BOARD MODEL PAPER-1

MATHS-2A

(Board of Intermediate Education -Model Paper)

Time : 3 Hours **Max.Marks : 75**

SECTION -A

I. Answer ALL the following Very Short Answer Questions: 10 × 2 = 20

1. Find the square root of –5+12i.

2. If $z_1 = -1$, $z_2 = i$ then find $\text{Arg}\left(\frac{z_1}{z_2}\right)$

3. Find the value of $(1+i)^{16}$.

4. If α,β are the roots of the equation $ax^2+bx+c=0$, then find the value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

5. Find the algebraic equation whose roots are two times the roots of $x^5-2x^4+3x^3-2x^2+4x+3=0$

6. Find the number of ways of arranging the letters of the word "INTERMEDIATE"

7. If ${}^nP_r = 5040$ and ${}^nC_r = 210$ find n and r.

8. If $(1+x+x^2)^n = a_0+a_1x+a_2x^2+.....+a_{2n}x^{2n}$ then find the value of $a_0+a_2+a_4+...+a_{2n}$.

9. The variance of 20 observations is 5. If each observation is multiplied by 2, then find the new variance of the resulting observations.

10. A Poisson variable satisfies P(x=1)=P(x=2), find P(X=5)

SECTION-B

II. Answer any FIVE of the following Short Answer Questions: 5 × 4 = 20

11. If z=x+iy and if the point P in the Argand plane represents z, find the locus of z satisfying the equation $|z-2-3i|=5$

12. Find the range of $\frac{x+2}{2x^2+3x+6}$

13. If the letters of the word MASTER are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word "REMAST"

14. Find the number of ways of selecting a cricket team of 11 players from 7 batsmen and 6 bowlers such that there will be atleast 5 bowlers in the team.

15. Resolve $\frac{x^2-3}{(x+2)(x^2+1)}$ into partial fractions.

16. Two persons A and B are rolling a die on the condition that the person who gets 3 will win the game. If A starts the game, then find the probabilities of A and B respectively to win the game.

17. A problem in calculus is given to two students A andB whose chances of solving it are 1/3, 1/4 respectively. Find the probability of the problem being solved if both of them try independently.

SECTION-C

III. Answer any FIVE of the following Long Answer Questions: 5×7 = 35

18. Find all the roots of the equation $x^{11}-x^7+x^4-1=0$

19. Solve $x^4-10x^3+26x^2-10x+1=0$

20. If n is a positive integer and x is any nonzero real number, then prove that

$$C_0+C_1\frac{x}{2}+C_2\frac{x^2}{3}+C_3\frac{x^3}{4}+\ldots+C_n\frac{x^n}{n+1}=\frac{(1+x)^{n+1}-1}{(n+1)x}$$

21. If $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+\ldots\ldots$ then prove that $9x^2+2x=11$

22. Calculate the variance and standard deviation for the following distribution:

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

23. The probabilities of three events A,B,C are such that P(A)=0.3, P(B)=0.4, P(C)=0.8, $P(A\cap B)=0.08$, $P(A\cap C)=0.28$, $P(A\cap B\cap C)=0.09$ and $P(A\cup B\cup C)\geq 0.75$, show that $P(B\cap C)$ lies in the interval [0.23, 0.48]

24. A random variable x has the following probability distribution

$X=x_i$	0	1	2	3	4	5	6	7
$P(X=x_i)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) the mean (iii) $P(0<X<5)$

SOLUTIONS

SECTION -A

1. Find the square root of –5 + 12i.

Sol: Let $-5+12i = a+bi \Rightarrow a=-5, b=12$ Now,

$$r=\sqrt{a^2+b^2}=\sqrt{(-5)^2+12^2}=\sqrt{25+144}=\sqrt{169}=13$$

$$\therefore \sqrt{a+ib}=\pm\left(\sqrt{\frac{r+a}{2}}+i\sqrt{\frac{r-a}{2}}\right) \Rightarrow \sqrt{-5+12i}=\pm\left(\sqrt{\frac{13-5}{2}}+i\sqrt{\frac{13+5}{2}}\right)=\pm\left(\sqrt{\frac{8}{2}}+i\sqrt{\frac{18}{2}}\right)$$

$$=\pm(\sqrt{4}+i\sqrt{9})=\pm(2+3i)$$

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2. If $z_1 = -1$, $z_2 = i$ then find $\text{Arg}\left(\frac{z_1}{z_2}\right)$

Sol: We know that $\text{Arg}(-1)=\pi$, $\text{Arg}\, i=\pi/2$ $\quad \therefore \text{Arg}\left(\frac{z_1}{z_2}\right)=\text{Arg}z_1-\text{Arg}z_2=\pi-\frac{\pi}{2}=\frac{\pi}{2}$

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3. Find the value of $(1 + i)^{16}$.

Sol: $1+i=\sqrt{2}\left[\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right]=\sqrt{2}\left[\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right]$

$$\therefore (1+i)^{16}=\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^{16}=(\sqrt{2})^{16}\left[\cos 16\frac{\pi}{4}+i\sin 16\frac{\pi}{4}\right]=2^8(\cos 4\pi+i\sin 4\pi)=256[1+i(0)]=256$$

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4. If α,β are the roots of the equation $ax^2+bx+c=0$, then find the value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

Sol: α, β are roots of $ax^2+bx+c=0 \Rightarrow \alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

$$\therefore \frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}=\frac{\left(-\frac{b}{a}\right)^2-2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}=\frac{\frac{b^2}{a^2}-\frac{2c}{a}}{\frac{c^2}{a^2}}=\frac{\frac{b^2-2ac}{a^2}}{\frac{c^2}{a^2}}=\frac{b^2-2ac}{c^2}$$

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5. Find the algebraic equation whose roots are two times the roots of $x^5-2x^4+3x^3-2x^2+4x+3=0$

Sol : Let $f(x)= x^5-2x^4+3x^3-2x^2+4x+3$. The required equation is $f(x/2)=0$

$$\Rightarrow\left(\frac{x}{2}\right)^5-2\left(\frac{x}{2}\right)^4+3\left(\frac{x}{2}\right)^3-2\left(\frac{x}{2}\right)^2+4\left(\frac{x}{2}\right)+3=0\Rightarrow\frac{1}{2^5}\left[x^5-4x^4+12x^3-16x^2+64x+96\right]=0$$

$$\Rightarrow x^5-4x^4+12x^3-16x^2+64x+96=0$$

6. Find the number of ways of arranging the letters of the word "INTERMEDIATE"

Sol: The given word INTERMEDIATE contains 12 letters in which 2 'I's are alike, 2'T's are alike, 3'E's are alike and rest are different.

$\therefore$ The number of arrangements $= \dfrac{n!}{p!q!r!} = \dfrac{(12)!}{2!.2!.3!}$

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7. If $^nP_r = 5040$ and $^nC_r = 210$ find n and r.

Sol: We know that $\dfrac{^nP_r}{^nC_r} = r! \Rightarrow r! = \dfrac{^nP_r}{^nC_r} = \dfrac{5040}{210} = 24 = 4!$ $\quad \therefore r! = 4! \Rightarrow r = 4$

$^nP_4 = 5040 = \times 10 \times 504 = 10 \times 9 \times 56 = 10 \times 9 \times 8 \times 7 = {}^{10}P_4$ $\quad \therefore n = 10$

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8. If $(1+x+x^2)^n = a_0+a_1x+a_2x^2+.....+a_{2n}x^{2n}$ then find the value of $a_0+a_2+a_4+...+a_{2n}$.

Sol: Given that $a_0+a_1x+a_2x^2+..........+a_{2n}x^{2n} = (1+x+x^2)^n$ ..... (A)

Put x=1 in (A), we get $a_0+a_1+a_2+..........+a_{2n} = (1+1+1^2)^n = 3^n$

$\Rightarrow a_0+a_1+a_2+..........+a_{2n} = 3^n$ ....(1)

Hence (i) is proved.

Put $x = -1$ in (A), we get $a_0-a_1+a_2-a_3+.........+a_{2n} = (1-1+1)^n = (1)^n = 1$

$\Rightarrow a_0-a_1+a_2-a_3+.........+a_{2n} = 1$.......... (2)

Now (1) + (2) $\Rightarrow 2(a_0+a_2+..........+a_{2n}) = 3^n+1 \Rightarrow a_0+a_2+..........+a_{2n} = \dfrac{3^n+1}{2}$

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9. The variance 20 observations is 5. If each observation is multiplied by 2, then find the new variance of the resulting observations.

Sol: Let $x_1, x_2,....x_{20}$ be the given observations.

$\therefore$ mean $\bar{x} = \dfrac{x_1+x_2+......+x_{20}}{20}$ ....(1)

If each observation is multiplied by 2, then the new observations are $2x_1, 2x_2, ..., 2x_{20}$.

$\therefore$ New mean $\bar{y} = \dfrac{2x_1+2x_2+......+2x_{20}}{20} = 2\dfrac{(x_1+x_2+......+x_{20})}{20} = 2(\bar{x})$ from (1)

$\therefore \bar{y} = 2\bar{x} \Rightarrow \bar{x} = \dfrac{\bar{y}}{2}$

Given that n=20 and variance =5

$\Rightarrow \dfrac{1}{20}\sum_{i=1}^{20}(x_i-\bar{x})^2 = 5 \Rightarrow \sum_{i=1}^{20}(x_i-\bar{x})^2 = 20\times 5 = 100$. But $\bar{x} = \dfrac{\bar{y}}{2}$

$\therefore \sum_{i=1}^{20}\left(\dfrac{y_i}{2}-\dfrac{\bar{y}}{2}\right)^2 = 100 \quad \Rightarrow \dfrac{1}{4}\sum_{i=1}^{20}(y_i-\bar{y})^2 = 100 \quad \Rightarrow \sum_{i=1}^{20}(y_i-\bar{y})^2 = 400$

$\therefore$ The variance of the resulting observations $= \dfrac{1}{n}\Sigma(y_i-\bar{y})^2 = \dfrac{1}{20}\times 400 = 20$

10. A Poisson variable satisfies P(x=1)=P(x=2), find P(X=5)

Sol: We have $P(X=r)=\frac{e^{-\lambda}\lambda^{r}}{r!}, \lambda>0$

Given that P(X=1)=P(X=2) $\Rightarrow \frac{\lambda e^{-\lambda}}{1!}=\frac{\lambda^{2}e^{-\lambda}}{2!} \Rightarrow \lambda^{2}=2\lambda \Rightarrow \lambda(\lambda-2)=0 \Rightarrow \lambda=2\ (\because \lambda>0)$

$\therefore P(X=5)=\frac{e^{-2}2^{5}}{5!}$

SECTION-B

11. If z=x+iy and if the point P in the Argand plane represents z, find the locus of z satisfying the equation |z–2–3i|=5

Sol: $z=x+iy \Rightarrow |z-2-3i|=5 \Rightarrow |x+iy-2-3i|=5 \Rightarrow |(x-2)+i(y-3)|=5$

$\Rightarrow \sqrt{(x-2)^{2}+(y-3)^{2}}=5 \Rightarrow (x-2)^{2}+(y-3)^{2}=25$

$\therefore$ This locus represents a circle with centre (2,3) and radius 5.

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12. Find the range of $\frac{x+2}{2x^{2}+3x+6}$

Sol: Let 'y' be a real value of the given expression

$\Rightarrow y=\frac{x+2}{2x^{2}+3x+6} \Rightarrow y(2x^{2}+3x+6)=x+2 \Rightarrow 2yx^{2}+3yx+6y-x-2=0$

$\Rightarrow 2yx^{2}+(3y-1)x+(6y-2)=0$ .... (1)

But x is real and (1) is a quadratic equation in x

$\therefore \Delta=b^{2}-4ac\geq 0 \Rightarrow (3y-1)^{2}-4(2y)(6y-2)\geq 0 \Rightarrow (9y^{2}-6y+1)-48y^{2}+16y\geq 0$

$\Rightarrow -39y^{2}+10y+1\geq 0 \Rightarrow 39y^{2}-10y-1\leq 0 \Rightarrow 39y^{2}-13y+3y-1\leq 0$

$\Rightarrow 13y(3y-1)+1(3y-1)\leq 0 \Rightarrow (13y+1)(3y-1)\leq 0 \Rightarrow \frac{-1}{13}\leq y\leq \frac{1}{3} \Rightarrow y\in\left[-\frac{1}{13},\frac{1}{3}\right]$

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13. If the letters of the word MASTER are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word "REMAST"

Sol: The alphabetical order of the letters of the word MASTER is

A,E,M,R,S,T

The number of words that begin with A – – – – – = 5! = 120

The number of words that begin with E – – – – – = 5! = 120

The number of words that begin with M – – – – – = 5! = 120

The number of words that begin with RA – – – – = 4! = 24

The number of words that begin with REA – – – = 3! = 6

The next word is REMAST = 1! = 1

$\therefore$ Rank of the word REMAST = 3(120) + 24 + 6 + 1

= 360 + 24 +6 + 1 = 391

14. Find the number of ways of selecting a cricket team of 11 players from 7 batsmen and 6 bowlers such that there will be atleast 5 bowlers in the team.

Sol: A Team of 11 players with atleast 5 bowlers can be selected in the following compositions:

Bowlers(6)	Batsmen(7)	No. of selections
5	6	${}^6C_5 \times {}^7C_6 = 6 \times 7 = 42$
6	5	${}^6C_6 \times {}^7C_5 = 1 \times 21 = 21$

$\because {}^7C_5 = {}^7C_2 = \frac{7\times 6}{2\times 1} = 21$

$\therefore$ the total number of selections=42+21= 63

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15. Resolve $\frac{x^2 - 3}{(x+2)(x^2+1)}$ into partial fractions.

Sol: Let $\frac{x^2-3}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$

$\Rightarrow A(x^2+1)+(Bx+C)(x+2) = x^2-3$ ....(1)

Putting $x=-2$ in (1)we get $A(4+1)+(Bx+C)(0) = 4-3 \Rightarrow 5A = 1 \Rightarrow A = 1/5$

Putting $x = 0$ in(1)we get $A + 2C = -3 \Rightarrow C = -8/5$

Comparing the coefficients of x^2, we get $A + B = 1 \Rightarrow B = 1 - A = 1 - \frac{1}{5} = \frac{4}{5}$

$\therefore \frac{x^2-3}{(x+2)(x^2+1)} = \frac{1}{5(x+2)} + \frac{4x-8}{5(x^2+1)}$

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16. Two persons A and B are rolling a die on the condition that the person who gets 3 will win the game. If A starts the game, then find the probabilities of A and B respectively to win the game.

Sol: Let p denote the probability of getting 3 when a die is rolled $\Rightarrow p = \frac{1}{6}$

Let q denote the probability of not getting 3 $\Rightarrow q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

As A starts the game, A wins if he toss 3 in the 1st or 3rd or 5th or ......turns.

$\therefore$Probability of A's winning = p+qqp+qqqqp+....=$p + q^2p + q^4p + \ldots$

$= \frac{p}{1-q^2} = \frac{1/6}{1-(5/6)^2} = \frac{6}{36-25} = \frac{6}{11}$ $\left[\because S_\infty = \frac{a}{1-r}\right]$

$\therefore$ Probability of B's winning $P(B) = 1 - P(A) = 1 - \frac{6}{11} = \frac{5}{11}$

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17. A problem in calculus is given to two students A andB whose chances of solving it are 1/3, 1/4 respectively. Find the probability of the problem being solved if both of them try independently.

Sol: Let A,B denote the events of solving the problem by A, B respectively $\Rightarrow P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$

The probability that the problem will be solved is $P(A \cup B)$

$\therefore P(A \cup B) = 1 - P(\bar{A} \cap \bar{B}) = 1 - P(\bar{A}).P(\bar{B}) = 1 - \left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right) = 1 - \left(\frac{2}{3}\right)\left(\frac{3}{4}\right) = 1 - \frac{1}{2} = \frac{1}{2}$

SECTION-C

III. Answer any FIVE of the following Long Answer Questions: 5×7=35

18. Find all the roots of the equation $x^{11}-x^7+x^4-1=0$

Sol: Given $x^{11}-x^7+x^4-1=0 \Rightarrow x^7(x^4-1)+1(x^4-1)=0 \Rightarrow (x^4-1)(x^7+1)=0$

Case (i) : $x^4-1=0 \Rightarrow x^4=1=(\cos 0+i\sin 0) \Rightarrow x^4=(\cos 2k\pi+i\sin 2k\pi)$

$\therefore x=(\cos 2k\pi+i\sin 2k\pi)^{1/4}=(\text{cis}2k\pi)^{1/4}$

$\Rightarrow x=\text{cis}\left(\frac{2k\pi}{4}\right)=\text{cis}\frac{k\pi}{2}, k=0,1,2,3$

By putting k = 0,1,2,3 we get $x=\text{cis}0=1, \text{cis}\frac{\pi}{2}=i, \text{cis}\pi=-1, \text{cis}\frac{3\pi}{2}=-i$

Case (ii) : $x^7+1=0 \Rightarrow x^7=-1=\cos\pi+i\sin\pi \Rightarrow x^7=\cos(2k\pi+\pi)+i\sin(2k\pi+\pi)$

$\therefore x=[\cos(2k+1)\pi+i\sin(2k+1)\pi]^{1/7}=(\text{cis}(2k+1)\pi)^{1/7}$

$\Rightarrow x=\text{cis}(2k+1)\frac{\pi}{7}, k=0,1,2,3,4,5,6$

By putting k=0,1,2,3,4,5,6 we get $x=\text{cis}\frac{\pi}{7}, \text{cis}\frac{3\pi}{7}, \text{cis}\frac{5\pi}{7}, \text{cis}\pi, \text{cis}\frac{9\pi}{7}, \text{cis}\frac{11\pi}{7}, \text{cis}\frac{13\pi}{7}$

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19. Solve $x^4-10x^3+26x^2-10x+1=0$

Sol: The degree of the given equation is n=4, which is Even. Also $a_k=a_{n-k}\ \forall\ k=0,1,2,3,4$
Hence the given equation is a reciprocal equation of class I of even degree, which is a S.R.E

Now, dividing the equation by x^2, we get $x^2-10x+26-\frac{10}{x}+\frac{1}{x^2}=0$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)-10\left(x+\frac{1}{x}\right)+26=0$ ....(1)

Put $x+\frac{1}{x}=y \Rightarrow x^2+\frac{1}{x^2}=y^2-2$

$(1) \Rightarrow (y^2-2)-10y+26=0 \Rightarrow y^2-10y+24=0$

$\Rightarrow y^2-6y-4y+24=0 \Rightarrow y(y-6)-4(y-6)=0 \Rightarrow (y-4)(y-6)=0$

$\Rightarrow y^2-6y-4y+24=0 \Rightarrow y(y-6)-4(y-6)=0 \Rightarrow (y-4)(y-6)=0$

Now, $y=4 \Rightarrow x+\frac{1}{x}=4 \Rightarrow x^2+1=4x \Rightarrow x^2-4x+1=0$

$\Rightarrow x=\frac{4\pm\sqrt{(4)^2-4.1.1}}{2.1}=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm 2\sqrt{3}}{2}=2\pm\sqrt{3}$

Now, $y=6 \Rightarrow x+\frac{1}{x}=6 \Rightarrow \frac{x^2+1}{x}=6 \Rightarrow x^2+1=6x \Rightarrow x^2-6x+1=0$

$\Rightarrow x=\frac{6\pm\sqrt{36-4}}{2}=\frac{6\pm\sqrt{32}}{2}=\frac{6\pm 4\sqrt{2}}{2}=3\pm 2\sqrt{2}$

$\therefore$ the roots of the given equation are $2\pm\sqrt{3}, 3\pm 2\sqrt{2}$

20. If n is a positive integer and x is any non-zero real number, then prove that

$$C_0 + C_1\frac{x}{2} + C_2\frac{x^2}{3} + C_3\frac{x^3}{4} + \dots + C_n\frac{x^n}{n+1} = \frac{(1+x)^{n+1}-1}{(n+1)x}$$

Sol: Method-I:

Let $S = C_0 + C_1.\frac{x}{2} + C_2.\frac{x^2}{3} + \dots + C_n.\frac{x^n}{n+1} = {}^nC_0 + {}^nC_1\frac{x}{2} + {}^nC_2\frac{x^2}{3} + \dots\dots + {}^nC_n.\frac{x^n}{n+1}$

$\Rightarrow x.S = {}^nC_0.x + {}^nC_1\frac{x^2}{2} + {}^nC_2.\frac{x^3}{3} + \dots + {}^nC_n.\frac{x^{n+1}}{n+1}$

$\Rightarrow (n+1)xS = \frac{n+1}{1}\cdot {}^nC_0.x + \frac{n+1}{2}.{}^nC_1.x^2 + \frac{n+1}{3}.{}^nC_2x^3 + \dots + \frac{n+1}{n+1}{}^nC_n.x^{n+1}$

$= {}^{n+1}C_1.x + {}^{n+1}C_2x^2 + {}^{n+1}C_3x^3 + \dots + {}^{n+1}C_{n+1}.x^{n+1}$ $\left(\because \left(\frac{n+1}{r+1}\right).{}^nC_r = {}^{(n+1)}C_{r+1}\right)$

$\Rightarrow (n+1)xS = (1+x)^{n+1} - 1$ $(\because {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_nx^n = (1+x)^n - 1)$

$\therefore S = \frac{(1+x)^{n+1}-1}{(n+1).x}$

Method-II:

$L.H.S = C_0 + \frac{C_1}{2}x + \frac{C_2}{3}x^2 + \dots + \frac{C_n}{n+1}x^n = {}^nC_0 + \frac{{}^nC_1}{2}x + \frac{{}^nC_2}{3}x^2 + \dots + \frac{{}^nC_n}{n+1}x^n$

$= 1 + \frac{n}{(1)2}x + \frac{n(n-1)}{(1.2)3}x^2 + \dots + \frac{1}{n+1}x^n$

$= \frac{1}{(n+1)x}\left[(n+1)x + \frac{(n+1)nx^2}{1.2} + \frac{(n+1)n(n-1)}{1.2.3}x^3 + \dots + x^{n+1}\right]$ (multiplying and dividing by $(n+1)x$)

$= \frac{1}{(n+1)x}\left[{}^{n+1}C_1x + {}^{n+1}C_2x^2 + {}^{n+1}C_3x^3 + \dots + {}^{n+1}C_{n+1}x^{n+1}\right]$

$= \left[\frac{{}^{n+1}C_0 + {}^{n+1}C_1x + {}^{n+1}C_2x^2 + {}^{n+1}C_3x^3 + \dots + {}^{n+1}C_{n+1}x^{n+1} - {}^{(n+1)}C_0}{(n+1)x}\right] = \frac{(1+x)^{n+1}-1}{(n+1)x} = R.H.S$

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21. If $x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + \dots\dots$ then prove that $9x^2+24x=11$

Sol: Given that $x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + \dots\dots = \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + \frac{1.3.5.7}{4!}\left(\frac{1}{3}\right)^4 + ..$

Adding $1 + \frac{1}{3}$ on both sides, we have $1 + \frac{1}{3} + x = 1 + \frac{1}{1!}\left(\frac{1}{3}\right) + \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + \dots\dots$

Comparing the above series with $1 + \frac{p}{1!}\left(\frac{y}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{y}{q}\right)^2 + \dots = (1-y)^{-p/q}$

we get $p = 1, p+q = 3 \Rightarrow 1+q = 3 \Rightarrow q = 2$

Also $\frac{y}{q}=\frac{1}{3}\Rightarrow y=\frac{q}{3}=\frac{2}{3}$

$$\therefore 1+\frac{1}{3}+x=(1-y)^{-p/q}=\left(1-\frac{2}{3}\right)^{-1/2}=\left(\frac{1}{3}\right)^{-1/2}=(3)^{1/2}=\sqrt{3}$$

$$\Rightarrow \frac{4}{3}+x=\sqrt{3}\Rightarrow x=\sqrt{3}-\frac{4}{3}=\frac{3\sqrt{3}-4}{3}=\frac{3\sqrt{3}-4}{3}\Rightarrow 3x=3\sqrt{3}-4\Rightarrow 3x+4=3\sqrt{3}$$

$$\Rightarrow (3x+4)^2=(3\sqrt{3})^2\Rightarrow 9x^2+24x+16=27\Rightarrow 9x^2+24x=11$$

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22. Calculate the variance and standard deviation for the following distribution:

Class	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
Frequency	**3**	**7**	**12**	**15**	**8**	**3**	**2**

Sol: Here h=10. We take the assumed mean A = 65. Here, C=10. Then $d_i=\frac{x_i-65}{10}$.

We form the following table with the given data.

Class interval(C.I)	**frequency (f_i)**	**Midpoint of C.I. (x_i)**	$d_i=\frac{x_i-65}{10}$	d_i^2	f_id_i	$f_id_i^2$
30-40	3	35	–3	9	–9	27
40-50	7	45	–2	4	–14	28
50-60	12	55	–1	1	–12	12
60-70	15	65	0	0	0	0
70-80	8	75	1	1	8	8
80-90	3	85	2	4	6	12
90-100	2	95	3	9	6	18
	$\Sigma f_i=50=N$				$\Sigma f_id_i=-15$	$\Sigma f_id_i^2=105$

Here, N=50, $\Sigma f_id_i=-15$, $\Sigma f_id_i^2=105$

Variance $\sigma^2=C^2\left[\frac{\Sigma f_id_i^2}{N}-\left(\frac{\Sigma f_id_i}{N}\right)^2\right]$

$$=10^2\left[\frac{105}{50}-\left(\frac{-15}{50}\right)^2\right]=100\left[\frac{105}{50}-\frac{225}{2500}\right]=100\left[\frac{(105)(50)-225}{2500}\right]=\frac{100(5025)}{2500}=201$$

Standard deviation $\sigma=\sqrt{201}=14.18$

23. The probabilities of three events A,B,C are such that P(A)=0.3, P(B)=0.4, P(C)=0.8, $P(A\cap B)=0.08$, $P(A\cap C) = 0.28$, $P(A\cap B\cap C)=0.09$ and $P(A\cup B\cup C)\geq 0.75$, show that $P(B\cap C)$ lies in the interval [0.23, 0.48]

Sol: Given that $P(A\cup B\cup C)\geq 0.75$; We know that $P(A\cup B\cup C)\leq 1$

$\therefore\ 0.75\leq P(A\cup B\cup C)\leq 1$

$\Rightarrow 0.75\leq P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)\leq 1$

$\Rightarrow 0.75\leq 0.3+0.4+0.8-0.08-0.28-P(B\cap C)+0.09\leq 1$

$\Rightarrow 0.75\leq 1.23-P(B\cap C)\leq 1$

$\Rightarrow -0.75\geq P(B\cap C)-1.23\geq -1$

$\Rightarrow 0.48\geq P(B\cap C)\geq 0.23$

$\Rightarrow 0.23\leq P(B\cap C)\leq 0.48$

$\therefore$ $P(B\cap C)$ lies in the interval [0.23, 0.48]

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24. A random variable x has the following probability distribution

$X=x_i$	0	1	2	3	4	5	6	7
$P(X=x_i)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) the mean (iii) P(0<X<5)

Sol : We know that the sum of the probabilities $\Sigma P(X=x_i)=1$

$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1 \Rightarrow 10k^2+9k=1 \Rightarrow 10k^2+9k-1=0$

$\Rightarrow 10k^2+10k-k-1=0 \Rightarrow 10k(k+1)-1(k+1)=0 \Rightarrow (10k-1)(k+1)=0$

$\Rightarrow k=1/10$, (since $k>0$)

(i) $k=1/10$

(ii) Mean $=\sum_{i=1}^{n} x_i.P(X=x_i)$

Mean(μ) $=0(0)+1(k)+2(2k)+3(2k)+4(3k)+5(k^2)+6(2k^2)+7(7k^2+k)$

$=0+k+4k+6k+12k+5k^2+12k^2+49k^2+7k=66k^2+30k$

$$=66\left(\frac{1}{100}\right)+30\left(\frac{1}{10}\right)=0.66+3=3.66$$

(iii) $P(0<x<5)=P(X=1)+P(X=2)+P(X=3)+P(X=4)=k+2k+2k+3k=8k=8\left(\frac{1}{10}\right)=\frac{8}{10}=\frac{4}{5}$