# WELCOME

## STAR 'QR CODE'
## DIGITAL CONTENT MATERIAL

## SYSTEM OF CIRCLES - INDEX

# 2.SYSTEM OF CIRCLES

| SECTIONS | *No. of periods* (6 to 7) | *Weightage in IPE* $1 \times 2 + 1 \times 4 = 6$ |
|---|---|---|
| **1. Angle between two intersecting circles** | 2 | 2 |
| **2. Radical axis, radical centre** | 3 | 4 |
| **3. Coaxial system of circles** | 2 | 4 |



*In the first section, angle between two intersecting circles is defined. Formulas to find the angle between intersecting circles are derived and hence the conditions for the* orthogonality *of two circles are obtained.*

*An application of the concept of power of a point w.r.t a circle is the* radical axis *of two circles and* radical centre *of three circles and these are discussed in section-2.*

*A family of circles having some common property in terms of their radical axes is known as* coaxial system of circles. *The standard form of a coaxial system is derived in Section-3.*

## SYNOPSIS POINTS

1. If $\theta$ is the angle between two intersecting circles of radii $r_1$, $r_2$ and d is the distance between the centres then $\cos\theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1r_2}$

2. Two circles with radii $r_1, r_2$ intersect orthogonally if and only if $r_1^2 + r_2^2 = d^2$.

3. The orthogonal condition for the circles $S' = x^2+y^2+2g_1x+2f_1y+c_1=0$, $S'' = x^2+y^2+2g_2x+2f_2y+c_2=0$ is $2g_1g_2+2f_1f_2=c_1+c_2$ (or) $2g_12g_2+2f_12f_2=2(c_1+c_2)$.

4. The equation of the radical axis of the circles S=0 and S'=0 is S−S'=0 .

5. If two circles intersect then their common chord is nothing but the radical axis.

6. If two circles touch each other then their common tangent is nothing but the radical axis.

7. If a circle S=0 cuts 3 circles S'=0, S''=0, S'''=0 orthogonally then the centre of the circle S=0 is the radical centre of the 3 circles and its radius equal to the length of the tangent from this radical centre to any of the given 3 circles.

8. If S=0 is a circle of a coaxial system and L=0 is the common radical axis of the system then the equation of the coaxial system passing through the points of intersection of S=0 & L=0 is $S+\lambda L=0$, $\lambda$ is a parameter.

9. If $S \equiv x^2+y^2+2gx+2fy+c=0$ and $L \equiv lx+my+n=0$ are the equations of a circle and a straight line respectively intersecting each other , then S+kL=0 represents a circle passing through the points of intersection of S=0 and L=0 for all real values of k.

# ADDITIONAL QUESTIONS WITH SOLUTIONS

**1** **Find the equation of the circle which intersects the circle $x^2 + y^2 - 6x + 4y - 3 = 0$ orthogonally and passes through the point (3, 0) and touches Y–axis.**

**Sol:** The equation of the required circle is taken as $S= x^2 + y^2 + 2gx + 2fy + c = 0$ ..........(1)

S=0 is orthogonal to $x^2 + y^2 - 6x + 4y - 3 = 0$

$\therefore$ 2gg' + 2ff' = c+c' $\Rightarrow -6g + 4f = c - 3$..........(2)

S = 0 passes through the point (3, 0) $\Rightarrow 9+0+6g+0+c = 0 \Rightarrow 6g + c = -9$ .............(3)

S = 0 touches y-axis $\Rightarrow f^2 = c$ ............. (4)

(2) + (3) $\Rightarrow -6g + 4f + 6g + c = c - 3 - 9 \Rightarrow 4f = -12 \Rightarrow f = -3$

From (4) $\Rightarrow (-3)^2 = c \Rightarrow c = 9$

From (3) $\Rightarrow 6g + 9 = -9 \Rightarrow 6g = -18 \Rightarrow g = -3$

Substituting the values of g, f, c in (1) we get the equation of the required circle as

$x^2+y^2-6x-6y+9=0$

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**2** **Find the equation of the circle which cuts the circles $x^2 + y^2 - 4x - 6y + 11 = 0$ and $x^2 + y^2 - 10x - 4y + 21 = 0$ orthogonally and has the diameter along the straight line $2x + 3y = 7$.**

**Sol:** The equation of the required circle is taken as $S=x^2+y^2+2gx+2fy+c=0$......(1)

S=0 is orthogonal to $x^2+y^2-4x-6y+11=0$

$\therefore 2gg'+2ff'=c+c' \Rightarrow 2g(-2)+2f(-3)=c+11 \Rightarrow -4g-6f=c+11$ ......(2)

S=0 is orthogonal to $x^2+y^2-10x-4y+21=0$

$\therefore 2gg'+2ff'=c+c' \Rightarrow 2g(-5)+2f(-2)=c+21 \Rightarrow -10g-4f=c+21$ ......(3)

The centre of the circle S = 0 is C = (–g, –f) lies on the line 2x + 3y = 7

$\Rightarrow -2g - 3f = 7 \Rightarrow 2g+3f+7 = 0$........(4)

(2) – (3) $\Rightarrow 6g-2f = -10 \Rightarrow 3g-f+5 = 0$ .....(5)

(4) + (5)×3 $\Rightarrow 2g + 3f + 7 + 9g - 3f +15 = 0 \Rightarrow 11g +22 = 0 \Rightarrow g = -2$

From (4) $\Rightarrow 2(-2)+3f+7 = 0 \Rightarrow -4 + 3f + 7 = 0 \Rightarrow 3f = -3 \Rightarrow f = -1$

Put g = –2 and f = –1 in (2) $\Rightarrow -4(-2)-6(-1)=c+11 \Rightarrow 14 = c + 11 \Rightarrow c = 3$

Substituting the values of g,f,c in (1) we get the equation of the required circle as

$x^2 + y^2 - 4x - 2y + 3 = 0$

**3** **If two points P,Q are conjugate w.r.t a circle $S \equiv x^2+y^2+2gx+2fy+c=0$, prove that the circle on PQ as diameter cuts the circle S=0 orthogonally.**

**Sol:** Let $P=(x_1,y_1)$, $Q=(x_2,y_2)$ and $S \equiv x^2+y^2+2gx+2fy+c=0$

The polar of $P(x_1,y_1)$ w.r.t S=0 is $S_1=0 \Rightarrow xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$......(1)

P,Q are conjugate w.r.t S=0 $\Rightarrow$ Q lies on the polar of P

From (1), $x_1x_2+y_1y_2+g(x_1+x_2)+f(y_1+y_2)+c=0$.........(2)

The equation of the circle having PQ as diameter is

$S' \equiv (x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0 \Rightarrow S' \equiv x^2+y^2-(x_1+x_2)x-(y_1+y_2)y+(x_1x_2+y_1y_2)=0$

Comparing the above equation with $S' \equiv x^2+y^2+2g'x+2f'y+c'=0$ we get

$2g'=-(x_1+x_2)$, $2f'=-(y_1+y_2)$, $c'=x_1x_2+y_1y_2$

Now $2gg'+2ff' = -g(x_1+x_2)-f(y_1+y_2) = c+(x_1x_2+y_1y_2)$, (from (2))

$= c+c'$

$\therefore$ The circles S=0, S'=0 cut orthogonally.

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**4** **If the equations of two circles whose radii are a, a' are S = 0 and S' = 0, then show that the circles $\frac{S}{a}+\frac{S'}{a'}=0$ and $\frac{S}{a}-\frac{S'}{a'}=0$ intersect orthogonally.**

**Sol:** Let '2d' be the distance between the centres of the circles S= 0 and S' = 0. Take the line joining the centres as X-axis and the point midway between the centres as origin.

The centres of the two circles are (d, 0) and (–d, 0) also the radii a and a'

$\therefore S = (x-d)^2+y^2 = a^2 \Rightarrow S = x^2+y^2-2dx+d^2-a^2 = 0$

$S' = (x+d)^2+y^2 = a^2 \Rightarrow S = x^2+y^2+2dx+d^2-a^2 = 0$

$\therefore$ The two circles $\frac{S}{a}\pm\frac{S'}{a'}=0$ becomes $Sa' \pm S'a = 0$

$\therefore$ Circle a' S + a S' becomes $(a'+a)(x^2+y^2+d^2)-2dx(a'-a)-aa'(a'+a)=0$

$$\Rightarrow x^2+y^2-2d\left(\frac{a'-a}{a'+a}\right)x+d^2-aa'=0$$

Similarly circle a' S – a S' = 0 becomes $x^2+y^2-2d\left(\frac{a'+a}{a'-a}\right)x+d^2+aa'=0$

$$2g_1g_2+2f_1f_2=2\left[-d.\frac{a'-a}{a'+a}\right]\left[-d.\frac{a'+a}{a'-a}\right]+0=2d^2=c_1+c_2$$

$\therefore$ The circles $S/a \pm S'/a'=0$ intersect orthogonally.