WELCOME

STAR 'QR CODE'

DIGITAL MATERIAL

MATHEMATICAL INDUCTION -INDEX

1) Introduction Page to M.I 02

2) Additional Q's with Solutions 03 - 7

2.MATHEMATICAL INDUCTION

1. INTRODUCTION PAGE

Sections	No. of periods (6)	Weightage in IPE [1x7 =7]
1. Mathematical Induction	6	7

A given statement can be proved or disproved by means of various methods like; analytic method, synthetic method, inductive method, deductive method, disproof by counter example, Reductio-ad-absurdum etc.,

Mathematical Induction is also such a method of proof.

Various "principles of Mathematical Induction "are derived from certain axioms on the set of natural numbers N={1,2,3,....} called Peano's axioms and Well ordering Principle.

• The Peano's axioms:

1. $1 \in N$ (1 is a natural number and $N \neq \phi$)
2. $n \in N \Rightarrow n+1 \in N$ (every natural number has a successor)
3. $n \in N \Rightarrow n+1 \neq 1$ (1 is not a successor to any natural number)
4. $m,n \in N, m+1=n+1 \Rightarrow m=n$ (If two successors are equal then the naturals are equal)
5. If $A \subset N$ such that (i) $1 \in A$ (ii) $n \in A \Rightarrow n+1 \in A$ then $A=N$

   (The principle of finite Mathematical Induction is derived from this axiom)

• Well ordering Principle: *Any non-empty set of natural numbers has a least element.*

This principle of finite Mathematical Induction is considered to be one of the best and powerful (infinitely many cases can be verified at a stroke!?) method of proving a given result. It has got wide scope of applications in various parts of Mathematics.

The major restriction to be considered, while applying this principle is that "The given statement is generally defined on the set of Natural numbers $N=\{1,2,3,....\}$".

It should be noted that, the principle of Mathematical Induction is not applicable to answer the question such as: $1+2+.....+n$ *is how much? But, it proves a given statement like;* $1+2+....+n=\frac{n(n+1)}{2}$ *(i.e., both L.H.S and R.H.S must be supplied).*

Thus, the principle of Mathematical Induction is only a method of proof for a known or guessed formula and is not a tool for discovering such formula.

Certain problems on "divisibility" are proved by using the principle of Mathematical Induction, with the help of "Division Algorithm".

Division Algorithm: *If 'a' is a positive integer and 'b' is an integer then ∃ a unique pair of integers q,r such that* $b=aq+r, 0 \le r < a$.

Here, if $r=0$ *then* $b=aq$ *and we say that b is divisible by a (or) b is a multiple of a (or) a divides b (or) a is a factor of b (or) a is a divisor of b.*

Note : *0 is divisible by any non-zero number, but division by 0 is undefined.*

ADDITIONAL QUESTIONS WITH SOLUTIONS

1. Using the Principle of M.I, prove that $4^3 + 8^3 + 12^3 + \ldots$ n terms $= 16n^2 (n + 1)^2$

Sol : 4,8,12 .... are in A.P. Hence the n^{th} term of the given series is $(4n)^3$

Let $S(n) : 4^3 + 8^3 + 12^3 + \ldots\ldots + (4n)^3 = 16n^2 (n + 1)^2$

Step 1: LH.S of $S(1) = 4^3 = 64$

R.H.S of $S(1) = 16.1^2 (1 + 1)^2 = 16(2^2) = 16(4) = 64$

$\therefore$ L.H.S of S (1) = R.H.S of S(1)

$\Rightarrow$ S(1) is true

Step 2: Assume that S(k) is true for $k \in N$

$S(k) : 4^3 + 8^3 + 12^3 + \ldots\ldots + (4k)^3 = 16k^2 (k +1)^2$ .... (1)

Step 3: Now, we show that S(k + 1) is true

$S(k + 1) : [4^3 + 8^3 + 12^3 + \ldots\ldots (4k)^3] + [4(k + 1)]^3 = 16 (k + 1)^2 (k + 1 + 1)^2.$

L.H.S of $S(k + 1) = [4^3 + 8^3 + \ldots\ldots (4k)^3] + [4(k + 1)^3]$

$= 16 k^2 (k +1)^2 + 64 (k + 1)^3,$ [From (1)]

$= 16(k+1)^2 [k^2 + 4(k + 1)]$

$= 16 (k+1)^2 [k^2 + 4k + 4]$

$= 16 (k + 1)^2 (k + 2)^2 = 16(k + 1)^2 (k + 1 + 1)^2$

= R.H.S of S(k + 1)

$\therefore$ S (k + 1) is true whenever S(k) is true

Hence, by the Principle of Mathematical Induction, the given statement in true, $\forall n \in N$.

2. P.T 2.3+3.4+4.5+.... upto n terms $=\dfrac{n(n^2+6n+11)}{3}$

Sol: n^{th} term of the given series is (n+1)(n+1)

Let $S(n): 2.3+3.4+4.5..................+(n+1)(n+2)=\dfrac{n(n^2+6n+11)}{3}$

Step-1: L.H.S of S(1) =2.3=6

$$\text{R.H.S of } S(1) = \frac{1(1^2+6(1)+11)}{3}=\frac{18}{3}=6$$

$\therefore$ L.H.S of S(1) = R.H.Sof S(1) $\Rightarrow$ S(1) is true.

Step-2: Assume that S(k) is true for $k\in N$

$$S(k)=2.3+3.4+4.5..................+(k+1)(k+2)=\frac{k(k^2+6k+11)}{3} \quad ......(1)$$

Step-3: We show that S(k+1) is true

$$S(k+1)=2.3+3.4+4.5.....+(k+1)(k+2)+(k+2)(k+3)=\frac{(k+1)[(k+1)^2+6(k+1)+11)}{3}$$

L.H.S of S(k+1)= $2.3+3.4+4.5.....+(k+1)(k+2)+(k+2)(k+3)$

$$=\frac{k(k^2+6k+11)}{3}+(k+2)(k+3)=\frac{k^3+6k^2+11k+3k^2+15k+18}{3}$$

$$=\frac{k^3+9k^2+26k+18}{3}=\frac{(k+1)(k^2+8k+18)}{3}=\frac{(k+1)[(k+1)^2+6(k+1)+11]}{3}$$

=R.H.S of S(k+1)

$\therefore$ L.H.S of S(k+1)=R.H.S of S(k+1) $\Rightarrow$ S(k+1) is true whenever S(k) is true

Hence, by the principle of Mathematical Induction, S(n) is true for all $n\in N$

3. **Show that** $2+7+12+.......+(5n-3)=\frac{n(5n-1)}{2}$

Sol: Let S(n) : $2+7+12+.......+(5n-3)=\frac{n(5n-1)}{2}$

Step 1: L.H.S of S(1) = 2;

R.H.S of S(1) $=\frac{1(5(1)-1)}{2}=\frac{4}{2}=2$

$\therefore$ L.H.S of S(1) = R.H.S of S(1)

$\Rightarrow$ S(1) is true.

Step 2: Assume that S(k) is true, for $k\in N$

S(k) : $2+7+12+.......+(5k-3)=\frac{k(5k-1)}{2}$ ....(1)

Step 3: Now, we show that S(k+1) is true

On adding (5(k+1)–3)=5k+2 to both sides of (1), we get

L.H.S of S(k+1) $=2+7+12+.......+(5k-3)+(5k+2)=\frac{k(5k-1)}{2}+(5k+2)$

$$=\frac{k(5k-1)+2(5k+2)}{2}$$

$$=\frac{5k^2-k+10k+4}{2}=\frac{5k^2+9k+4}{2}$$

$$=\frac{(k+1)(5k+4)}{2}=\frac{(k+1)(5(k+1)-1)}{2}$$

= R.H.S of S(k+1)

$\therefore$ L.H.S of S(k+1)=R.H.S of S(k+1) $\Rightarrow$ S(k+1) is true whenever S(k) is true

Hence, by the principle of Mathematical Induction , S(n) is true, for all $n\varepsilon N$

4. Use Mathematical Induction to prove that $\sum_{k=1}^{n}(2k-1)^2 = \frac{n(2n-1)(2n+1)}{3}$ $\forall n \in N$

Sol: Let $S(n): \sum_{k=1}^{n}(2k-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

$\Rightarrow S(n): 1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

Step 1: L.H.S of S(1)=1^2=1

R.H.S of S(1)=$\frac{1(2(1)-1)(2(1)+1)}{3} = \frac{1(3)}{3} = 1$

$\therefore$ L.H.S of S(1) = R.H.S of S(1) $\Rightarrow$ S(1) is true

Step 2: Assume that S(k) is true for $k \in N$

$S(k) = 1^2 + 3^2 + 5^2 + ... + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$ .....(1)

Step 3: Now, we show that S(k+1) is true

$S(k+1): 1^2 + 3^2 + 5^2 + ... + (2k-1)^2 + (2k+1)^2 = \frac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$

L.H.S of S(k+1)=$[1+3+5+....+(2k-1)^2]+(2k+1)^2$

$= \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$ [From (1)]

$= \frac{k(2k-1)(2k+1) + 3(2k+1)^2}{3}$

$= \frac{(2k+1)[k(2k+1) + 3(2k+1)]}{3} = \frac{(2k+1)[2k^2 - k + 6k + 3]}{3}$

$= \frac{(2k+1)[2k^2 + 5k + 3]}{3} = \frac{(2k+1)(2k+3)(k+1)}{3} = \frac{(k+1)(2k+1)(2k+3)}{3}$

$= \frac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$ = R.H.S of S(k+1)

$\therefore$ L.H.S of S(k+1)=R.H.S of S(k+1)

$\Rightarrow$ S(k+1) is true whenever S(k) is true

Hence by the principal of Mathematical induction S(n) is true $\forall n \in N$

5. Show that $1^2 + 2^2 + ... + n^2 > \frac{n^3}{3}$ for $n \in N$

Sol: Let S(n): $1^2 + 2^2 + ... + n^2 > \frac{n^3}{3}$, $n \in N$

Step 1: L.H.S of S(1) $= 1^2 = 1 > \frac{1}{3}$ = R.H.S of S(1) $\therefore$ S(1) is true

Step 2: Assume that S(k) is true for $k \in N$

$$S(k) : 1^2 + 2^2 + ... + k^2 > \frac{k^3}{3} \quad .....(1)$$

Step 3: Now, we show that S(k+1) is true

$$S(k+1): 1^2 + 2^2 + ... + k^2 + (k+1)^2 > \frac{(k+1)^3}{3}$$

$$\text{L.H.S of } S(k+1) = [1^2 + 2^2 + ... + k^2] + (k+1)^2 > \frac{k^3}{3} + (k+1)^2 \quad \text{[From (1)]}$$

$$= \frac{k^3 + 3(k+1)^2}{3} = \frac{k^3 + 3(k^2 + 2k + 1)}{3} = \frac{k^3 + 3k^2 + 6k + 1}{3} > \frac{k^3 + 3k^2 + 3k + 1}{3}$$

$$= \frac{(k+1)^3}{3} = \text{R.H.S of } S(k+1)$$

$\therefore$ S(k+1) is true whenever S(k) is true

$\therefore$ By the principle of mathematical induction, S(n) is true for all $n \in N$