WELCOME

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DIGITAL CONTENT MATERIAL

CIRCLES-INDEX

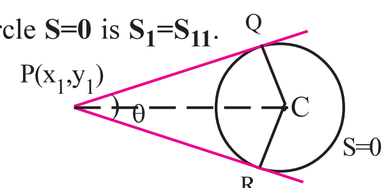
1.THE CIRCLE

SECTIONS	*No. of periods (25 to 33)*	*Weightage in IPE* $2\times2+1\times4+2\times7=22$
1. Equation of a circle in various forms	3 to5	2 or 7
2. Equation of a circle with given ends of a diameter	1	2
3. Circle and a line, Length of Chord, tangential condition	5 to 6	2 or 7
4. Notation	1	
5. Relative positions of a point and a circle, Length of tangent, Angle between the tangents	3	2 or 7
6. Equation of a Chord in various forms	2 to 3	2 or 7
7. Equation of a Tangent in various forms	3 to 4	2 or 7
8. Relative positions of two circles, Common tangents & centres of similitude	2 to 3	7
9. Pole & Polar, conjugate point, conjugate lines, inverse point	5 to 6	2 or 7

- 'Mathematics is the Queen of Sciences' & 'Circle is the Queen of Curves'.
- The study of Circles as a part of pure geometry is dealt with in the lower classes.
- In this chapter, an analytical approach is developed to translate the geometrical concepts into algebraic equations.
- One of the major concerns in this chapter is to find the equation of the circle satisfying the given conditions.
- In order to abridge lengthy expressions, a notational approach in the form of S, S_1, S_{11} & S_{12} is introduced.
- The algebraic treatment is given to the concepts related to Tangent, Normal, Chord of contact of a circle.
- Analytical treatment is given to the fundamental concepts like; relative positions of a point and a circle, relative positions of a line and a circle and relative positions of two circles.
- Concepts of pole, polar, conjugate points, conjugate lines, inverse points etc., are newly introduced.
- Parametric treatment is given wherever needed.

SYNOPSIS POINTS

1. The equation of the circle with **centre C(a,b)** & **radius r** is $(x-a)^2+(y-b)^2=r^2$ and its **parametric form** is $x=a+r\cos\theta,\ y=b+r\sin\theta$.
2. The equation of the circle in the **standard form** is $x^2+y^2=r^2$ and the **parametric point** is $\theta(r\cos\theta,\ r\sin\theta)$
3. The equation of the circle in the **general form** is $x^2+y^2+2gx+2fy+c=0$ & its **centre** $C=(-g,-f)$ and **radius** $r=\sqrt{g^2+f^2-c}$
4. The equation of the circle with $(x_1,y_1), (x_2,y_2)$ as **ends of a diameter** is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$ (or) $x^2+y^2-(x_1+x_2)x-(y_1+y_2)y+x_1x_2+y_1y_2=0$
5. If **p** is the **perpendicular distance** from the centre of a circle of **radius r**, to a chord then the **length of the chord** is $2\sqrt{r^2-p^2}$
6. The **x-intercept** of $x^2+y^2+2gx+2fy+c=0$ is $2\sqrt{g^2-c}$ and **y-intercept** is $2\sqrt{f^2-c}$
7. The **tangential condition** for a circle and a line is **p=r**, where p is the perpendicular distance from the centre of the circle to the given line and r is the radius of the circle.
8. The condition for the circle $x^2+y^2+2gx+2fy+c=0$ to **touch**
   (i) the **X-axis** is $c=g^2$ (ii) the **Y-axis** is $c=f^2$ (iii) **both the axes** is $c=g^2=f^2$
9. *Notation*: $S\equiv x^2+y^2+2gx+2fy+c$; $S_{11}\equiv x_1^2+y_1^2+2gx_1+2fy_1+c$
   $S_1\equiv xx_1+yy_1+g(x+x_1)+f(y+y_1)+c$ (or) $S_1\equiv(x_1+g)x+(y_1+f)y+x_1g+y_1f+c$
10. A point $P(x_1,y_1)$ and a **circle S=0** are given then the **point P lies**
    (i) **on the circle** if $S_{11}=0$ (ii) **inside** the circle if $S_{11}<0$ (iii) **out side** the circle if $S_{11}>0$
11. (i) the **equation of the chord of contact of** (x_1,y_1) w.r.to **S=0**
    (ii) the **equation of tangent at** (x_1,y_1) on the circle **S=0**
    (iii) the **equation of polar of** (x_1,y_1) w.r.to the circle **S=0**
    } $S_1=0$
12. The **equation of the chord** with (x_1,y_1) as **mid point** to the circle **S=0** is $S_1=S_{11}$.



13. (i) the **length** of the **tangent** from $P(x_1,y_1)$ to S=0 is $\sqrt{S_{11}}$
    (ii) the equation of the **pair of tangents** is $S_{11}S=S_1^2$
    (iii) $\tan\theta=\dfrac{2r\sqrt{S_{11}}}{S_{11}-r^2}$ (or) $\tan\dfrac{\theta}{2}=\dfrac{r}{\sqrt{S_{11}}}$ (iv) **Area** of the $\Delta PQC=\dfrac{1}{2}r\sqrt{S_{11}}$
14. The **equation of the tangent** at $(r\cos\theta,\ r\sin\theta)$ on $x^2+y^2=r^2$ is $x\cos\theta+y\sin\theta=r$
15. The **tangential condition** for the line (i) $y=mx+c$ and the circle $x^2+y^2=r^2$ is $c^2=r^2(1+m^2)$
    (ii) $lx+my+n=0$ and the circle $x^2+y^2=r^2$ is $n^2=r^2(l^2+m^2)$
16. (i) the **point of contact** of the **tangent** $lx+my+n=0$ w.r.to $x^2+y^2=r^2$
    (ii) the **pole** of the **line** $lx+my+n=0$ w.r.to the circle $x^2+y^2=r^2$
    } $\left(\dfrac{-lr^2}{n},\dfrac{-mr^2}{n}\right)$
17. If (x_1,y_1) is the **pole** of $lx+my+n=0$ w.r.t a circle with centre (a,b) then $\dfrac{x_1-a}{l}=\dfrac{y_1-b}{m}=\dfrac{-r^2}{la+mb+n}$
18. The condition for the lines $l_1x+m_1y+n_1=0$ and $l_2x+m_2y+n_2=0$ to be **conjugate** w.r.t the circle $x^2+y^2=r^2$ is $n_1n_2=r^2(l_1l_2+m_1m_2)$

ADDITIONAL QUESTIONS WITH SOLUTIONS

1 **Find the centre of the circle passing through the points (0,0), (2, 0), (0, 2)**

Sol: Let A=(0,0), B=(2,0), C=(0,2)

Let $S(x_1,y_1)$ be the centre of the circle $\Rightarrow$ SA=SB=SC

Now, SA=SB $\Rightarrow SA^2=SB^2 \Rightarrow (x_1-0)^2+(y_1-0)^2=(x_1-2)^2+(y_1-0)^2$

$\Rightarrow x_1^2+y_1^2=x_1^2-4x_1+4+y_1^2 \Rightarrow -4x_1+4=0 \Rightarrow x_1=1$

Also, SA = SC $\Rightarrow SA^2=SC^2$

$\Rightarrow (x_1-0)^2+(y_1-0)^2=(x_1-0)^2+(y_1-2)^2 \Rightarrow x_1^2+y_1^2=x_1^2+y_1^2-4y_1+4$

$\Rightarrow -4y_1+4=0 \Rightarrow y_1=1$

$\therefore$ Centre of circle $S(x_1,y_1)=(1,1)$

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2 **Suppose a point (x_1, y_1) satisfies $x^2+y^2+2gx+2fy+c=0$ then show that it represents a circle whenever g, f and c are real**

Sol: In the given equation $x^2+y^2+2gx+2fy+c=0$..........(1) we have

(i) coefficient of x^2 = coefficient of y^2 (ii) coefficient of xy = 0

(1) represents a real circle if radius $r \geq 0 \Rightarrow g^2+f^2-c \geq 0$

Let (x_1, y_1) be a point on the circle $x^2+y^2+2gx+2fy+c=0$,

Then we have $x_1^2+y_1^2+2gx_1+2fx_1+c=0$.

Adding g^2+f^2 on both sides we have $(x_1^2+y_1^2+2gx_1+2fx_1+c)+g^2+f^2=0+g^2+f^2$

$\Rightarrow x_1^2+y_1^2+2gx_1+2fx_1+g^2+f^2=g^2+f^2-c \Rightarrow (x_1^2+2gx_1+g^2)+(y_1^2+2fx_1+f^2)=g^2+f^2-c$

$\Rightarrow (x_1+g)^2+(y_1+f)^2 \geq 0$. So $g^2+f^2-c \geq 0$ where g, f and c are real

$\therefore$ The given equation represents a real circle .

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3 **Find the equation of the circle passing through (0, 0) and making intercept 6 unit on X-axis and intercept 4 units on Y-axis**

Sol: Let the required circle cuts the x-axis at A and the y-axis at B such that OA =6, OB=4 .

Let C be the centre of the circle and P,Q be the mid points of OA, OB.

Then OP=OA/2=6/2=3; CP=QO=OB/2 = 4/2 = 2

Now $OC=\sqrt{OP^2+PC^2}=\sqrt{3^2+2^2}=\sqrt{9+4}=\sqrt{13}$

Radius of the circle $\sqrt{13}$, Centres of the circles are $C=(\pm 3, \pm 2)$

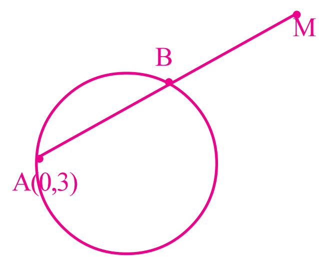
Equation of the required circles is $(x \pm 3)^2+(y \pm 2)^2=(\sqrt{13})^2 \Rightarrow x^2+y^2 \pm 6x \pm 4y=0$

4 **From the point A(0,3) on the circle $x^2+4x+(y-3)^2=0$ a chord AB is drawn and extended to a point M such that AM=2AB. Find the equation of the locus of M.**

Sol: Given A = (0,3).

Let M= (x_1,y_1) be a point such that AM= 2AB $\Rightarrow$ AB+BM= AB+AB$\Rightarrow$AB= BM

$\Rightarrow$B is mid point of AM $\Rightarrow B=\left(\frac{x_1}{2},\frac{y_1+3}{2}\right)$



But B is a point on the circle $x^2+4x+(y-3)^2=0$

$$\Rightarrow\left(\frac{x_1}{2}\right)^2+4\left(\frac{x_1}{2}\right)+\left(\frac{y_1+3}{2}-3\right)^2=0$$

$$\Rightarrow\frac{x_1^2}{4}+2x_1+\frac{y_1^2-6y_1+9}{4}=0\Rightarrow x_1^2+y_1^2+8x_1-6y_1+9=0$$

Hence the equation of locus of M is $x^2+y^2+8x-6y+9=0$, which is again a circle.

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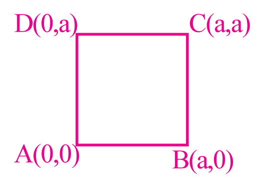
5 **If ABCD is a square then show that the points A, B, C and D are concyclic**

Sol: We know that any three non-collinear points are always concyclic.

Consider a square of side a .

Let the 4 vertices be A(0, 0), B(a, 0), C(a,a), D(0, a)



Consider the general equation of the circle

$S= x^2 + y^2 + 2gx + 2fy + c = 0$

Let S=0 passes through three non-collinear points A(0, 0), B(a, 0), D(0, a) .

Put A (0, 0) in S=0 $\Rightarrow 0 + 0 + 2g(0) + 2f(0) + c = 0 \Rightarrow c = 0$

Put B (a, 0) in S=0 $\Rightarrow a^2 + 0 + 2g(a) + 2f(0) + 0 = 0$

$\Rightarrow a^2 + 2ag = 0 \Rightarrow a + 2g = 0 \Rightarrow g=-\frac{a}{2}$

Put D (0, a) in S=0 $\Rightarrow 0 + a^2 + 2g(0) + 2f(a) + 0 = 0$

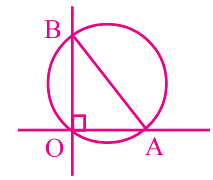
$\Rightarrow a^2 + 2af = 0 \Rightarrow a + 2f = 0 \Rightarrow f=-\frac{a}{2}$

Substituting the values of g, f and c in S=0, we get $x^2 + y^2 - ax - ay = 0$

Substituting the remaining point C (a, a) in the above equation, we have $a^2 + a^2 - a^2 - a^2 = 0$

C also lies on the circle $\therefore$ A, B, C, D are concyclic

6 **Find the equation of the circum-circle of the triangle formed by the line ax+by+c=0 and the coordinate axes.**



Sol: The line ax+by+c=0 cuts the axes at $A\left(\frac{-c}{a},0\right)$, $B\left(0,\frac{-c}{b}\right)$

For the required circle A,B are ends of a diameter.

Hence its equation is $\left(x+\frac{c}{a}\right)(x-0)+\left(y+\frac{c}{b}\right)(y-0)=0 \quad \Rightarrow x^2+y^2+\frac{c}{a}x+\frac{c}{b}y=0$

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7 **Show that the locus of the point of intersection of the lines x cos α + y sin α = a, x sin α – y cos α = b (α is a parameter) is a circle.**

Sol: Let $P(x_1,y_1)$ be the point of intersection of the given lines $x\cos\alpha+y\sin\alpha=a$, $x\sin\alpha-y\cos\alpha=b$

$\therefore\ x_1\cos\alpha+y_1\sin\alpha=a$ ....(1) and $x_1\sin\alpha-y_1\cos\alpha=b$ ....(2) .

To eliminate α, square & add (1), (2); $(x_1\cos\alpha+y_1\sin\alpha)^2+(x_1\sin\alpha-y_1\cos\alpha)^2=a^2+b^2$

$$x_1^2\cos^2\alpha+y_1^2\sin^2\alpha+2x_1y_1\cos\alpha\sin\alpha+x_1^2\sin^2\alpha+y_1^2\cos^2\alpha-2x_1y_1\cos\alpha\sin\alpha=a^2+b^2$$

$$\Rightarrow x_1^2(\cos^2\alpha+\sin^2\alpha)+y_1^2(\sin^2\alpha+\cos^2\alpha)=a^2+b^2$$

$\Rightarrow x_1^2+y_1^2=a^2+b^2 \qquad \therefore$ Equation of locus of $P(x_1,y_1)$ is $x^2+y^2=a^2+b^2$.

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8 **Show that the locus of a point such that the ratio of distance of it from two given points is constant k (≠ ± 1) is a circle.**

Sol: Let $P(x_1, y_1)$ be a point on the locus. Let A(a, 0), B(–a, 0) be two given points

The distance from P to A is $PA=\sqrt{(x_1-a)^2+y_1^2}$

The distance from P to B is $PB=\sqrt{(x_1+a)^2+y_1^2}$

Given that $PA : PB = k : 1 \Rightarrow \frac{PA}{PB}=\frac{k}{1}\Rightarrow PA=k\,PB \Rightarrow PA^2=k^2.PB^2$

$$\Rightarrow (x_1-a)^2+y_1^2=k^2[(x_1+a)^2+y_1^2]\Rightarrow(1-k^2)(x_1^2+y_1^2+a^2)+(-1-k^2)(2ax_1)=0$$

Dividing the above equation by $(1-k^2)$ $(\because k\neq\pm1)$, we have

$$x_1^2+y_1^2-2\left(\frac{1+k^2}{1-k^2}\right)ax_1+a^2=0$$

$\therefore$ Locus of $P(x_1,y_1)$ is $x^2+y^2-2\left(\frac{1+k^2}{1-k^2}\right)ax+a^2=0$ which represents a circle.

9 **Find the equation of the circle with centre (–3, 4) and touching y – axis**

Sol: Perpendicular distance from C(–3, 4) to y – axis is p = | x - coordinate of centre| = |–3| = 3

Since the circle touches the y – axis, we have $r = p \Rightarrow r = 3$

$\therefore$ The equation of the circle with centre (–3, 4) and radius 3 is $(x + 3)^2 + (y - 4)^2 = 9$

$\Rightarrow x^2 + 6x + 9 + y^2 - 8y + 16 - 9 = 0 \Rightarrow x^2 + y^2 + 6x - 8y + 16 = 0$

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10 **If $x^2 + y^2 = c^2$ and $\frac{x}{a}+\frac{y}{b}=1$ intersect at A and B, then find $\overline{AB}$. Hence deduce the condition, that the line touches the circle.**

Sol: Given circle is $x^2 + y^2 = c^2$. Its centre C = (0, 0) and radius r = c

The given line is $\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{bx+ay}{ab}=1 \Rightarrow bx+ay=ab \Rightarrow bx+ay-ab=0$

Perpendicular distance from centre (0,0) to the line bx+ay–ab=0 is $p=\left|\frac{ab}{\sqrt{a^2+b^2}}\right|$

Length of the chord $= 2\sqrt{r^2-p^2} = 2\sqrt{c^2-\frac{a^2b^2}{a^2+b^2}}$

Hence if length of chord =0 then the chord becomes a tangent.

$$\Rightarrow c^2-\frac{a^2b^2}{a^2+b^2}=0 \Rightarrow c^2=\frac{a^2b^2}{a^2+b^2}$$

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11 **The line y = mx + c and the circle $x^2 + y^2 = a^2$ intersect at A and B. If AB = 2λ then show that $c^2 = (1 + m^2)(a^2 - \lambda^2)$**

Sol: Given circle is $x^2 + y^2 = a^2$. Its centre C= (0, 0) and radius r = a

The given line is $y = mx + c \Rightarrow mx - y + c = 0$

Perpendicular distance from the centre (0,0) to the line mx – y + c = 0 is $p=\frac{|c|}{\sqrt{m^2+1}}$

Given length of chord AB = 2λ

$$\Rightarrow \not{2}\sqrt{r^2-p^2}=\not{2}\lambda \Rightarrow r^2-p^2=\lambda^2 \Rightarrow r^2-\lambda^2=p^2 \Rightarrow a^2-\lambda^2=\frac{c^2}{m^2+1}$$

$\Rightarrow c^2 = (a^2 - \lambda^2)(1 + m^2)$

$\therefore$ The required condition is $c^2 = (1 + m^2)(a^2 - \lambda^2)$

12 **Find the equation of tangents of the circle $x^2 + y^2 - 10 = 0$ at the points whose abscissae are 1.**

Sol: Equation of the given circle is $x^2 + y^2 - 10 = 0$

Let the point on the circle with abscissae 1 be taken as $P(1,y_1)$

$\therefore\ 1^2 + y_1^2 - 10 = 0 \Rightarrow 1 + y_1^2 = 10 \Rightarrow y_1^2 = 9 \Rightarrow y_1 = \pm 3$

$\therefore$ The co-ordinates of P are (1, 3) and (1, –3)

Equation of the tangent at P(1, 3) on the circle $S = x^2 + y^2 - 10 = 0$ is $S_1=0$

$\Rightarrow xx_1 + yy_1 - 10 = 0 \Rightarrow x(1) + y(3) - 10 = 0 \Rightarrow x + 3y - 10 = 0$

Equation of the tangent at P(1, –3) on the circle $S=x^2 + y^2 - 10 = 0$ is $S_1=0$

$\Rightarrow xx_1 + yy_1 - 10 = 0 \Rightarrow x(1) + y(-3) - 10 = 0 \Rightarrow x - 3y - 10 = 0$

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13 **Find the equation of tangents of the circle $x^2 + y^2 - 8x - 2y + 12 = 0$ at the points whose ordinates are 1.**

Sol: Equation of the given circle is $x^2 + y^2 - 8x - 2y + 12 = 0$

Let the point on the circle with ordinate 1 be taken as $P(x_1,1)$

$x_1^2 + 1 - 8x_1 - 2 + 12 = 0 \Rightarrow x_1^2 - 8x_1 + 11 = 0$

$$\Rightarrow x_1 = \frac{8 \pm \sqrt{64-44}}{2} = \frac{8 \pm \sqrt{20}}{2} = \frac{8 \pm 2\sqrt{5}}{2} = 4 \pm \sqrt{5} \Rightarrow x_1 = 4+\sqrt{5},\ x_2 = 4-\sqrt{5}$$

$\therefore$ The co-ordinates of P are $(4+\sqrt{5}, 1)$ and Q $(4-\sqrt{5}, 1)$

Equation of the tangent at $P(4+\sqrt{5}, 1)$ is $x(4+\sqrt{5}) + y.1 - 4(x + 4+\sqrt{5}) - (y+1) + 12 = 0$

$\Rightarrow 4x + \sqrt{5}x + y - 4x - 16 - 4\sqrt{5} - y - 1 + 12 = 0 \Rightarrow \sqrt{5}x - 5 - 4\sqrt{5} = 0 \Rightarrow \sqrt{5}\left(x - \sqrt{5} - 4\right) = 0$

$\Rightarrow x - \sqrt{5} - 4 = 0 \Rightarrow x = 4+\sqrt{5}$

Equation of the tangent at Q $(4-\sqrt{5}, 1)$ is $x(4-\sqrt{5}) + y.1 - 4(x + 4-\sqrt{5}) - (y+1) + 12 = 0$

$\Rightarrow 4x - \sqrt{5}x + y - 4x - 16 + 4\sqrt{5} - y - 1 + 12 = 0 \Rightarrow -\sqrt{5}x - 5 + 4\sqrt{5} = 0 \Rightarrow -\sqrt{5}(x + \sqrt{5} - 4) = 0$

$\Rightarrow x + \sqrt{5} - 4 = 0 \Rightarrow x = 4 - \sqrt{5}$

14 **Find the equation of the circle passing through (–1, 0) and touching x + y – 7 = 0 at (3, 4).**

Sol: Let the equation of the required circle be $S = x^2+y^2+2gx+2fy+c=0$

(–1,0) lies on S=0 $\Rightarrow 1+0+2g(-1)+0+c=0 \Rightarrow -2g+c+1=0 \Rightarrow 2g - c = 1$ ....(1)

The equation of the tangent at (3, 4) on the circle S= 0 is $S_1=0$

$\Rightarrow xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0 \Rightarrow x(3)+y(4)+g(x+3)+f(y+4)+c=0$

$\Rightarrow 3x + 4y + gx + 3g + fy + 4f + c = 0 \Rightarrow (3 + g)x + (4 + f)y + (3g + 4f + c) = 0$

Comparing the above equation with the given line x + y – 7 = 0,

we have $\frac{3+g}{1} = \frac{4+f}{1} = \frac{3g+4f+c}{-7} \Rightarrow 3 + g = 4 + f \Rightarrow g - f = 1$ .... (2)

and $\frac{3+g}{1} = \frac{3g+4f+c}{-7} \Rightarrow -21 - 7g = 3g + 4f + c \Rightarrow 10g + 4f + c = -21$ ....(3)

(1) + (3) $\Rightarrow 12g + 4f = -20 \Rightarrow 3g + f = -5$ .... (4)

(2) + (4) $\Rightarrow 4g = -4 \Rightarrow g = -1$. From (2) we get $-1 - f = 1 \Rightarrow f = -2$

From (1), $-2 - c = 1 \Rightarrow c = -3$

Substituting g= –1, f= –2, c= –3 in S=0 we have $x^2 + y^2 - 2x - 4y - 3 = 0$

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15 **Find the equations of the circles passing through (1,–1), touching the lines 4x + 3y + 5 = 0 and 3x – 4y – 10 = 0**

Sol: The given two lines are perpendicular.

So the centre of the circle lies on the bisectors of angles between the given lines.

The bisectors of the angles between the given lines are $\frac{4x+3y+5}{5} \pm \frac{3x-4y-10}{5} = 0$

$\Rightarrow (4x+3y+5)+(3x-4y-10)=0$ or $(4x+3y+5)-(3x-4y-10)=0$

$\Rightarrow 7x-y-5=0$ or $x+7y+15=0 \Rightarrow y=7x-5$ or $x = -(7y+15)$

Case (i): If the centre lies on x = – (7y+15), then the centre can be taken as (–7k–15,k)

Applying the tangential condition p=r, we have

$\frac{|4(-7k-15)+3k+5|}{5} = \sqrt{(-7k-15-1)^2+(k+1)^2} \Rightarrow (-25k - 55)^2 = 25\,(-(7k + 16))^2 + (k + 1)^2$

$\Rightarrow [-5(5k + 11)]^2 = 25[(7k + 16)^2+(k + 1)^2]$

$\Rightarrow 25(25k^2 + 110k + 121) = 25(49k^2 + 224k + 256 + k^2 + 2k + 1)$

$\Rightarrow 25k^2 + 116k + 136 = 0$. $\because \Delta < 0$, k is imaginary. So case (i) is not possible

Case (ii): $\therefore$ The centre of the circle lies on y = 7x – 5 and hence it is of the form (k, 7k –5)

$$\therefore \left|\frac{4k+3(7k-5)+5}{5}\right| = \sqrt{(k-1)^2+(7k-5+1)^2} \Rightarrow \left|\frac{5(5k-2)}{5}\right| = \sqrt{(k-1)^2+(7k-4)^2}$$

$\Rightarrow (5k-2)^2 = (k-1)^2 + (7k-4)^2 \Rightarrow 25k^2 - 20k + 4 = k^2 - 2k + 1 + 49k^2 - 56k + 16$

$\Rightarrow 25k^2 - 38k + 13 = 0 \Rightarrow (25k-13)(k-1) = 0 \Rightarrow k = 1$ or $k = 13/25$

If k = 1, then centre = (1, 2), radius = $\sqrt{(1-1)^2+(2+1)^2} = 3$

Equation of the circle is $(x-1)^2 + (y-2)^2 = 3^2 \Rightarrow x^2 + y^2 - 2x - 4y - 4 = 0$

If $k = \frac{13}{25}$, then centre $= \left(\frac{13}{25}, \frac{-34}{25}\right)$

$$\text{radius} = \sqrt{\left(\frac{13}{25}-1\right)^2 + \left(-\frac{34}{25}+1\right)^2} = \sqrt{\frac{144}{625}+\frac{81}{625}} = \sqrt{\frac{225}{625}} = \frac{15}{25} = \frac{3}{5}$$

Equation of the circle is $\left(x - \frac{13}{25}\right)^2 + \left(y + \frac{34}{25}\right)^2 = \left(\frac{3}{5}\right)^2$

$$\Rightarrow x^2 - \frac{26}{25}x + \frac{169}{625} + y^2 + \frac{68}{25}y + \frac{1156}{625} = \frac{9}{25} \Rightarrow 25(x^2+y^2) - 26x + 68y + 44 = 0$$

$\therefore$ Required circles are $x^2 + y^2 - 2x - 4y - 4 = 0$, $25(x^2 + y^2) - 26x + 68y + 44 = 0$

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16 **If the parametric values of two points A and B lying on the circle $x^2+y^2-6x+4y-12=0$ are 30° and 60° respectively then find the equation of the chord joining A and B.**

Sol: Comparing the given equation with $x^2+y^2+2gx+2fy+c=0$, we get g = –3, f=2, c = –12

$$r = \sqrt{g^2+f^2-c} = \sqrt{(-3)^2+2^2-(-12)} = \sqrt{9+4+12} = \sqrt{25} = 5$$

$\therefore$ The equation of the chord joining the points $\theta_1 = 30°, \theta_2 = 60°$ is given by

$$(x+g)\cos\left(\frac{\theta_1+\theta_2}{2}\right) + (y+f)\sin\left(\frac{\theta_1+\theta_2}{2}\right) = r\cos\left(\frac{\theta_1-\theta_2}{2}\right)$$

$$\Rightarrow (x-3)\cos\left(\frac{60°+30°}{2}\right) + (y+2)\sin\left(\frac{60°+30°}{2}\right) = 5\cos\left(\frac{60°-30°}{2}\right)$$

$$\Rightarrow (x-3)\cos 45° + (y+2)\sin 45° = 5\cos 15° \Rightarrow \frac{(x-3)}{\sqrt{2}} + \frac{(y+2)}{\sqrt{2}} = \frac{5(\sqrt{3}+1)}{2\sqrt{2}}$$

$$\Rightarrow (x-3) + (y+2) = \frac{5(\sqrt{3}+1)}{2} \quad \Rightarrow 2x + 2y - (7 + 5\sqrt{3}) = 0$$

17 **Find the slope of the polar of (1,3) with respect to the circle $x^2+y^2-4x-4y-4=0$. Also find the distance from the centre to it.**

Sol: Equation of the circle is $S=x^2+y^2-4x-4y-4=0 \Rightarrow$ centre $C=(2,2)$

Polar of $P(1,3)$ w.r.to $S=0$ is $S_1=0$

$\Rightarrow 1(x)+3(y)-2(1+x)-2(3+y)-4=0 \Rightarrow x+3y-2-2x-6-2y-4=0 \Rightarrow -x+y-12=0$

$\Rightarrow x-y+12=0$...(1)

Slope of the above line is 1.

Perpendicular distance from C(2,2) to (1) is $P=\frac{|2-2+12|}{\sqrt{1+1}}=\frac{12}{\sqrt{2}}=\frac{6\times 2}{\sqrt{2}}=6\sqrt{2}$ units.

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18 **Find the coordinates of the point of intersection of tangents at the points where $x+4y-14=0$ meets the circle $x^2+y^2-2x+3y-5=0$.** EAM Q

Sol: The given line is nothing but the chord to the given circle.

In this case, the required point is nothing but the pole of the given line w.r.t the given circle.

Comparing $x+4y-14=0$ with $lx+my+n=0$, we get $l=1$, $m=4$, $n=-14$

Given circle is $S=x^2+y^2-2x+3y-5=0 \Rightarrow g=-1$, $f=3/2$ and $c=-5$

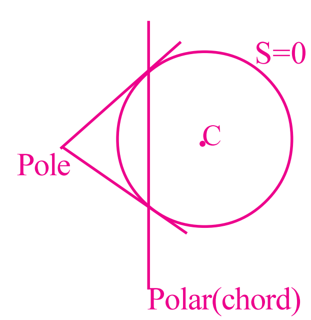
Radius $r=\sqrt{g^2+f^2-c}=\sqrt{(-1)^2+\left(\frac{3}{2}\right)^2+5}=\sqrt{1+\frac{9}{4}+5}=\sqrt{\frac{9+24}{4}}=\sqrt{\frac{33}{4}} \Rightarrow r^2=\frac{33}{4}$

Now, $N=lg+mf-n=1(-1)+4\left(\frac{3}{2}\right)-(-14)=-1+6+14=19$

$\therefore$ Pole $=\left(-g+\frac{lr^2}{N},-f+\frac{mr^2}{N}\right)=\left(1+\frac{1\left(\frac{33}{4}\right)}{19},\frac{-3}{2}+\frac{4\left(\frac{33}{4}\right)}{19}\right)$

$=\left[1+\frac{33}{4(19)},\frac{-3}{2}+\frac{4(33)}{4(19)}\right]=\left[\frac{4(19)+33}{4(19)},\frac{-6(19)+4(33)}{4(19)}\right]$

$=\left[\frac{76+33}{76},\frac{-114+132}{76}\right]=\left[\frac{109}{76},\frac{18}{76}\right]=\left[\frac{109}{76},\frac{9}{38}\right]$



$\therefore$ The required point $=\left(\frac{109}{76},\frac{9}{38}\right)$

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19 **If ax+by+c=0 is the polar of (1,1) with respect to the circle $x^2+y^2-2x+2y+1=0$ and H.C.F of a,b,c is equal to one then find $a^2+b^2+c^2$.**

Sol: Equation of the circle is $S=x^2+y^2-2x+2y+1=0$

Polar of $P(1,1)$ w.r.to $S=0$ is $S_1=0 \Rightarrow 1.x+1.y-1(1+x)+1(1+y)+1=0$

$\Rightarrow x+y-1-x+1+y+1=0 \Rightarrow 2y+1=0$............(1)

But the given equation of the polar is $ax+by+c=0$...........(2)

Since H.C.F of a,b,c is 1, comparing (1) and (2) we get $a=0$, $b=2$, $c=1$.

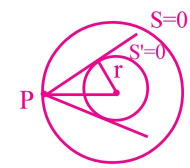
$\therefore a^2+b^2+c^2=0+4+1=5$

20 **From a point on the circle $x^2+y^2+2gx+2fy+c=0$ two tangents are drawn to the circle $x^2+y^2+2gx+2fy+c\sin^2\alpha+(g^2+f^2)\cos^2\alpha=0$, prove that the angle between them is 2α.**

EAM Q

Sol: Let $P(x_1,y_1)$ be a point on the circle $S\equiv x^2+y^2+2gx+2fy+c=0 \Rightarrow x_1^2+y_1^2+2gx_1+2fy_1+c=0$

$\Rightarrow x_1^2+y_1^2+2gx_1+2fy_1=-c$.......(1)



The radius of the circle $S'\equiv x^2+y^2+2gx+2fy+c\sin^2\alpha+(g^2+f^2)\cos^2\alpha=0$ is

$$r=\sqrt{g^2+f^2-c\sin^2\alpha-(g^2+f^2)\cos^2\alpha}$$

$$=\sqrt{(g^2+f^2)(1-\cos^2\alpha)-c\sin^2\alpha}=\sqrt{(g^2+f^2-c)\sin^2\alpha}=\sqrt{(g^2+f^2-c)}\,.\sin\alpha$$

Length of the tangent from $P(x_1,y_1)$ to $S'=0$ is

$$\sqrt{S_{11}}\equiv\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c\sin^2\alpha+(g^2+f^2)\cos^2\alpha},\ \text{[Using (1)]}$$

$$=\sqrt{-c+c\sin^2\alpha+(g^2+f^2)\cos^2\alpha}=\sqrt{(g^2+f^2)\cos^2\alpha-c(1-\sin^2\alpha)}=\sqrt{(g^2+f^2-c)}\,.\cos\alpha$$

$$\therefore\ \tan\theta=\frac{r}{\sqrt{S_{11}}}=\frac{\sqrt{(g^2+f^2-c)}.\sin\alpha}{\sqrt{(g^2+f^2-c)}.\cos\alpha}=\frac{\sin\alpha}{\cos\alpha}=\tan\alpha\Rightarrow\theta=\alpha$$

Hence, the angle between the two tangents is $2\theta=2\alpha$.