SOLVED MODEL PAPER-4

MATHS-2A

Time: 3 Hours Max. Marks : 75

SECTION -A

I. Answer ALL the following Very Short Answer Questions: 10 × 2=20

1. If z_1 =(3, 5), z_2 = (2, 6) find $z_1 . z_2$.
2. Express 1–i in modulas - amplitude form.
3. Find the value of $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5-\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$
4. For what values of x, the expression $15+4x-3x^2$ is negative?
5. If a,b,c are the roots of $x^3-px^2+qx-r=0$ then find $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$
6. Find the number of permutations that can be made by using all the letters of the word INTERMEDIATE
7. If ${}^nC_5={}^nC_6$ then find ${}^{13}C_n$
8. If the coefficients of $(2r+4)^{th}$ term and $(3r+4)^{th}$ term in the expansion of $(1+x)^{21}$ are equal, then find r.
9. Find the variance and standard deviation for the discrete data : 5,12,3,18,6,8,2,10
10. If the mean and variance of a bionomial variable X are 2.4 and 1.44 respectively, find $P(1<x\le 4)$

SECTION-B

II. Answer any FIVE of the following Short Answer Questions: 5×4=20

11. Determine the locus of z, $z\neq -2i$ such that $\text{Re}\left(\frac{z-4}{z-2i}\right)=0$
12. Solve $2x^4+x^3-11x^2+x+2=0$.
13. Find the sum of all 4-digit numbers that can be formed using the digits 0, 2, 4, 7, 8 without repetition.
14. Find the numerically greatest term in the expansion of $(2+3x)^{10}$ where x=11/8.
15. Resolve $\frac{x^2-x+1}{(x+1)(x-1)^2}$ into partial fractions.
16. A speaks truth in 75% of the cases and B in 80% of the cases.

    What is the probability that their statements about an incident do not match.
17. If A,B are events with P(A)= 0.5, P(B)=0.4 and $P(A\cap B)=0.3$, find the

    probability that i) A does not occur ii) neither A nor B occurs.

SECTION-C

III. Answer any FIVE of the following Long Answer Questions: **5×7=35**

18. If m,n are integers and x=cosα + i sinα, y =cosβ + i sinβ, then prove that

$$x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta) \text{ and } x^m y^n - \frac{1}{x^m y^n} = 2i\sin(m\alpha + n\beta)$$

19. Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$, given that $2 + i\sqrt{3}$ is a root of the equation.

20. Find the sum of the series $1 - \frac{4}{5} + \frac{4.7}{5.10} - \frac{4.7.10}{5.10.15} + ...$

21. Show that $C_0 + \frac{3}{2}.C_1 + \frac{9}{3}.C_2 + \frac{27}{4}.C_3 + ... + \frac{3^n}{n+1}.C_n = \frac{4^{n+1} - 1}{3(n+1)}$.

22. Find the variance and standard deviation of the following frequency distribution.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

23. Three boxes numbered I, II, III contains the balls as follows:

	White	Black	Red
I	1	2	3
II	2	1	1
III	4	5	3

One box is randomly selected and a ball is drawn from it. If the ball is red, then find the probability that it is from box II.

24. A cubical die is thrown. Find the mean and variance of X, giving the number on the face that shows up.

SPECIAL NOTES

SOLUTIONS

SECTION -A

1. If z_1 =(3, 5), z_2 = (2, 6) find $z_1 . z_2$.

Sol: Given that z_1 =(3, 5) = 3+ 5i and z_2 = (2, 6)= 2 + 6i

$z_1.z_2 = (3+5i).(2+6i) = 6 + 10i+18i+30i^2 = 6 + 28i + 30(-1) = -24 + 28i = (-24, 28)$

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2. Express 1–i in modulas - amplitude form.

Sol: Let $1 - i = x + iy \Rightarrow x = 1, y = -1$

$$\therefore r = \sqrt{x^2 + y^2} = \sqrt{(1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

Now, $\tan\theta = \frac{y}{x} \Rightarrow \tan\theta = \frac{-1}{1} = -1 = \tan\left(-\frac{\pi}{4}\right) \Rightarrow \theta = \frac{-\pi}{4}$ [$\because$ (1,–1) lies in Q_4]

$\therefore$ the polar form is $r(\cos\theta + i\sin\theta) = \sqrt{2}\left(\cos\left(\frac{-\pi}{4}\right) + i\sin\left(\frac{-\pi}{4}\right)\right) = \sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$

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3. Find $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 - \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$

Sol: $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 - \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^5 - \left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^5$

$= \left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) - \left(\cos\frac{5\pi}{6} - i\sin\frac{5\pi}{6}\right) = 2i\sin\frac{5\pi}{6} = 2i\sin\left(\pi - \frac{\pi}{6}\right) = 2i\sin\frac{\pi}{6} = 2i\left(\frac{1}{2}\right) = i$

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4. For what values of x, the expression $15+4x-3x^2$ is negative?

Sol: Let $f(x) = 15+4x-3x^2 = -3x^2+4x+15 \Rightarrow a = -3, b = 4, c = 15$

$\Delta = b^2 - 4ac = (4)^2 - 4(-3)(15) = 16+180 = 196 > 0$ Thus $\Delta > 0$

$\therefore$ the roots are real

Now, $-3x^2+4x+15=0 \Rightarrow 3x^2-4x-15=0 \Rightarrow 3x^2-9x+5x-15=0 \Rightarrow 3x(x-3)+5(x-3)=0$

$\Rightarrow (3x+5)(x-3)=0 \Rightarrow x = -5/3$ or 3

The coefficient of $x^2 = -3$, which is negative

$\therefore$ a, f(x) have the same sign for $x<\alpha$ or $x>\beta$ $\Rightarrow -3x^2+4x+15$ is negative for $x < -5/3$ or $x>3$

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5. If a,b,c are the roots of $x^3-px^2+qx-r=0$ then find $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$

Sol: Since a, b,c are roots of the given equation, we have $s_1=a+b+c=p, s_2=ab+bc+ca=q, s_3=abc=r$

$$\therefore \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} = \frac{a^2b^2+b^2c^2+c^2a^2}{a^2b^2c^2} = \frac{(ab+bc+ca)^2 - 2abc(a+b+c)}{(abc)^2} = \frac{q^2-2rp}{r^2}$$

6. Find the number of permutations that can be made by using all the letters of the word INTERMEDIATE

Sol: The given word INTERMEDIATE contains 12 letters in which 3 'E's are alike,2 'I's are alike, 2'T's are alike and rest are different.

$\therefore$ The number of arrangements $= \dfrac{n!}{p!q!r!} = \dfrac{12!}{3!2!2!}$

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7. If $^nC_5 = {^nC_6}$ then find $^{13}C_n$

Sol: We know $^nC_r = {^nC_s} \Rightarrow r+s=n$ (or) $r=s$ $\quad \therefore {^nC_5} = {^nC_6} \Rightarrow n = 5+6 = 11$

$$\therefore {^{13}C_n} = {^{13}C_{11}} = {^{13}C_{13-11}} = {^{13}C_2} = \frac{13\times 12}{1\times 2} = 78$$

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8. If the coefficients of $(2r+4)^{th}$ term and $(3r+4)^{th}$ term in $(1+x)^{21}$ are equal, then find r.

Sol: Coefficient of r^{th} term in $(1+x)^n$ is $^nC_{r-1}$

In $(1+x)^{21}$, the coefficient of $(2r+4)^{th}$ term= coefficient of $(3r+4)^{th}$ term

$\Rightarrow {^{21}C_{2r+3}} = {^{21}C_{3r+3}}$ $\quad \because {^nC_r} = {^nC_s} \Rightarrow r=s$ (or) $r+s=n$

$\therefore$ $2r+3 = 3r+3 \Rightarrow r = -6$. But r cannot be negative

$(2r+3)+(3r+3)=21 \Rightarrow 5r+6=21 \Rightarrow 5r=15 \Rightarrow r=3$

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9. Find the variance and standard deviation for the discrete data : 5,12,3,18,6,8,2,10

Sol: Mean of the given data, $\bar{x} = \dfrac{\sum x_i}{n} = \dfrac{5+12+3+18+6+8+2+10}{8} = \dfrac{64}{8} = 8$

The deviations of observations from the mean are

5–8= –3; 12–8=4; 3–8= –5; 18–8=10; 6–8= –2;8–8=0; 2–8= –6; 10–8=2

Hence, the absolute values of the deviations are 3, 4, 5, 10, 2, 0, 6, 2

$$\text{Variance } \sigma^2 = \frac{\Sigma(x_i-\bar{x})^2}{n} = \frac{3^2+4^2+5^2+10^2+2^2+0^2+6^2+2^2}{8}$$

$$= \frac{9+16+25+100+4+0+36+4}{8} = \frac{194}{8} = 24.25$$

Standard deviation $\sigma = \sqrt{24.25} \cong 4.95$

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10. If the mean and variance of a bionomial variable X are 2.4 and 1.44 respectively, find $P(1<x\le 4)$

Sol: Mean = np = 2.4 ....(1) Variance = npq = 1.44 ......(2)

Dividing (2) by (1), $\dfrac{npq}{np} = \dfrac{1.44}{2.4} = \dfrac{3}{5}$ $\quad \therefore q = \dfrac{3}{5} \Rightarrow p = 1-q = 1-\dfrac{3}{5} = \dfrac{2}{5}$

$(1) \Rightarrow n\left(\dfrac{2}{5}\right) = 2.4 \Rightarrow n = 2.4\left(\dfrac{5}{2}\right) = 6$ $\quad P(1<X\le 4) = P(X=2)+P(X=3)+P(X=4)$

$$= {^6C_2}.q^4.p^2 + {^6C_3}q^3.p^3 + {^6C_4}q^2.p^4 = {^6C_2}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^2 + {^6C_3}\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^3 + {^6C_4}\left(\frac{3}{5}\right)^2.\left(\frac{2}{5}\right)^4$$

$$= \frac{36}{15625}(135+120+60) = \frac{2268}{3125}$$

SECTION-B

11. Determine the locus of z, $z \neq -2i$ such that $Re\left(\frac{z-4}{z-2i}\right)=0$

Sol : $\frac{z-4}{z-2i}=\frac{(x+iy)-4}{(x+iy)-2i}$

$$=\frac{(x-4)+iy}{x+i(y-2)}=\frac{[(x-4)+iy][x-i(y-2)]}{[x+i(y-2)][x-i(y-2)]}=\frac{(x^2-4x+y^2-2y)+i(2x+4y-8)}{x^2+(y-2)^2}$$

$$=\frac{(x^2-4x+y^2-2y)}{x^2+(y-2)^2}+\frac{i(2x+4y-8)}{x^2+(y-2)^2}$$ Its real part $=0$

$$\Rightarrow \frac{x^2-4x+y^2-2y}{x^2+(y-2)^2}=0 \Rightarrow x^2-4x+y^2-2y=0 \quad \Rightarrow x^2+y^2-4x-2y=0$$

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12. Solve $2x^4 + x^3 - 11x^2 + x + 2 = 0$.

Sol : The degree of the given equation is n=4, which is Even. Also $a_k=a_{n-k}$ $\forall$ k=0,1,2,3,4

Hence the given equation is a reciprocal equation of class I of even degree, which is a S.R.E

Now, dividing the equation by x^2, we get $2x^2+x-11+\frac{1}{x}+\frac{2}{x^2}=0$

$$\Rightarrow 2\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)-11=0 \quad ....(1)$$

Put $x+\frac{1}{x}=y$ then $x^2+\frac{1}{x^2}=y^2-2$

$(1) \Rightarrow 2(y^2-2)+y-11=0 \Rightarrow 2y^2+y-15=0$

$\Rightarrow 2y^2+6y-5y-15=0 \Rightarrow 2y(y+3)-5(y+3)=0 \Rightarrow (2y-5)(y+3)=0$

$\Rightarrow 2y-5=0$ (or) $y+3=0 \Rightarrow y=\frac{5}{2}$ (or) $y=-3$

If $y=\frac{5}{2}$ then $x+\frac{1}{x}=\frac{5}{2}=2\frac{1}{2}=2+\frac{1}{2} \Rightarrow x=2$ or $\frac{1}{2}$

Now, $\mathbf{y}=-3 \Rightarrow \mathbf{x}+\frac{1}{\mathbf{x}}=-3 \Rightarrow \mathbf{x}^2+1=-3\mathbf{x} \Rightarrow \mathbf{x}^2+3\mathbf{x}+1=0$

$$\Rightarrow x=\frac{-3\pm\sqrt{(-3)^2-4.1.1}}{2.1}=\frac{-3\pm\sqrt{9-4}}{2}=\frac{-3\pm\sqrt{5}}{2}$$

$\therefore$ the roots of the given equation are $2,\frac{1}{2},\frac{-3+\sqrt{5}}{2},\frac{-3-\sqrt{5}}{2}$

13. Find the sum of all 4-digit numbers that can be formed using the digits 0, 2, 4, 7, 8 without repetition.

Sol : If zero is one among the given n digits, then the sum of the r-digited numbers that can be formed using the given 'n' distinct digits $(r \le n \le 9)$ is

$^{(n-1)}P_{(r-1)} \times$ Sum of the digits $\times 111.......1$ (r times) $- {}^{(n-2)}P_{(r-2)} \times$ Sum of the digits $\times 111.......1$ [(r–1) times]

Hence n=5, r=4, digits are (0,2,4,7,8) Hence the sum of all 4 digited numbers that can be formed using the digits {0,2,4,78} without repetition is

$$= {}^{(5-1)}P_{(4-1)} \times (0+2+4+7+8) \times (1111) - {}^{(5-2)}P_{(4-2)} \times (0+2+4+7+8) \times (111)$$

$$= {}^{4}P_3(21) \times 1111 - {}^{3}P_2(21)(111) = 24 \times 21 \times 1111 - 6(21)(111)$$

$$= 21(26664) - 21(666) = 21(26664 - 666) = 21\ (25998) = 5{,}45{,}958$$

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14. Find the numerically greatest term in the expansion of $(2+3x)^{10}$ where x=11/8.

Sol: We have $(2+3x)^{10} = (2)^{10}\left(1+\frac{3x}{2}\right)^{10}$

Comparing $\left(1+\frac{3x}{2}\right)^{10}$ with $(1+X)^n$, we get n=10 and $|X| = \left|\frac{3x}{2}\right| = \left|\frac{3}{2}.\frac{11}{8}\right| = \frac{33}{16}$

$$\therefore \frac{(n+1)|X|}{|X|+1} = \frac{(10+1)\frac{33}{16}}{\frac{33}{16}+1} = \frac{11.\frac{33}{16}}{\frac{49}{16}} = \frac{363}{49} = \frac{343+20}{49} = \frac{7(49)+20}{49} = 7 + \frac{20}{49} = 7\frac{20}{49}$$

The integral part of the above number is 7. $\therefore T_{7+1}$ is the numerically greatest term in $(2+3x)^{10}$.

$$\therefore T_8 = T_{7+1} = {}^{10}C_7 . 2^{10-7}(3x)^7 = {}^{10}C_7 . 2^3\left(3.\frac{11}{8}\right)^7 = 8.\ {}^{10}C_7\left(\frac{33}{8}\right)^7$$

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15. Resolve $\frac{x^2-x+1}{(x+1)(x-1)^2}$ into partial fractions.

Sol: Let $\frac{x^2-x+1}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

$$\therefore \frac{x^2-x+1}{(x+1)(x-1)^2} = \frac{A(x-1)^2 + B(x+1)(x-1) + C(x+1)}{(x+1)(x-1)^2}$$

$\Rightarrow A(x-1)^2 + B(x+1)(x-1) + C(x+1) = x^2 - x + 1$ ....(1)

Putting x = 1 in (1), we get $1 = A(1-1)^2 + B(2)(0) + C(1+1) \Rightarrow 2C = 1 \Rightarrow C = 1/2$

Putting x = –1 in (1), we get $= A(1-(-1))^2 + B(-1+1)(-1-1) + C(-1+1) = 3 \Rightarrow 4A = 3 \Rightarrow A = 3/4$

Equating the coefficients of x^2, we get A+B=1 $\Rightarrow B = 1 - A = 1 - \frac{3}{4} = \frac{1}{4}$

$$\therefore \frac{x^2-x+1}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{3}{4(x+1)} + \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}$$

16. A speaks truth in 75% of the cases and B in 80% of the cases. What is the probability that their statements about an incident do not match.

Sol: Let A,B denote the events of speaking truth by A,B respectively

$$P(A)=\frac{75}{100}=\frac{3}{4};\ P(B)=\frac{80}{100}=\frac{4}{5}$$

$$\therefore P(\bar{A})=1-P(A)=1-\frac{3}{4}=\frac{1}{4};\quad P(\bar{B})=1-P(B)=1-\frac{4}{5}=\frac{1}{5}$$

Let E be the event that A and B contradict to each other

$$\Rightarrow P(E)=P\left[(A\cap\bar{B})\cup(\bar{A}\cap B)\right]=P(A\cap\bar{B})+P(\bar{A}\cap B)$$

$$=P(A)P(\bar{B})+P(\bar{A})P(B)\quad [\because A,B \text{ are independent}]$$

$$=\frac{3}{4}\times\frac{1}{5}+\frac{1}{4}\times\frac{4}{5}=\frac{7}{20}$$

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17. If A,B are events with P(A)= 0.5, P(B)=0.4 and P(A∩B)=0.3, find the probability that i) A does not occur ii) neither A nor B occurs.

Sol: Given P(A)= 0.5, P(B)=0.4 and P(A∩B)=0.3,

(i) $P(\bar{A})=1-P(A)=1-0.5=0.5$

(ii) $P(\bar{A}\cap\bar{B})=P(\overline{A\cup B})=1-P(A\cup B)=1-[P(A)+P(B)-P(A\cap B)]$

$= 1-(0.5+0.4-0.3) = 1-0.6 = 0.4$

SECTION-C

18. If m,n are integers and x=cosα + i sinα, y =cosβ + i sinβ, then prove that $x^m y^n+\frac{1}{x^m y^n}=2\cos(m\alpha+n\beta)$ **and** $x^m y^n-\frac{1}{x^m y^n}=2i\sin(m\alpha+n\beta)$

Sol: Given $x=\cos\alpha+i\sin\alpha,\ y=\cos\beta+i\sin\beta$

$$\Rightarrow x^m=(\cos\alpha+i\sin\alpha)^m=\cos m\alpha+i\sin m\alpha$$

$$y^n=(\cos\beta+i\sin\beta)^n=\cos n\beta+i\sin n\beta$$

$$\therefore x^m y^n=(\cos m\alpha+i\sin m\alpha)(\cos n\beta+i\sin n\beta)=(\text{cis}m\alpha)(\text{cis}m\beta)=\text{cis}(m\alpha+n\beta)$$

$$=\cos(m\alpha+n\beta)+i\sin(m\alpha+n\beta)\ ....(1)$$

Hence $\frac{1}{x^m y^n}=\cos(m\alpha+n\beta)-i\sin(m\alpha+n\beta)$ ....(2)

By adding (1) and (2), we get, $x^m y^n+\frac{1}{x^m y^n}=2\cos(m\alpha+n\beta)$

By subtracting (2) from (1), we get, $x^m y^n-\frac{1}{x^m y^n}=2i\sin(m\alpha+n\beta)$

19. Solve the equation $x^4-4x^2+8x+35=0$, given that $2+i\sqrt{3}$ is a root of the equation.

Sol: Let $f(x)=x^4-4x^2+8x+35$

$2+i\sqrt{3}$ is a root of $f(x)=0 \Rightarrow 2-i\sqrt{3}$ is also a root of $f(x)=0$

The sum of roots $(2+i\sqrt{3})+(2-i\sqrt{3})=4$ and product of roots $(2+i\sqrt{3})(2-i\sqrt{3})=4+3=7$

$\therefore$ the equation with roots $2+\sqrt{3}, 2-\sqrt{3}$ is x^2–(sum of roots)x + product=0 $\Rightarrow x^2-4x+7=0$

$\Rightarrow x^2-4x+7$ is a factor of f(x)

	1	0	–4	8	35
4	0	4	16	20	0
–7	0	0	–7	–28	–35
	1	4	5	0	0

Now, $x^2+4x+5=0 \Rightarrow x=\dfrac{-4\pm\sqrt{16-20}}{2}=\dfrac{-4\pm\sqrt{-4}}{2}=\dfrac{-4\pm 2i}{2}=-2\pm i$

$\therefore$ the roots of the given equation are $2+i\sqrt{3},\ 2-i\sqrt{3},-2+i,-2-i$

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20. Find the sum of the series $1-\frac{4}{5}+\frac{4.7}{5.10}-\frac{4.7.10}{5.10.15}+.....$

Sol: Let $S=1-\dfrac{4}{5}+\dfrac{4.7}{5.10}-\dfrac{4.7.10}{5.10.15}+.....=1-\dfrac{4}{1}\left(\dfrac{1}{5}\right)+\dfrac{4.7}{1.2}\left(\dfrac{1}{5}\right)^2-\dfrac{4.7.10}{1.2.3}\left(\dfrac{1}{5}\right)^3+.....$

Now comparing the above series with

$$1-\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2-\frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3+....=(1+x)^{-\frac{p}{q}}$$

we get p=4, p+q=7 $\Rightarrow$4+q=7 $\Rightarrow$ q=3 Also, we have $\dfrac{x}{q}=\dfrac{1}{5}\Rightarrow x=\dfrac{q}{5}=\dfrac{3}{5}$

$$\therefore\ S=(1+x)^{\frac{-p}{q}}=\left(1+\frac{3}{5}\right)^{\frac{-4}{3}}=\left(\frac{8}{5}\right)^{\frac{-4}{3}}=\left(\frac{5}{8}\right)^{\frac{4}{3}}=\frac{5^{4/3}}{8^{4/3}}=\frac{\sqrt[3]{5^4}}{(2^3)^{4/3}}=\frac{\sqrt[3]{625}}{2^4}=\frac{\sqrt[3]{625}}{16}$$

21. Show that $C_0+\frac{3}{2}.C_1+\frac{9}{3}.C_2+\frac{27}{4}.C_3+............+\frac{3^n}{n+1}.C_n=\frac{4^{n+1}-1}{3(n+1)}$.

Sol: $S=C_0+C_1.\frac{3}{2}+C_2.\frac{3^2}{2}+C_3.\frac{3^3}{4}+............+C_n.\frac{3^n}{n+1}$...............(1)

$\Rightarrow 3S=C_0.3+C_1.\frac{3^2}{2}+C_2.\frac{3^3}{2}+C_3.\frac{3^4}{4}+............+C_n.\frac{3^{n+1}}{n+1}$...............(2)

$\Rightarrow (n+1)3S=(n+1)C_0.3+(n+1)C_1.\frac{3^2}{2}+(n+1)C_2.\frac{3^3}{2}+(n+1)C_3.\frac{3^4}{4}+............+(n+1)C_n.\frac{3^{n+1}}{n+1}$

$=({}^{n+1}C_1).3+({}^{n+1}C_2).3^2+({}^{n+1}C_3).3^3+.........+{}^{(n+1)}C_{n+1}.3^{n+1}\quad\left(\because\left(\frac{n+1}{r+1}\right).{}^nC_r={}^{(n+1)}C_{r+1}\right)$

$=(1+3)^{n+1}-1=4^{n+1}-1\quad(\because {}^nC_1x+{}^nC_2x^2+.....+{}^nC_nx^n=(1+x)^n-1)$

$\therefore S=\frac{4^{n+1}-1}{3(n+1)}$

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22. Find the variance and standard deviation of the following frequency distribution.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Sol: Here N= Σf_i=3+5+9+5+4+3+1= 30

Also Σf_ix_i=4(3)+8(5) + 11(9)+ 17(5) + 20(4)+24(3)+32(1)= 420

$\therefore \bar{x}=\frac{\Sigma f_ix_i}{N}=\frac{420}{30}=14$

x_i	f_i	f_ix_i	$x_i-\bar{x}$	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
4	3	12	–10	100	300
8	5	40	–6	36	180
11	9	99	–3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	15	324	324
		Σf_ix_i=420			$\Sigma f_i(x_i-\bar{x})^2$=1374

Variance $(\sigma^2)=\frac{1}{N}\Sigma f_i(x_i-\bar{x})^2=\frac{1}{30}(1374)=45.8$

Standard Deviation $\sigma=\sqrt{45.8}=6.77$

23. Three boxes numbered I, II, III contains the balls as follows:

	White	Black	Red
I	1	2	3
II	2	1	1
III	4	5	3

One box is randomly selected and a ball is drawn from it. If the ball is red, then find the probability that it is from box II.

Sol: Let B_1, B_2, B_3 be the events of selecting boxes B_1, B_2, B_3 and R be the event of getting drawing a red ball

$\therefore P(B_1)=1/3, P(B_2)=1/3, P(B_3)=1/3,$

$$P\left(\frac{R}{B_1}\right)=\frac{3}{6}=\frac{1}{2}, P\left(\frac{R}{B_2}\right)=\frac{1}{4} \text{ and } P\left(\frac{R}{B_3}\right)=\frac{3}{12}=\frac{1}{4}$$

$\therefore$ by Baye's theorem, the required probability is

$$P\left(\frac{B_2}{R}\right)=\frac{P(B_2)P\left(\frac{R}{B_2}\right)}{P(B_1)P\left(\frac{R}{B_1}\right)+P(B_2)P\left(\frac{R}{B_2}\right)+P(B_3)P\left(\frac{R}{B_3}\right)}$$

$$=\frac{\frac{1}{3}\times\frac{1}{4}}{\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{4}}=\frac{\frac{1}{4}}{\frac{1}{2}+\frac{1}{4}+\frac{1}{4}}=\frac{1}{4}$$

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24. A cubical die is thrown. Find the mean and variance of X, giving the number on the face that shows up.

Sol: Let S be the sample space of throwing a die and X be the random variable.

Then P(X) is given by the following table.

$X=x_i$	1	2	3	4	5	6
$P(X=x_i)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Mean of X is $\mu=\sum_{i=1}^{6} X_i.P(X=x_i)$

$$=1.\frac{1}{6}+2.\frac{1}{6}+3.\frac{1}{6}+4.\frac{1}{6}+5.\frac{1}{6}+6.\frac{1}{6}=\frac{1}{6}(1+2+3+4+5+6)=\frac{21}{6}=\frac{7}{2}$$

Variance of X is $\sigma^2=\sum_{i=1}^{6} x_i^2.P(X=x_i)-\mu^2$

$$=1^2.\frac{1}{6}+2^2.\frac{1}{6}+3^2.\frac{1}{6}+4^2.\frac{1}{6}+5^2.\frac{1}{6}+6^2.\frac{1}{6}-\left(\frac{7}{2}\right)^2$$

$$=\frac{1}{6}(1+4+9+16+25+36)-\frac{49}{4}=\frac{91}{6}-\frac{49}{4}=\frac{182-147}{12}=\frac{35}{12}$$