SOLVED MODEL PAPER-2

MATHS-2A

Time : 3 Hours Max.Marks : 75

SECTION -A

I. Answer ALL the following Very Short Answer Questions: 10 × 2 = 20

1. Find the real and imaginary parts of the complex number $\frac{a+ib}{a-ib}$
2. Write the conjugate of (3+4i)(2–3i)
3. If A,B,C are angles of a triangle, x=cisA, y=cisB, Z=cisC, then find the value of xyz.
4. For what values of m, $x^2+(m+3)x+(m+6)=0$ will have equal roots?
5. If 1,1,α are the roots of $x^3-6x^2+9x-4=0$ then find α.
6. Find the number of diagonals of a polygon with 12 sides.
7. If ${}^{(n+1)}P_5 : {}^{n}P_6 = 2:7$ find n.
8. Find the term independent of x in $\left(\frac{3}{\sqrt[3]{x}}+5\sqrt{x}\right)^{25}$
9. Find the mean deviation about mean for the data 3,6,10,4,9,10
10. For a binomial distribution with mean 6 and variance 2.Find the first two terms of the distribution.

SECTION-B

II. Answer any FIVE of the following Short Answer Questions: 5 × 4 = 20

11. If $(x-iy)^{1/3} = a-ib$, then show that $\frac{x}{a}+\frac{y}{b}=4(a^2-b^2)$
12. If x is a real number, find the range of $\frac{2x^2-6x+5}{x^2-3x+2}$
13. If the letters of the word 'MASTER' are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word "MASTER"
14. Find the number of ways of selecting 11 member cricket team from 7 batsmen, 6 bowlers and 2 wicket keepers so that the team contains 2 wicket keepers and atleast 4 bowlers.
15. Resolve $\frac{2x+3}{5(x+2)(2x+1)}$ into partial fractions.
16. If one ticket is randomly selected from tickets numbered 1 to 30, then find the probability that the number on the ticket is (i) a multiple of 5 or 7 (ii) a multiple of 3 or 5.
17. Bag B_1 contains 4 white and 2 black balls. Bag B_2 contains 3 white and 4 black balls. A bag is drawn at random and a ball is chosen at random from it. What is the probability that the ball is white?

SECTION-C

III. Answer any FIVE of the following Long Answer Questions: **$5 \times 7 = 35$**

18. If n is a positive integer then show that $(1+i)^n+(1-i)^n=2^{(n+2)/2}\cos(n\pi/4)$

19. Solve $3x^3-26x^2+52x-24=0$, given that the roots are in geometric progression.

20. Prove that $C_0.C_r+C_1.C_{r+1}+C_2.C_{r+2}+....+C_{n-r}.C_n={}^{2n}C_{(n+r)}$ for $0\le r\le n$

    Hence deduce that (i) $C_0^2+C_1^2+C_2^2+......+C_n^2={}^{2n}C_n$

    (ii) $C_0C_1+C_1C_2+C_2C_3+.....+C_{n-1}.C_n={}^{2n}C_{n+1}$

21. If $x=\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+...\infty$, then find $3x^2+6x$

22. Find the mean deviation about the mean for the following continuous distribution:

Sales (in Rs. thousand)	40-50	50-60	60-70	70-80	80-90	90-100
Number of companies	5	15	25	30	20	5

23. State and prove addition theorem on Probability.

24. A random variable X has its range {0,1,2} and the probabilities are $P(X=0)=3c^3$, $P(X=1)=4c-10c^2$, $P(X=2)=5c-1$ where 'c' is a constant, find (i) c (ii) $P(0<x<3)$ (iii) $P(1<x\le 2)$ (iv) $P(x<1)$

SPECIAL NOTES

SOLUTIONS

SECTION -A

1. Find the real and imaginary parts of the complex number $\frac{a+ib}{a-ib}$

Sol: $\frac{a+ib}{a-ib}=\frac{(a+ib)}{(a-ib)}\frac{(a+ib)}{(a+ib)}=\frac{(a^2-b^2)+2abi}{a^2+b^2}=\frac{a^2-b^2}{a^2+b^2}+\frac{2ab}{a^2+b^2}i$

$\therefore$ Real part $=\frac{a^2-b^2}{a^2+b^2}$; Imaginary part $=\frac{2ab}{a^2+b^2}$

---

2. Write the conjugate of (3+4i)(2–3i)

Sol: (3+4i)(2–3i) =[(3)(2)–(4)(–3)]+ [4(2)+3(–3)]i = (6+12)+(8–9)i = 18–i

$\therefore$ the conjugate of 18–i is 18+i

---

3. Express the complex number $-\sqrt{3}+i$ in modulus-amplitude form.

Sol: Let $-\sqrt{3}+i=x+iy$. $\Rightarrow x=-\sqrt{3},\ y=1$

$\therefore r=\sqrt{x^2+y^2}=\sqrt{(-\sqrt{3})^2+(1)^2}=\sqrt{3+1}=\sqrt{4}=2$

$\tan\theta=\frac{y}{x}\Rightarrow\tan\theta=\left(-\frac{1}{\sqrt{3}}\right)=\tan\frac{5\pi}{6}\Rightarrow\theta=\frac{5\pi}{6}$ $[\because(-\sqrt{3},1)$ lies in $Q_2]$

$\therefore -\sqrt{3}+i$ modulus-amplitude form $r(\cos\theta+i\sin\theta)=2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

---

4. For what values of m, $x^2+(m+3)x+(m+6)=0$ will have equal roots?

Sol: The given equation is $x^2+(m+3)x+(m+6)=0$

Comparing the above equation with $ax^2+bx+c=0$ we get a=1, b= m+3, c = m+6

If the roots of the given equation are equal then $\Delta=b^2-4ac=0$

$\therefore \Delta=(m+3)^2-4(1)(m+6)=0\Rightarrow(m^2+6m+9)-4m-24=0\Rightarrow m^2+2m-15=0$

$\Rightarrow(m+5)(m-3)=0\Rightarrow m=-5,3$

---

5. If 1,1,α are the roots of $x^3-6x^2+9x-4=0$ then find α.

Sol: Comparing the given equation with $a_0x^3+a_1x^2+a_2x+a_3=0$ we get, $a_0=1$, $a_1=-6$, $a_2=9$, $a_3=-4$

Product of roots $1.1.\alpha=S_3=\frac{-a_3}{a_0}=\frac{4}{1}$ $\therefore\ \alpha=4$.

---

6. Find the number of diagonals of a polygon with 12 sides.

Sol: The number of diagonals in a 'n' -gon $=\frac{n(n-3)}{2}=\frac{12\times9}{2}=54$

---

7. If ${}^{(n+1)}P_5:{}^{n}P_6=2:7$ find n.

Sol: ${}^{(n+1)}P_5 : {}^{n}P_6 = 2 : 7 \Rightarrow 2\,{}^{n}P_6 = 7\,{}^{(n+1)}P_5$

$\Rightarrow 2n(n-1)(n-2)(n-3)(n-4)(n-5) = 7(n+1)n(n-1)(n-2)(n-3)$

$\Rightarrow 2(n-4)(n-5) = 7(n+1) \Rightarrow 2(n^2 - 9n + 20) = 7n + 7 \Rightarrow 2n^2 - 18n + 40 = 7n+7$

$\Rightarrow 2n^2 - 25n + 33 = 0 \Rightarrow 2n^2 - 22n - 3n + 33 = 0 \Rightarrow 2n(n-11) - 3(n-11) = 0$

$\Rightarrow (n-11)(2n-3) = 0 \Rightarrow n = 11$ (n cannot be equal to 3/2)

8. Find the term independent of x in $\left(\frac{3}{\sqrt[3]{x}} + 5\sqrt{x}\right)^{25}$

Sol: The general term of $\left(\frac{3}{\sqrt[3]{x}} + 5\sqrt{x}\right)^{25}$ is

$$T_{r+1} = {}^{25}C_r \left(\frac{3}{\sqrt[3]{x}}\right)^{25-r} \left(5\sqrt{x}\right)^r = {}^{25}C_r 3^{25-r} 5^r x^{\frac{-25+r}{3} + \frac{r}{2}}$$

To get the term independent of x, we put

$$\frac{-25+r}{3} + \frac{r}{2} = 0 \Rightarrow \frac{25-r}{3} = \frac{r}{2} \Rightarrow 50 - 2r = 3r \Rightarrow 5r = 50 \Rightarrow r = 10$$

$\therefore$ from (1), the term independent of x is ${}^{25}C_{10} 3^{25-10} 5^{10} = {}^{25}C_{10} 3^{15} 5^{10}$

---

9. Find the mean deviation about mean for the data 3,6,10,4,9,10

Sol: The given data is 3,6,10,4,9,10. Here n= 6

Mean of the given data is $\bar{x} = \frac{3+6+10+4+9+10}{6} = \frac{42}{6} = 7$

The deviations of the observations from the mean (i.e., $x_i - \bar{x}$) are

3–7 = –4; 6–7 = –1; 10–7=3; 4–7=–3; 9–7=2; 10–7=3

Hence, the absolute values of the deviations are 4, 1, 3, 3, 2 3

$\therefore$ The mean deviation about mean is

$$M.D. = \frac{\Sigma|x_i - \bar{x}|}{6} = \frac{4+1+3+3+2+3}{6} = \frac{16}{6} = 2.67$$

---

10. For a binomial distribution with mean 6 and variance 2. Find the first two terms of the distribution.

Sol: Given that mean np=6, variance npq=2

$$\therefore (np)q = 2 \Rightarrow 6(q) = 2 \Rightarrow q = \frac{2}{6} = \frac{1}{3} \quad \Rightarrow p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$$

Now, $np = 6 \Rightarrow n.\frac{2}{3} = 6 \Rightarrow n = \frac{18}{2} = 9$

$\therefore$ the first 2 terms of the distribution are $P(X=0) = {}^{9}C_0 \left(\frac{1}{3}\right)^9 = \frac{1}{3^9}$;

SECTION-B

11. If $(x-iy)^{1/3} = a - ib$, then show that $\frac{x}{a}+\frac{y}{b}=4(a^2-b^2)$

Sol: Given that $(x-iy)^{1/3} = a-ib \Rightarrow x-iy = (a-ib)^3$

$\Rightarrow x-iy = a^3-3a^2bi+3ai^2b^2-i^3b^3 = a^3-3a^2bi-3ab^2+ib^3$

$= (a^3-3ab^2)-i(3a^2b-b^3)$

$(a-b)^3=a^3-3a^2b+3ab^2-b^3$

Equating real parts on both sides, we get $x=a^3-3ab^2 = a(a^2-3b^2) \Rightarrow \frac{x}{a}=a^2-3b^2$

Equating imaginary parts on both sides, we get $y=3a^2b-b^3= b(3a^2-b^2) \Rightarrow \frac{y}{b}=3a^2-b^2$

$\therefore \frac{x}{a}+\frac{y}{b}=(a^2-3b^2)+(3a^2-b^2)=4a^2-4b^2=4(a^2-b^2)$

---

12. If x is a real number, find the range of $\frac{2x^2-6x+5}{x^2-3x+2}$

Sol: Let 'm' be a real value of the given expression $\Rightarrow m = \frac{2x^2-6x+5}{x^2-3x+2}$

$\Rightarrow mx^2-3xm+2m = 2x^2-6x+5 \Rightarrow mx^2-3xm+2m-2x^2+6x-5=0$

$\Rightarrow x^2(m-2)+x(6-3m)+(2m-5) = 0$ -------(1)

(1)is a quadratic equation in x and $x\in R \Rightarrow b^2-4ac\geq 0$

$\Rightarrow (6-3m)^2-4(m-2)(2m-5)\geq 0 \Rightarrow (36-36m+9m^2)-4(2m^2-9m+10)\geq 0$

$\Rightarrow 36-36m+9m^2-8m^2+36m-40 \geq 0$

$\Rightarrow m^2-4 \geq 0 \Rightarrow (m+2)(m-2) \geq 0 \Rightarrow m\in(-\infty, -2]\cup[2,\infty)$

$\therefore$ Range of the given expression is $(-\infty, -2]\cup [2,\infty)$

---

13. If the letters of the word 'MASTER' are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word "MASTER"

Sol: The alphabetical order of the letters of the word MASTER is

A,E,M,R,S,T

The number of words that begin with A – – – – – = 5! = 120

The number of words that begin with E – – – – – = 5! = 120

The number of words that begin with MAE – – – = 3! = 6

The number of words that begin with MAR – – – = 3! = 6

The number of words that begin with MASE – – = 2! = 2

The number of words that begin with MASR – – = 2! = 2

The next word is MASTER = 1! = 1

$\therefore$ Rank of the word MASTER = 2(120) + 2(6) + 2(2) + 1

= 240 + 12 + 4 + 1 = 257

14. Find the number of ways of selecting 11 member cricket team from 7 batsmen, 6 bowlers and 2 wicket keepers so that the team contains 2 wicket keepers and atleast 4 bowlers.

Sol: A team of 11 players with 2 wicket keepers and atleast 4 bowlers can be selected in the following compositions.

Keepers(2)	Bowlers (6)	Batsmen (7)	No. of selections
2	4	5	${}^2C_2 \times {}^6C_4 \times {}^7C_5 = 1 \times 15 \times 21 = 315$
2	5	4	${}^2C_2 \times {}^6C_5 \times {}^7C_4 = 1 \times 6 \times 35 = 210$
2	6	3	${}^2C_2 \times {}^6C_6 \times {}^7C_3 = 1 \times 1 \times 35 = 35$

$\therefore$ the total number of selections=315+210+35=560

---

15. Resolve $\frac{2x+3}{5(x+2)(2x+1)}$ into partial fractions.

Sol: Let $\frac{2x+3}{(x+2)(2x+1)} = \frac{A}{x+2} + \frac{B}{2x+1} = \frac{A(2x+1)+B(x+2)}{(x+2)(2x+1)}$

$\therefore\ A(2x+1)+B(x+2) = 2x+3 .....(1)$

Putting $x = -2$ in (1) we get $A(2(-2)+1)+B[0]=2(-2)+3 \Rightarrow -3A=-1 \Rightarrow A=1/3$

Comparing the coefficients of constant terms on both sides of (1), we get A+2B =3

$\Rightarrow \frac{1}{3}+2B=3 \Rightarrow 2B = 3-\frac{1}{3}=\frac{8}{3} \Rightarrow B=\frac{4}{3}$

$\Rightarrow \frac{2x+3}{(x+2)(2x+1)} = \frac{A}{x+2}+\frac{B}{2x+1} = \frac{1}{3(x+2)}+\frac{4}{3(2x+1)}$

$\therefore \frac{2x+3}{5(x+2)(2x+1)} = \frac{1}{5}\left[\frac{1}{3(x+2)}+\frac{4}{3(2x+1)}\right] = \frac{1}{15(x+2)}+\frac{4}{15(2x+1)}$

---

16. If one ticket is randomly selected from tickets numbered 1 to 30, then find the probability that the number on the ticket is (i) a multiple of 5 or 7 (ii) a multiple of 3 or 5.

Sol: If a number is selected from 1 to 30 then $n(S)={}^{30}C_1=30$

(i) Let A be event of getting a multiple of 5 $\Rightarrow A = \{5,10,15,20,25,30\} \Rightarrow n(A)=6$

Let B be event of getting a multiple of 7 $\Rightarrow B = \{7,14,21,28\} \Rightarrow n(B)=4$

Here $A \cap B = \phi$

$\therefore P(A\cup B) = P(A)+P(B) = \frac{6}{30}+\frac{4}{30}=\frac{10}{30}=\frac{1}{3}$

(ii) Let A be event of getting a multiple of 3 $\Rightarrow A = \{3,6,9,12,15,18,21,24,27,30\} \Rightarrow n(A)=10$

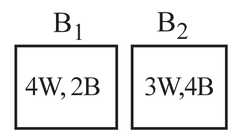
Let B be event of getting a multiple of 5 $\Rightarrow B = \{5,10,15,20,25,30\} \Rightarrow n(B)=6$

Here $A\cap B=\{15, 30\} \Rightarrow n(A\cap B)=2$

$\therefore P(A\cup B) = P(A)+P(B)-P(A\cap B) = \frac{10}{30}+\frac{6}{30}-\frac{2}{30}=\frac{10+6-2}{30}=\frac{14}{30}=\frac{7}{15}$

17. Bag B_1 contains 4 white and 2 black balls. Bag B_2 contains 3 white and 4 black balls. A bag is drawn at random and a ball is chosen at random from it. What is the probability that the ball is white?

Sol: Let B_1, B_2 denote the events of choosing bag B_1, bag B_2 respectively.



Let E be the event of choosing a white ball

Then $P(B_1)=\frac{1}{2}, P(B_2)=\frac{1}{2}, P(E|B_1)=\frac{4}{6}=\frac{2}{3}, P(E|B_2)=\frac{3}{7}$

$\therefore P(E)=P(B_1).P(E|B_1)+P(B_2).P(E|B_2)$

$$=\frac{1}{2}.\frac{2}{3}+\frac{1}{2}.\frac{3}{7}=\frac{1}{2}\left(\frac{2}{3}+\frac{3}{7}\right)=\frac{1}{2}\left(\frac{14+9}{21}\right)=\frac{1}{2}.\frac{23}{21}=\frac{23}{42}$$

SECTION-C

18. If n is a positive integer then show that $(1+i)^n+(1-i)^n=2^{(n+2)/2}\cos(n\pi/4)$

Sol: First we find the mod-amp form of (1+i).

Let $x+iy=1+i \Rightarrow x=1, y=1$

$$\therefore\ r=\sqrt{x^2+y^2}=\sqrt{1^2+1^2}=\sqrt{2};\quad \tan\theta=\frac{y}{x}=\frac{1}{1}=1=\tan\frac{\pi}{4}\Rightarrow\theta=\frac{\pi}{4}$$

$\therefore$ mod– Amp form of $1+i$ is $r(\cos\theta+i\sin\theta)=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$$\Rightarrow(1+i)^n=\left(\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right)^n$$

$$=(\sqrt{2})^n\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^n=2^{n/2}\left(\cos n\frac{\pi}{4}+i\sin n\frac{\pi}{4}\right)\quad\text{..... (1) (by Demoivre's theorem)}$$

Similarly, $(1-i)^n=2^{n/2}\left(\cos n\frac{\pi}{4}-i\sin n\frac{\pi}{4}\right)$ ..... (2)

Adding (1) & (2), we get $(1+i)^n+(1-i)^n$

$$=2^{n/2}\left(\left(\cos n\frac{\pi}{4}+i\sin n\frac{\pi}{4}\right)+\left(\cos n\frac{\pi}{4}-i\sin n\frac{\pi}{4}\right)\right)$$

$$=2^{\frac{n}{2}}.2\cos n\frac{\pi}{4}=2^{\frac{n}{2}+1}.\cos\frac{n\pi}{4}=2^{\frac{n+2}{2}}.\cos\frac{n\pi}{4}$$

19. Solve $3x^3-26x^2+52x-24=0$, given that the roots are in geometric progression.

Sol: Let the roots of $3x^3-26x^2+52x-24=0$ in G.P be taken as a/r, a, ar

$\therefore$ Product of roots $S_3=\left(\frac{a}{r}\right)(a)(ar)=\frac{24}{3}=8 \Rightarrow a^3=8 \Rightarrow a=2$

$S_1=\frac{a}{r}+a+ar=\frac{26}{3} \Rightarrow 2\left(\frac{1}{r}+1+r\right)=\frac{26}{3} \Rightarrow 2\left(\frac{1+r+r^2}{r}\right)=\frac{26}{3}$

$\Rightarrow\left(\frac{1+r+r^2}{r}\right)=\frac{13}{3} \Rightarrow 3(r^2+r+1)=13r \Rightarrow 3r^2+3r+3=13r \Rightarrow 3r^2-10r+3=0$

$\Rightarrow 3r^2-9r-r+3=0 \Rightarrow 3r(r-3)-1(r-3)=0 \Rightarrow (3r-1)(r-3)=0 \Rightarrow r=3 \text{ or } \frac{1}{3}$

$\therefore$ the roots are $\left(\frac{a}{r}\right),(a),(ar) \Rightarrow \frac{2}{3}, 2, 2(3) \Rightarrow \frac{2}{3}, 2, 6$

---

20. Prove that $C_0.C_r+C_1.C_{r+1}+C_2.C_{r+2}+....+C_{n-r}.C_n={}^{2n}C_{(n+r)}$ for $0\le r\le n$

Hence deduce that (i) $C_0^2+C_1^2+C_2^2+......+C_n^2={}^{2n}C_n$

(ii) $C_0C_1+C_1C_2+C_2C_3+.....+C_{n-1}.C_n={}^{2n}C_{n+1}$

Sol: We have $(1+x)^n=C_0+C_1x+C_2x^2+.......+C_rx^r+C_{r+1}x^{r+1}+C_{r+2}x^{r+2}+.......+C_nx^n$ .....(1)

$\Rightarrow (x+1)^n=C_0x^n+C_1x^{n-1}+C_2x^{n-2}+.......+C_{n-r}x^{n-r}+......+C_n$ ....(2)

Multiplying (2) and (1), we get

$(C_0x^n+C_1x^{n-1}+C_2x^{n-2}+....+C_{n-r}x^{n-r}+C_n)(C_0+C_1x+C_2x^2+......+C_rx^r+C_{r+1}x^{r+1}+C_{r+2}x^{r+2}+....+C_nx^n)$

$=(x+1)^n(1+x)^n=(1+x)^{2n}$

Comapring the coefficient of x^{n+r} both sides, we get

$C_0C_r+C_1C_{r+1}+C_2C_{r+2}+....+C_{n-r}C_n={}^{2n}C_{n+r}$

(i) On substituting r=0 , we get $C_0^2+C_1^2+C_2^2+......+C_n^2={}^{2n}C_n$

(ii) On substituting r=1, we get $C_0C_1+C_1C_2+C_2C_3+.....+C_{n-1}.C_n={}^{2n}C_{n+1}$

---

21. If $x=\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+...\infty$, then find $3x^2+6x$

Sol: $x=\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+...\infty$

Adding 1 on both sides, we have $1+x=1+\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+...\infty$

$=1+\frac{1}{1!}\left(\frac{1}{5}\right)+\frac{1.3}{2!}\left(\frac{1}{5}\right)^2+\frac{1.3.5}{3!}\left(\frac{1}{5}\right)^3+...\infty$

Comparing the above series with $1+\frac{p}{1!}\left(\frac{y}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{y}{q}\right)^2+.......=(1-y)^{-p/q}$

we get p =1, p+q =3 ⇒q =2 and $\frac{y}{q}=\frac{1}{5}\Rightarrow y=\frac{q}{5}=\frac{2}{5}$

$$\therefore 1+x=(1-y)^{\frac{-p}{q}}=\left(1-\frac{2}{5}\right)^{\frac{-1}{2}}=\left(\frac{3}{5}\right)^{\frac{-1}{2}}=\left(\frac{5}{3}\right)^{\frac{1}{2}}=\sqrt{\frac{5}{3}}$$

$$\Rightarrow(1+x)^2=\frac{5}{3}\Rightarrow 1+2x+x^2=\frac{5}{3}\Rightarrow 3+6x+3x^2=5\Rightarrow 3x^2+6x=2$$

---

22. Find the mean deviation about the mean for the following continuous distribution:

Sales (in Rs. thousand)	40-50	50-60	60-70	70-80	80-90	90-100
Number of companies	5	15	25	30	20	5

Sol: We form the following table from the given data

Sales	Number of companies(f_i)	Midpoints of class interval(x_i)	$f_i x_i$	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
40-50	5	45	225	26	130
50-60	15	55	825	16	240
60-70	25	65	1625	6	150
70-80	30	75	2250	4	120
80-90	20	85	1700	14	280
90-100	5	95	475	24	120
	$\Sigma f_i=100=N$		$\Sigma f_i x_i=7100$		$\Sigma f_i\lvert x_i-\bar{x}\rvert=1040$

Here $N=\Sigma f_i=100$ and Mean $\bar{x}=\frac{\Sigma f_i x_i}{N}=\frac{7100}{100}=71$

∴ Mean deviation about the mean $M.D=\frac{\Sigma f_i\lvert x_i-\bar{x}\rvert}{N}=\frac{1040}{100}=10.4$

---

23. State and prove addition theorem on Probability.

Sol: Statement:If E1,E2 are the 2events of a sample space S then P(E1∪E2)=P(E1)+P(E2)–P(E1∩E2)

Case (i): When $E_1\cap E_2=\phi$

$E_1\cap E_2=\phi \Rightarrow P(E_1\cap E_2)=0$

$\therefore P(E_1\cup E_2) = P(E_1)+P(E_2)$ [∵ from the union axiom]

$= P(E_1)+P(E_2)-0 = P(E_1)+P(E_2)-P(E_1\cap E_2)$

Case (ii) : When $E_1\cap E_2\neq\phi$

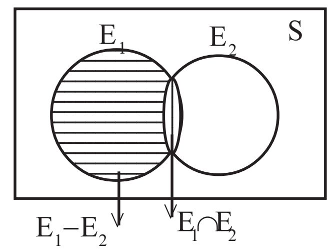
$E_1\cup E_2$ can be expressed as union of 2 mutually exclusive events E_1-E_2, E_2

Hence, $E_1\cup E_2=(E_1-E_2)\cup E_2$ also $(E_1-E_2)\cap E_2=\phi$

$\therefore$ $P(E_1 \cup E_2)=P[(E_1-E_2)\cup E_2]=P(E_1-E_2)+P(E_2)$ ....(1) [$\because$ from the union axiom]

Also, E_1 can be expressed as union of 2 mutually exclusive events E_1-E_2, $E_1\cap E_2$

Hence, $E_1=(E_1-E_2)\cup(E_1\cap E_2)$, also $(E_1-E_2)\cap(E_1\cap E_2)=\phi$



$\therefore$ $P(E_1)=P((E_1-E_2)\cup(E_1\cap E_2))=P(E_1-E_2)+P(E_1\cap E_2)$

$\Rightarrow P(E_1-E_2)=P(E_1)-P(E_1\cap E_2)$

$\therefore$ from (1), $P(E_1\cup E_2) = (P(E_1)-P(E_1\cap E_2))+P(E_2)$

$= P(E_1)+P(E_2)-P(E_1\cap E_2)$

---

24. A random variable X has its range{0,1,2} and the probabilities are $P(X=0)=3c^3$, $P(X=1)=4c-10c^2$, $P(X=2)=5c-1$ where 'c' is a constant, find (i) c (ii) $P(0<x<3)$ (iii) $P(1<x\le 2)$ (iv) $P(x<1)$

Sol: (i) We know that $\Sigma P(X=xi)=1$

$\Rightarrow 3c^3+4c-10c^2+5c-1=1 \Rightarrow 3c^3-10c^2+9c-1=1 \Rightarrow 3c^3-10c^2+9c-2=0$

Here, the sum of the coefficients is 3–10+9–2=0. Hence 1 is a root of the above equation.

$\therefore$ By synthetic division, we have

1	3	–10	9	–2
	0	3	–7	2
	3	–7	2	0

$\therefore 3c^3-10c^2+9c-2=(c-1)(3c^2-7c+2)=(c-1)[3c^2-6c-c+2]$

$=(c-1)[3c(c-2)-1(c-2)]=(c-1)(c-2)(3c-1)$

Now, $3c^3-10c^2+9c-2=0 \Rightarrow (c-1)(c-2)(3c-1)=0 \Rightarrow c=1, 2, \frac{1}{3}$

If, c=1 then , $P(X=0)=3c^3=3.1^3=3>1$, which is impossible

If c=2 then, $3c^3=3(2)^3=24>1$, which is also impossible.

$\therefore$ c=1/3 is the only possible value.

(ii) $P(0<X<3)=P(X=1)+P(X=2)=(4c-10c^2)+(5c-1)=9c-10c^2-1$

$=9\left(\frac{1}{3}\right)-10\left(\frac{1}{3}\right)^2-1=\frac{9}{3}-\frac{10}{9}-1=3-\frac{10}{9}-1=2-\frac{10}{9}=\frac{8}{9}$

(iii) $P(1<x\le 2)=P(X=2)=5c-1=5\left(\frac{1}{3}\right)-1=\frac{5}{3}-1=\frac{2}{3}$

(iv) $P(X<1)=P(X=0)=3c^3=3\left(\frac{1}{3}\right)^3=3.\frac{1}{27}=\frac{1}{9}$