$ then $2^{x-1} = 2 = 2^1 \Rightarrow x - 1 = 1 \Rightarrow x = 2$

\therefore The roots are $2, 1$.

11 Solve $7^{1+x} + 7^{1-x} = 50$ for real x .

Sol: The given equation can be written as $7 \cdot 7^x + \frac{7}{7^x} - 50 = 0$

$$\Rightarrow 7 \cdot 7^x \cdot 7^x + 7 - 50 \cdot 7^x = 0 \Rightarrow 7 \cdot 7^{2x} - 50 \cdot 7^x + 7 = 0$$

Put $7^x = t$, then we get $7t^2 - 50t + 7 = 0$

$$\Rightarrow 7t^2 - 49t - t + 7 = 0 \Rightarrow 7t(t-7) - (t-7) = 0 \Rightarrow (7t-1)(t-7) = 0 \Rightarrow t=7, 1/7$$

If $t=7$ then $7^x = 7 \Rightarrow x = 1$.

If $t = 1/7$ then $7^x = 7^{-1} \Rightarrow x = -1$

\therefore The roots are $-1, 1$.

APPLICATIONS ON QUADRATIC EQUATIONS

12 Prove that there is a unique pair of consecutive positive odd integers such that the sum of their squares is 290 and find it.

Sol: Let the pair of consecutive positive odd integers be $x, x+2$.

$$\therefore x^2 + (x+2)^2 = 290 \dots\dots(1)$$

$$\Rightarrow x^2 + (x^2 + 4x + 4) = 290 \Rightarrow 2x^2 + 4x - 286 = 0 \Rightarrow x^2 + 2x - 143 = 0 \Rightarrow x^2 + 13x - 11x - 143 = 0$$

$$\Rightarrow x(x+13) - 11(x+13) = 0 \Rightarrow (x-11)(x+13) = 0 \Rightarrow x = 11, -13$$

Here 11 is the only positive odd integer satisfying equation(1)

\therefore The unique pair of integers is $\{11, 13\}$

13 The cost of a piece of cable wire is Rs. 35/-. If the length of the piece of wire is 4 meters more and each meter costs Rs. 1/- less, the cost would remain unchanged. What is the length of the wire?

Sol: Let the length of the piece of wire be ' l ' meters and the cost of each meter be Rs. x /-

From the initial condition $lx = 35 \dots\dots\dots(1)$

Also $(l+4)(x-1) = 35$

$$\Rightarrow lx - l + 4x - 4 = 35 \dots\dots\dots(2)$$

$$\text{From (1) and (2); } 35 - l + 4x - 4 = 35 \Rightarrow 4x = l + 4 \Rightarrow x = \frac{l+4}{4}$$

On substituting this value of ' x ' in (1), we get

$$l \left(\frac{l+4}{4} \right) = 35 \Rightarrow l(l+4) = 35(4) \Rightarrow l^2 + 4l - 140 = 0 \Rightarrow l^2 + 14l - 10l - 140 = 0$$

$$\Rightarrow l(l+14) - 10(l+14) = 0 \Rightarrow (l+14)(l-10) = 0 \Rightarrow l = -14 \text{ and } l = 10$$

But the length cannot be negative. So $l=10$.

\therefore The length of the piece of wire is $l=10$ m.

- 14** One fourth of a herd of goats was seen in the forest. Twice the square root of the number in the herd had gone up the hill and the remaining 15 goats were on the bank of the river. Find the total number of goats.

Sol: Let the number of goats in the herd be 'x'
 The number of goats seen in the forest = $\frac{x}{4}$,
 The number of goats gone upto the hill = $2\sqrt{x}$
 The number of goats on the bank of a river = 15

$$\therefore \frac{x}{4} + 2\sqrt{x} + 15 = x \Rightarrow x + 8\sqrt{x} + 60 = 4x \Rightarrow 3x - 8\sqrt{x} - 60 = 0$$

$$\text{Put } \sqrt{x} = y \Rightarrow 3y^2 - 8y - 60 = 0 \Rightarrow 3y^2 - 18y + 10y - 60 = 0 \Rightarrow 3y(y - 6) + 10(y - 6) = 0$$

$$\Rightarrow (3y + 10)(y - 6) = 0 \Rightarrow y = 6, -\frac{10}{3}$$

$$\text{But } y \text{ cannot be negative. So } y = 6 \Rightarrow \sqrt{x} = 6 \Rightarrow x = 36$$

$$\therefore \text{Total number of goats} = 36$$

- 15** In a cricket match Anil took one wicket less than twice the number of wickets taken by Ravi. If the product of the number of wickets taken by them is 15. Find the number of wickets taken by each of them.

Sol: Let the number of wickets taken by Anil and Ravi be x and y respectively.

$$\text{Then } x = 2y - 1 \dots\dots (1) \text{ and } xy = 15 \dots\dots (2)$$

$$\text{From (1) and (2), } (2y - 1)y = 15 \Rightarrow 2y^2 - y - 15 = 0 \Rightarrow 2y^2 - 6y + 5y - 15 = 0$$

$$\Rightarrow 2y(y - 3) + 5(y - 3) = 0 \Rightarrow (y - 3)(2y + 5) = 0 \Rightarrow y = 3, -5/2$$

Since the number of wickets cannot be negative, we have $y = 3$.

$$\text{From (2), we get } 3x = 15 \Rightarrow x = 5$$

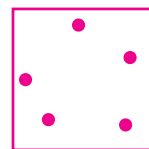
\therefore The wickets taken by Anil and Ravi are 5 and 3 respectively.

- 16** Some points on a plane are marked and they are connected pairwise by line segments. If the total number of line segments formed is 10, find the number of marked points on the plane.

Sol: Let the number of points marked on the plane be 'n'.

Since each point is joined to the remaining (n-1) points,

the number of line segments having a given point as an end point is (n-1).



Hence the total number of line segments formed = $n(n-1)$.

But in this counting each line segment is counted exactly twice at each of its end points.

Hence the total number of line segments actually formed = $\frac{n(n-1)}{2}$

\therefore By hypothesis $\frac{n(n-1)}{2} = 10$

$$\Rightarrow n^2 - n - 20 = 0 \Rightarrow n^2 - 5n + 4n - 20 = 0 \Rightarrow n(n-5) + 4(n-5) = 0 \Rightarrow (n-5)(n+4) = 0$$

The roots of the equation $(n-5)(n+4) = 0$ are -4 and 5 .

But the number of wickets n cannot be negative. So $n=5$.

\therefore The number of points marked on the plane is 5 .

STAR-Q

PRACTICE EXERCISE

- Solve the following inequations by algebraic method.
 (i) $15x^2+4x-4 \leq 0$ (ii) $x^2-2x+1 < 0$ (iii) $2-3x-2x^2 \geq 0$ (iv) $x^2-4x-21 \geq 0$
- Solve the following inequations by graphical method.
 (i) $x^2-7x+6 > 0$ (ii) $4-x^2 > 0$ (iii) $15x^2+4x-4 \leq 0$ (iv) $x^2-4x-21 \geq 0$
- Find the set of values of x for which the inequalities $x^2-3x-10 < 0$, $10x-x^2-16 > 0$ hold simultaneously.
- Solve the following inequations
 - $\sqrt{3x-8} < -2$
 - $\sqrt{-x^2+6x-5} > (8-2x)$
 - $\sqrt{x^2-3x-10} > (8-x)$
- Solve the inequation $\sqrt{x+2} > \sqrt{8-x^2}$
 (*Hint: Solve $(x+2) > (8-x^2)$ and $8-x^2 \geq 0$*)
- Solve the inequation $\sqrt{(x-3)(2-x)} < \sqrt{4x^2+12x+11}$
 (*Hint: Solve $(x-3)(2-x) \geq 0$ and $4x^2+12x+11 > (x-3)(2-x)$*)
- Solve the inequation $\frac{\sqrt{6+x-x^2}}{2x+5} \geq \frac{\sqrt{6+x-x^2}}{x+4}$
 (*Hint: Solve $\frac{1}{2x+5} - \frac{1}{x+4} \geq 0$ and $6+x-x^2 \geq 0$*)

ANSWERS

- (i) $-2/3 \leq x \leq 2/5$ (ii) ϕ (iii) $-2 \leq x \leq 1/2$ (iv) $\{x/x \in (-\infty, -3] \cup [7, \infty)\}$
- (i) The values of x are left to 1 and right to 6 (ii) The values of x lie between -2 & 2
 (iii) The values of x lie between $-2/3$ and $2/5$, including both
 (iv) The values of x lies left to -3 and right to 7 including both
- $\{x \in \mathbb{R}: 2 < x < 5\}$ 4. (i) ϕ (ii) $(3, 5)$ (iii) $(74/13, \infty)$ 5. $(2, 2\sqrt{2})$
- $[2, 3]$ 7. $[-2, -1] \cup \{3\}$